Exercise 4 (Due by May 21)

1. Let $L := \{w_1w_2w_3w_4w_5| w_i \in \{a, b, c, d, e\}^* \text{ for } 1 \leq i \leq 5\}$, and there is a homomorphism $h$ from $w_i$ to $w_j$ for all $1 \leq i, j \leq 5$.

Show that $L$ is no TAL. (One can use the fact that $a^kb^kd^ke^k$ is no TAL).

Solution:

Let $L' := \{a^kB^kC^kD^kE^k\}$. Intersect $L$ with $L'$.

Together with the homomorphism from $w_i$ to $w_j$ for all $1 \leq i, j \leq 5$ which enforces that all $w_i$ have the same length, we get the language $L'' = \{a^kB^kC^kD^kE^k\}$.

Since TAL is closed under intersection with regular languages, $L''$ should be a TAL, but we know that it isn’t. Contradiction, therefore $L$ is no TAL either.

2. Give the elementary trees of an FTAG $G$ that generates (1), along the lines of the examples presented in the lecture:

(1) Where did John meet Sandy?

Also provide the derived tree and the derivation tree!

Solution:
3. Bonus exercise: Give a LTAG that generates the language $L = \{a^n b^n c^n d^n \mid n \geq 0\}$. Hint: There is more than one solution.

Solution:

There are mainly three different solutions depending on which dependencies one wants to generate.

(a) Cross-serial dependencies:

(b) Nested:
(c) doubly nested:

```
  S
   |
  c

  S_{NA}
   /\   /
  a  S  d
   |   /
  b  S_{NA}  c
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