Tense in Intensional Contexts: Two Semantic Accounts of Abusch’s Theory of Tense

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1. Introduction*

I became acquainted with Dorit Abusch's (1993) paper "Sequence of Tense Revisited: Two Semantic Accounts of Tense in Intensional Contexts" through a talk she gave at the summer school in Copenhagen. The essential insight I gained from the talk is that there cannot be any deictic tense in an intensional context. In intensional contexts, tenses behave like bound temporal variables. I believe that this insight is central for a correct understanding of the semantics of tenses. I am not aware that this point has been made in the literature before Abusch. If this is so, then the paper is a milestone in the literature about tense.

For me, Abusch's message was obscured by at least three factors. First, she doesn't speak about bound tense at all. This is my terminology, but it is faithful to what Abusch actually says, I believe. The second factor is that Abusch doesn't indicate a precise semantics for tenses. In her theory, tenses sometimes seem to denote times, sometimes relations between times. The reader is left alone to make sense of this. The third point has to do with English: Abusch wants to solve the so-called double access phenomenon, which refers to the fact that a present tense in an intensional context cannot express mere "contemporaneity" if it is embedded under a past. She believes that this fact is constitutive for the semantics of the present tense. Crosslinguistic evidence, e.g., from German, shows that the phenomenon is an idiosyncrasy of English and should be kept separate from the semantics of the present tense.

On the following pages, I want to make precise the semantic and syntactic assumptions underlying Abusch's account of tense in English. After the completion of the first draft, I became acquainted with Irene Heim's (1994) reconstruction of Abusch, which was conceptually rather different. I changed my original paper under the influence of hers. As a result, my proposal has become very similar to hers, but differences still exist, and they will become important when we consider the combination of tenses with framesetters and temporal adverbs of quantification. I haven't been able to handle all the relevant data in Heim's theory. My own treatment will show that the matter is quite complicated. In order to give a correct treatment of the semantics of tense, we have to say something about the role of the subjunctive and of aspevtual relations in the sense of (Klein 1994), as we will see.

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The theory may be summarized like this. In extensional contexts, tenses denote free time variables carrying presuppositions which restrict the assignment function: PRES must denote a time overlapping the speech time, PAST must denote a time before it. I will give a precise account of how this can be expressed. In intensional contexts, PRES and PAST will always denote bound variables. The presuppositions are cancelled. It will be a somewhat tedious business to derive this result from the meaning of tenses. In addition, there are relational tenses which are obtained from the denotational ones by moving the relational information from the presupposition to the content dimension. I call this process isomerism. The original contribution is the elaboration of a compositional semantics and syntax which regulates the interplay between framesetters, tenses, and temporal adverbs of quantification. I will argue that it is not possible to quantify over tenses. We quantify over event times introduced by aspects in the sense of (Klein 1994).

2. The essentials of Abusch's theory of tense

In this section I want to represent the essentials of Abusch's analysis of tense in English. Consider non-embedded tense first. (A seal of shape [A,n] refers to example #n of Abusch's paper.)

(2-1) a. John left  

(2-1b) is Abusch's LF for (2-1a). She uses the labels Pst for PAST and Prs for PRES. I have changed this in order to be closer to labels established in the literature. u is the time of utterance (or speech time), and t₂ is the time denoted by PAST. The LF is intended to capture the information "John leaves at t₂, where t₂ is before u". The tree suggests that PAST₂, the semantic tense, is an argument of the verb. The past morphology ("left") can be taken as a morphosyntactic reflex of the fact that a PAST argument is required. The question is, of course, what the meaning of PAST₂ is exactly. This issue is not addressed
in Abusch's paper, and in what follows we will consider two proposals to answer it.

An analogous analysis is assumed for the present tense; the difference is that \( \text{PRES}_i \) denotes a time which is not before \( u \), \( t_2 \nless u \). Therefore, we have the following meaning rules:

**Tenses in extensional contexts**

In an extensional context, \( \text{PAST}_i \) denotes a time \( t_i \) before \( u \), and \( \text{PRES}_i \) denotes a time \( t_i \) not before \( u \).

Let us call the time denoted by a (semantic) tense **reference time**. Furthermore, we say that a tense which relates its reference time to the speech time is a **deictic tense**. So far, Abusch's analysis of deictic tenses is standard. The novelty is the treatment of tenses in intensional contexts. Consider the example on page 31 of her paper:

(2-2) a. Mary believed that it was raining

Since this expresses so-called simultaneity, one would expect a logical form of the following kind:

(2-2) b. Mary \( \text{PAST}_2 \) believed that it \( \text{PAST}_2 \) was raining

Here, the reference time of the matrix past tense and that of the embedded past tense are the same. This, however, would give us the wrong reading for almost every case in which the sentence is used. Think of the following situation. It is five o'clock. The sky is dark and there is a gloom in the air. Mary thinks: "It's seven o'clock, and it is raining." If the LF (2-2b) were the correct one, the content of Mary's belief should be "It is raining at five o'clock" because Mary had her thought at five o'clock. Therefore, \( \text{PAST}_2 \) denotes five o'clock, and the *that*-clause must be evaluated with respect to that reference time. Obviously, this is the wrong prediction. The content of Mary's belief is rather something like "It is raining at seven o'clock". For an argument of this kind, *vide* von Stechow (1995).

At first sight, the argument sounds bewildering because it is at variance with most of the literature about tense, except Baker (1989/95). But a bit of reflection shows that the argument is sweeping: a subject virtually never knows at which time she is located. The argument should not be that surprising after all because it is similar to the considerations which motivated Lewis's (1989) de se
analysis of attitudes. The analysis given by Abusch to (2-2a) is therefore not (2-2b) but rather something like (2-2c):

\[(2-2) \quad \text{c. Mary \textsc{past}2 believed} \lambda_2[\text{that it \textsc{past}2 was raining}]\]

The important point is that the reference time of the embedded \textsc{past}2 is bound by the lambda operator. That's why I speak of \textbf{bound tense} in such a case. Binding by abstraction breaks any temporal relation between the reference time of the embedded tense and that of the matrix tense. If there is any sense in the notion of contemporaneity, it certainly cannot be understood as a temporal relation between the matrix tense and the embedded tense. Thus, coindexing the two makes no sense at all. Actually, (Abusch 1993) chooses a different index for the matrix tense. I didn't follow her in the discussion of the example because it could suggest that the sameness or difference of the index is at issue. This is not relevant, however. What matters is the intervening lambda abstraction.

What is the meaning of the embedded \textsc{past}2 then? Abusch holds the view that it expresses the information \(t_2 \not< t_2\) in this context. This means that we can paraphrase the intended reading of (2-2c) as (2-2d):

\[(2-2) \quad \text{d. Mary believes at } t_2, t_2 < u, \text{ the proposition} \lambda_2[\text{that it is raining at } t_2, \text{ where } t_2 \not< t_2]\]

Given that the information \(t_2 \not< t_2\) is a tautology, the paraphrase boils down to:

\[(2-2) \quad \text{e. Mary believes at } t_2, t_2 < u, \text{ the proposition} \lambda_2[\text{that it is raining at } t_2]\]

A present tense in an embedded context is represented in exactly the same way; i.e., sentence (2-3a) has roughly the LF (2-3b) and expresses the proposition (2-3c):

\[(2-3) \quad \text{a. Mary believes that it is raining} \]

b. Mary \textsc{pres}2 believes \(\lambda_2[\text{that it \textsc{pres}2 is raining}]\)

c. Mary believes at \(t_2, t_2 \not< u\), the proposition \(\lambda_2[\text{that it is raining at } t_2, \text{ where } t_2 \not< t_2]\)

Since the embedded clauses in (2-2b) and (2-3b) mean exactly the same thing, Abusch obviously has to make the assumption that \textsc{pres}i and \textsc{past}i can mean
the same thing in an intensional context. Summarizing the results of the
discussion, we obtain the following principles for tenses in intensional contexts:

**Tenses in intensional contexts**

1. No deictic or free tense in intensional contexts!
2. In an intensional context, both PRES\textsubscript{i} and PAST\textsubscript{i} express the
tautology that t\textsubscript{i} \not< t\textsubscript{i}.

A free tense is one with a free variable. It cannot occur in an intensional
context for the same reasons which exclude a deictic tense from these contexts.\textsuperscript{1} Therefore, every PRES\textsubscript{i} or PAST\textsubscript{i} is bound in an intensional context and
expresses a bound variable.

At this point, the question arises how PRES\textsubscript{i} and PAST\textsubscript{i} can both express
the same information in an intensional context. One could simply stipulate that.
But Abusch prefers to derive that from the meaning of tenses. She says roughly
this:

**Tense syntax and semantics**

1. *[Constraint]* PRES\textsubscript{i} and PAST\textsubscript{i} in extensional contexts are associated
   with the relations \not< and <, respectively. The associated relation
   applies to (t\textsubscript{i},u) in this case.
2. *[Relations]* Every tense in an intensional context inherits every
   relation associated to or inherited by a c-commanding tense.
3. *[PAST constraint]* In an intensional context, PAST\textsubscript{i} is associated
   with the relation \not< if at least one of the relations inherited is <. The
   associated relation applies to (t\textsubscript{i}, t\textsubscript{i}) in this case.
4. *[PRES constraint]* In an intensional context, none of the relations
   inherited by PRES\textsubscript{i} can be <, and PRES\textsubscript{i} is associated with \not<, and
   the associated relation applies to (t\textsubscript{i}, t\textsubscript{i}).

The four conditions explain the nature of the entries under a tense node as
illustrated in the tree (2-2f). The condition called constraint might more
specifically be named constraint on reference time. To illustrate the principles,

\textsuperscript{1}The point is certainly related to the claim that individuals denoted by proper names cannot be part
of the content of an attitude because we don't know the essence of the object denoted and therefore
don't know what a name denotes. This is one of the motivations for Haas-Spohn's (1995) theory of
subjective meaning.
consider the full representation of the LF (2-2c), which is taken from page 31 of (Abusch, 1993):

(2-2)

The relation associated with the matrix PAST₁ is <. It applies to (t₁, u) and is inherited by the lower PAST₂, which is in an intensional context. Therefore, condition 3 is fulfilled, and the relation associated with the lower PAST₂ is ¬<, which applies to (tᵢ, tᵢ). The store which collects the relations of a node is called Relations. Since every tense in an intensional context is bound, the LF is well-formed. The notation R^believed is a variable for either the relation < or ¬<. The hyperscript merely serves to distinguish between different R-variables. Similar remarks apply to the LF (2-3b). In the next section, we will say something about the lambda abstraction over worlds contained in the tree.

The attentive reader might have noticed a problem arising from Abusch’s treatment of the present tense which requires a further principle in order to make the theory descriptively adequate. Recall that the LF of the present sentence (2-3a) expresses the proposition:
(2-3)  c. Mary believes at t₂, t₂ ¬< u, the proposition λ₂[that it is raining at t₂, where t₂ ¬< t₂]

Constraining the relation between the reference time t₂ and the speech time u by the relation ¬< does not exclude that t₂ is after u. This is an undesirable result because sentence (2-3a) doesn't describe this kind of situation. To prevent this consequence, Abusch introduces a constraint which restricts the reference time of tenses:

[ULC] **Upper Limit Constraint:**  
Abusch, p.24: "the now of an epistemic alternative is an upper limit for the reference of tenses.[...] the local evaluation time is an upper limit for the reference of tenses."

For the time being, we ignore the first part of the constraint because the "subjective now" refers to the time at which the subject of an attitude localizes herself in the belief worlds. At the level of LF, this is always a bound time variable and the notion of reference is not relevant in most cases. What matters here is the second half of the constraint. By stipulation, the local evaluation time for tenses in extensional contexts is the speech time u. It follows that t₂ in (2-3c) cannot be later than u. Since t₂ cannot be earlier than u, it has to overlap u, the standard assumption for the meaning of PRES.

The reader may wonder how the ULC is compatible with the existence of the future tense. It is not consistent, and Abusch bites the bullet and holds the view that there is no future tense in English. A sentence like (2-4a) is roughly analyzed as (2-4b):

(2-4)  a. John will buy a fish  
       b. John PRES₁ will buy a fish

In other words, the so-called future is a combination of PRES+will, where PRES is the temporal argument of will. Abusch even claims that will is a modal verb because it licenses a bound tense in its complement, which is a diagnostic of the intensionality of the complement:
(2-5) a. John will buy a fish that is alive
   b. John \( \text{PRES}_1 \) will \( \lambda_2 \text{[buy a fish that \text{PRES}_2 \text{ is alive}]} \)
   c. \( \exists t_2 [t_2 > t_1, t_1 \nless u, \text{John buys at } t_2 \text{ a fish that is alive at } t_2] \)

   (2-5b) is supposed to capture the reading paraphrased in (2-5c), where the fish is alive at the time of buying. At the speech time \( u \), the fish need not even be born. Similarly, Ogihara's (1989) celebrated sentence (2-6a) is analyzed as (2-6b) in Abusch's theory:

(2-6) a. John said he would buy a fish that was alive
   b. John \( \text{PAST}_1 \) said \( \lambda_2 \text{[he \text{PAST}_2 \text{ would } \lambda_3 \text{[buy a fish that \text{PAST}_3 \text{ is alive}]}]} \)

   \textit{Would} is the past form of \textit{will}, of course. Both \( \text{PAST}_2 \) and \( \text{PAST}_3 \) can be interpreted as bound variables, since both inherit the precedence relation \(<\) associated with \( \text{PAST}_1 \) and can therefore be associated with the \( \nless\)-relation.

   It is a consequence of the system that a bound variable \( \text{PRES} \) cannot be embedded under \( \text{PAST} \). Consider the following two examples.

(2-7) a. Mary believed that it is raining
   b. \*Mary \( \text{PAST}_2 \) believed \( \lambda_2 \text{[that it } \text{PRES}_2 \text{ was raining]} \)

   The LF (2-7b) is not well-formed, because \( \text{PRES}_2 \) inherits the \(<\)-relation associated with \( \text{PAST}_1 \), a violation of the \( \text{PRES} \)-constraint. (2-7a) illustrates the double access phenomenon, which requires an LF that is different from (2-7b). The issue will be discussed in section 3.7.

   For similar reasons, (2-8b) is not a good LF for (2-8a) because the \( \text{PAST} \)-constraint is violated.

(2-8) a. Mary believes that it was raining
   b. \*Mary \( \text{PRES}_2 \) believes \( \lambda_2 \text{[that it } \text{PAST}_2 \text{ was raining]} \)

   The last remark concerns the distribution of bound tense. Abusch's principles restrict bound tenses to intensional contexts. An intensional context is created by an intensional predicate. If this predicate is a finite verb, it will have a tense. As a consequence, a tense in an intensional context is generally subordinate to some
other tense. The rules do not apply to subordinate tenses in general, because the inheritance mechanism is restricted to subordinate tenses in intensional contexts. In contrast to intensional contexts, contemporaneity can be expressed by coindexation in extensional contexts. The following example illustrates such a case:

(2-9)  a. Bill married a woman that was rich
       b. Bill PAST₁ married a woman that PAST₁ was rich

Here, the woman is rich at the time of the wedding but not necessarily so at the speech time u. The LF involves no bound PAST in the sense in which the term has been introduced, though PAST₁ is bound in the sense of GB-theory, a notion of binding which should not be confused with the "logical" notion used here.

This example completes the illustration of Abusch's theory of tense. I think that the architecture of the proposal has emerged, but many details still have to be elaborated. The latter is the purpose of the next sections.

3. My interpretation of Abusch

3.1. Semantics and syntax

I will represent my interpretation by means of a logical language. I choose a Ty2-language in the sense of (Gallin 1975). The difference between Montague's Intensional Logic and this language is that the language makes the parameters world and time explicit.

My interpretation will represent the meaning of Abusch's LF (2-1b), here repeated as (3-1a), by the formula (3-1b):

---

2Exceptions are constructions like:

I know that Mary was a strange child. But her desire to marry a man who resembled her is really bizarre.

For a discussion, vide (Abusch 1993, p.26 ff.).
(3-1) a. John PAST 2 left
b. \( \text{leave}(t_2; R^{leave}(t_2, t_0) \land R^{leave} = <)(John)(w_0) \)

PAST 2 is interpreted as the time variable \( t_2 \), but the assignment has to fulfill certain presuppositions, namely Abusch's reference constraint and the tense constraint, which appear after the semi-colon. We will give a precise account of the notation in a moment.

Following (Heim 1994), I adopt the convention that a free time variable \( t_0 \) refers to the speech time \( u \); i.e., for the contextually specified assignment \( g_c \), \( g_c(t_0) = u \). Henceforth, I will refer to \( t_0 \) as the distinguished time variable of the system. Similarly, \( w_0 \) is the distinguished world variable, which refers to the actual world. \( s \) and \( e \) denote types of worlds and individuals, respectively. The formula therefore presupposes that the predicate \( \text{leave} \) has the type \( \langle t, \langle e, \langle s, t \rangle \rangle \rangle \), and its meaning is:

**Meaning of** \( \text{leave} \):
\[
\|\text{leave}\|(t)(x)(w) = 1 \text{ iff } x \text{ leaves in } w \text{ at } t, \text{ for any world } w \text{ and time } t.
\]

Next, let me comment on my treatment of presuppositions. A presupposition of an expression is a formula of type \( t \), which is separated from the expression by a semi-colon. The semi-colon may be viewed as a version of Fabricius-Hansen's (1983) restriction operator. The operator is a filter on variable assignments which satisfy an expression. For instance, it can express the requirement that PAST 2 requires an assignment \( g \) such that \( g(\text{PAST 2}) \) is before the speech time \( u \). An unrestricted expression is satisfied by every variable assignment or by none. A restricted expression is satisfied by those assignments which satisfy the presupposition or by no assignment which satisfies the presupposition. The meaning of an unrestricted expression is a total function from assignments into truth-values; the meaning of a restricted expression is a partial function from assignments into truth-values.

**The Presupposition Operator:**
If \( \alpha \) is an expression of any type \( \tau \) and \( \pi \) is of type \( t \), then \( \alpha; \pi \) is of type \( \tau \). \( \|\alpha; \pi\|^{\tau} \) is defined only if \( \|\pi\|^{t} = 1 \). If this is so, \( \|\alpha; \pi\|^{\tau} = \|\alpha\|^{\tau} \).

It is an important property of the presupposition operator that variables occurring both in the content expression and in the presupposition can be bound by
the same lambda operator. In fact, this was one of Fabricius-Hansen's motivations to prefer this kind of analysis to earlier two-dimensional representations. We will make use of this property shortly.

We are now ready to evaluate (3-1c). Let \( g_c \) be the assignment determined by the context of utterance \( c \). Then

\[
\models \left( \text{leave}(t_2; R_{\text{leave}}(t_2, t_0) \wedge R_{\text{leave}} = <))(\text{John})(w_0) \right)^{g_c}
\]

is defined only if

\[
\models \begin{cases} 
R_{\text{leave}}(t_2, t_0) \wedge R_{\text{leave}} = < \iff g(R_{\text{leave}})(g(t_2), g(t_0)) \\
R_{\text{leave}} = < \iff g(t_2) < g(t_0) 
\end{cases}
\]

Given this, \( \models \text{leave}(t_2)(\text{John})(w_0)^{g_c} = 1 \) iff John leaves in the world \( g(w_0) \) at the time \( g(t_2) \). These truth conditions are adequate.

Next, let us elaborate on Abusch's LFs. Our precise notation for Abusch's tree (2-1b) will be this:

(3-1) c.

\[
\begin{array}{c}
\text{AgrP}(w_0) \\
\text{John} \\
\text{Agr'} \\
\text{Agr} \\
\text{TP} \\
\text{TIME-P} \\
\{R_{\text{leave}}\} \\
R_{\text{leave}}(t_2, t_0) \\
R_{\text{leave}} = < \\
\end{array}
\]

\[
\text{PAST}_2 \\
\text{T} \quad \text{Past} \quad \text{VP} \\
\text{leave}
\]

The notation distinguishes between morphological tense, i.e., the projection of the tense morphology Past and semantic tense, called TIME-P. The same distinction is made in (Stowell 1993) and (Zeller 1994). I am assuming that TIME-P is the specifier of TP.

The entries under TIME-P contain the presuppositions for the variables, which have been illustrated already. Recall that \( R_{\text{leave}} \) is a variable which ranges over the two temporal relations \(<\) and \( \neg<\) and which is restricted by the (reference) constraint \( R_{\text{leave}}(t_2, t_0) \) and the tense constraint \( R_{\text{leave}} = < \). The store \{ \}, called Relations, contains the relation variables of the node and the variables inherited from higher tense nodes if the tense is in an intensional context. Together, these conditions restrict the reference of the referential index of
PAST$_2$. In this particular case, Relations contains only the relational variable of the tense node itself.

The PAST- and the PRES-constraints introduced in the last section may be reformulated with reference to the store and the relation variables as follows:

**Tense constraints**

**PAST-constraint:** At least one variable in the store equals the precedence relation $<$.  
**PRES-constraint:** No variable in the store is identical to the precedence relation.

We have introduced the convention that the relation variable ranges over $<$ and $\neg<$ only. Therefore, every relation variable under a PRES-node equals $\neg<$. The meaning of the two tenses is now simply this:

**Meaning of tenses**

Let $\alpha$ be PRES or PAST. $\llbracket \alpha_i^k \rrbracket^s = g(i)$, iff $g$ satisfies the constraints of the node $k$.

If the constraints are not fulfilled, $\llbracket \alpha_i^k \rrbracket^s$ is not defined.

Thus, as far as the content is concerned, the past and the present tense mean the same. The difference comes with the difference of the constraints. It is clear that the tree (3-2) can be directly translated into the formula (3-2c), which gives a correct account of the truth conditions. In order to see how present in an extensional context works, consider an LF for *Mary is pregnant*:

(3-2)  
\begin{enumerate}
  \item a. Mary PRES$_1$ is pregnant  
  \item b. $\textit{be - pregnant}(t_1; R^{be}(t_1,t_0) \land R^{be} = \neg<)(\textit{Mary})(w_0)$
\end{enumerate}

The constraints on PRES$_1$ are found as a presupposition of the formula translating the LF. The ULC adds the further constraint that $t_1$ is not later than $t_0$. Therefore, the complete translation of the LF is (3-2c):

(3-2)  
\begin{enumerate}
  \item c. $\textit{be - pregnant}(t_1; R^{be}(t_1,t_0) \land R^{be} = \neg< \land \neg t_1 > t_0)(\textit{Mary})(w_0)$
\end{enumerate}

If we evaluate this, we find out that the formula is true if Mary is pregnant in the actual world at time $g(t_1)$, where $g(t_1)$ must overlap the speech time.
Before we go on, let us formulate our assumptions about the relation variables explicitly.

**R-association:**
Each tense node k is associated with one **relation variable** $R^k$ ranging over the relations precedence (\(<\)) and non-precedence (\(\neg<\)). Call $R^k$ **the relation variable of the node k**.

**Reference constraint:**
If the semantic tense of the node $k$ carries the index $i$, the first argument of $R^k$ is the time variable $t_i$. The second argument of $R^k$ is always $t_0$. This information is called **reference constraint**.

Our next concern will be to give a precise account of what Abusch says about tense in intensional contexts. The main ingredient of the theory is the inheritance mechanism for relation variables described on page 27 of her paper:

All operators with intensional arguments...introduce a relation variable relating their local temporal parameter with their local evaluation time. Such relations...are transmitted by a feature-passing mechanism to the intensional arguments.

(3-3a) is the LF for the sentence (2-2a), where irrelevant details have been omitted and AgrP is written as S.
Following (Heim 1994), we establish the convention that the complementizer of a that-clause, which is not a relative clause, contains $\lambda t_0 \lambda w_0$; i.e., it binds the "intensional parameters" time and world. Therefore, an intensional context can be characterized as the scope of these two binders. In the last section, the principle was introduced that free tense cannot occur in an intensional context. It follows that every tense in an intensional context must carry the index 0 and is therefore bound. We are now in a position to make Abusch's inheritance mechanism for relation variables precise:

**Rules of R-storage:**

a. The relation of $k$ is in the store (**R-association**).

b. The store of a bound tense node contains the relations of the tense node of the next higher embedding predicate in addition (**R-transmission**).

The translation of the tree into our logical language is this:
(3-3) b. \( \text{believe}(\lambda t_0 \lambda w_0 [\text{rain}(t_0; \pi_1)(w_0)](t_2; \pi_2)(\text{Mary})(w_0)), \)

where \( \pi_1 = R^{\text{rain}}(t_0, t_0) \wedge [R^{\text{bel}} =< \vee R^{\text{rain}} =<] \)

and \( \pi_2 = R^{\text{bel}}(t_2, t_0) \wedge R^{\text{bel}} =< \)

In order to evaluate the formula, we need a semantics for de dicto belief. To be sure, Abusch assumes a de se analysis in the style of (Lewis 1979). This, however, is not necessary for the discussion of the examples. The only thing that matters is that the belief-operator is a quantifier that quantifies over both worlds and times.

**Semantics for de dicto belief**

[H, 33]

\( \text{believe} \) is a symbol of type \( \langle \langle i, st \rangle, \{i, \{e, st\}\} \rangle \).

\( \| \text{believe} \|^r_c (P)(t)(x)(w) = 1 \) iff for every world \( w' \) and time \( t' \) not ruled out by what \( g(x) \) believes in \( g(w) \) at \( g(t) \), \( P(t)(w') = 1 \).

Let us evaluate the formula (3-1b) now.

\( \| \text{believe}(\lambda t_0 \lambda w_0 [\text{rain}(t_0; \pi_1)(w_0)](t_2; \pi_2)(\text{Mary})(w_0))^r_c \) is defined only if \( g_c(R^{\text{bel}})(g(t_2), g(t_0)) = 1 \) and \( g_c(R^{\text{bel}}) = <. \) These presuppositions imply that \( g_c(t_2) < g_c(t_0) \). If this is satisfied, then for every world \( w \) and for every time \( t \) such that \( w \) at \( t \) is not ruled out by what Mary believes in \( g(w) \) at \( g(t) \),

\( \| \lambda t_0 \lambda w_0 [\text{rain}(t_0; \pi_1)(w_0)](t_2; \pi_2)(\text{Mary})(w_0) \|^r_c \)

is true in \( w \) at \( t \). We first check, whether the presupposition is fulfilled for any such \( w \) and \( t \). This requires that

\( \| \lambda t_0 [R^{\text{rain}}(t_0, t_0) \wedge [R^{\text{bel}} =< \vee R^{\text{rain}} =<]] \|^r_c(t) = 1 \), for any of the ts mentioned. This is the case if \( g_c(R^{\text{rain}})(t, t) = 1 \) and \( g_c(R^{\text{rain}}) = < \) or \( g_c(R^{\text{bel}}) = <. \) The first conjunct requires that \( g_c(R^{\text{rain}}) \neq <. \) This is possible because the matrix presupposition tells us that \( g_c(R^{\text{rain}}) = <, \) which guarantees the truth of the second conjunct. From the global presupposition for relation variables, which states that their range is the set \( \{<, \neg<\} \), we deduce that \( g_c(R^{\text{rain}}) = \neg<. \) Therefore, the presupposition is met for any \( t \) whatsoever. This means that the subordinate clause doesn't have any presupposition!

Thus, the complete truth conditions of the formula are these: For every world \( w \) and every time \( t \) not ruled out by what Mary believes in the actual world at time \( g(t_2) \): it rains in \( w \) at time \( t \). The presupposition requires that \( g(t_2) \) is
before the speech time. Surely, this is a satisfying account of the truth conditions of the sentence.

3.2. Some practice

Before we discuss some further examples in order to get a feeling for the theory, let us introduce a notational convention for our (abbreviated) LFs: we will omit the world parameter. For embedded clauses this is possible anyway, because we can simply leave the world argument unsaturated. For instance, 

\[ \lambda w_0 \lambda t_0 [rain(t_0)(w_0)] \]

is equivalent to

\[ \lambda t_0 [rain(t_0)] \].

If a bound tense behaves like a bound variable, we can do the same with the bound tense, because the latter formula means the same as the formula \( rain \), which expresses a proposition. But tenses are time variables that have to fulfill presuppositions on assignments. And we have to check these. It is very tedious to calculate the presupposition exactly in the way exercised in the last section. There is a simpler, informal way to convince ourselves whether a bound tense is well-formed or not. For that purpose, we ignore the world parameter for extensional contexts as well. Unembedded sentences would express propositions as well in that case (cf. (Cresswell 1973)).

A sketch of a proof that the LF (3-3a), which we have discussed, expresses "simultaneity" runs like this:
This reasoning shows that the "simultaneous" interpretation is possible. The next example allows a bound variable reading for the embedded PAST as well. It is licensed by the PAST of the higher intensional verb:
Two remarks are in order. The first concerns the location of the tense argument in an infinitival. Following (Heim 1994), we associated it with \textit{to}. We stipulate that \textit{to} always denotes \(t_0\). Thus, \textit{to} is not the head of TP proper but only the marker of a tense argument. The PRO subject of the infinitival, a bound variable, is ignored in the representation. The second remark concerns QR. We will always distinguish between the referential index of the phrase moved and the "movement index". The latter is bound and has nothing to do with the former. This convention is due to Heim as well.

The next argument shows that we cannot have a bound variable reading for an embedded PAST if the main verb is in the present tense:
(3-6) Mary expects to marry a man who loved her
*Mary PRES₁ expects λ₀[ a man who PAST₀ loved her λ₂[ to₀ marry

\[
\begin{align*}
&\text{R}^{\text{exp}} \\
&\{\text{R}^{\text{exp}}\} \\
&\text{R}^{\text{exp}} \neq < \\
\hline
&\text{R}^{\text{love}}(t₁,t₀) \\
&\text{R}^{\text{love}}(t₀,t₀) \\
\end{align*}
\]

\[\therefore \text{R}^{\text{love}} \neq < \land \text{R}^{\text{exp}} = < \quad (\text{Contradiction to: } \text{R}^{\text{exp}} \neq <)\]

Since the embedded PAST is bound, it has to satisfy the reference constraint \(\text{R}^{\text{love}}(t₀,t₀)\). Therefore, \(\text{R}^{\text{love}}\) must be \(\neg<\), and the higher tense must be associated with the precedence relation. The higher tense, however, is PRES, and its tense constraint requires that it cannot be associated with the precedence relation. Thus, we have derived a contradiction.

On the other hand, we can have an absolute reading for the embedded tense of the last sentence if we scope the object over the matrix tense:

(3-7) Mary expects to marry a man who loved her

\[
\begin{align*}
&\text{R}^{\text{love}} \\
&\{\text{R}^{\text{love}}\} \\
&\text{R}^{\text{love}} = < \\
\hline
&\text{R}^{\text{exp}}(t₂,t₀) \\
&\text{R}^{\text{exp}}(t₁,t₀) \\
\end{align*}
\]

\[\therefore \text{R}^{\text{exp}} = \neg<\]

The sentence can have a "back-shifted" reading as well. We will return to such readings in section 3.6.

To be sure, the arguments sketched in this section are very informal. But they provide the reader with a quick method of checking the presuppositions for a particular tense.

3.3. Licensing bound tense

In this section, we will give a survey of the configurations in which bound tenses may occur. Furthermore, we repeat the argument that a bound tense is a bound
time variable on the content level. We already noticed that a bound variable
PAST must be licensed by a higher, in fact, a c-commanding PAST.

**Bound PAST licensing:**
A bound PAST requires a c-commanding PAST.

**Proof.** A bound PAST occurs in the following configuration:

...TENSE$_i$ Predicate$_1$$\lambda_0[...$PAST$_0$ Predicate$_2$...

We assume that TENSE is the nearest c-commanding tense. If TENSE is PAST,
then nothing remains to prove. Suppose, therefore, that TENSE = PRES. Then,
no relation in the store of TENSE is the precedence relation, by the tense constraint
of PRES. Therefore, no relation which PAST$_0$ inherits from TENSE$_i$ is the
precedence relation. The relation R associated with PAST$_0$ can't be the
precedence relation either because of the requirement that R(t$_0$,t$_0$) is true. It
follows that the store of PAST$_0$ doesn't contain any precedence relation, a
contradiction to the tense constraint of PAST. Therefore, TENSE = PAST. Q.E.D.

Due to the negative formulation of its tense constraint, a bound PRES
doesn't require a c-commanding PRES. As a matter of fact, there will, however, be
a c-commanding tense in most cases, because a bound PRES occurs in the
argument of an intensional predicate, which is usually a verb. Complements of
nouns might be problematic:

(3-8)  a. His belief $\lambda_0$[that he PAST$_0$ was successful] PAST$_1$ was strange
       b. His belief $\lambda_0$[that he PRES$_0$ is successful] PAST$_1$ was strange

(3-8b) cannot be used to express "simultaneity" of belief. The bound PRES$_0$
behaves as if it were c-commanded by PAST$_1$. We can obtain "simultaneity" if
we assume that the subject originates in the SpecA position and is reconstructed
at LF. I won't, however, try to solve this issue. A related construction is
discussed on page 26 of Abusch's paper.

The licensing condition for bound PRES is complementary: a bound PRES
must not be c-commanded by PAST. We can even have a slightly stronger result:

**Bound PRES constraint:**
A bound PRES under PAST leads to an inconsistency.
Proof. A bound PRES under PAST occurs in the constellation

\[ \ldots \text{PAST}_1 \text{ Predicate}_1 \ldots \lambda_0[\ldots \text{PRES}_0 \text{ Predicate}_2 \ldots] \]

with PAST$_1$ c-commanding PRES$_0$. PRES$_0$ inherits all the relations in the store of PAST$_1$, one of which must be the precedence relation $<$ due to the tense constraint of PAST. This, however, is excluded by the tense constraint of PRES. Q.E.D.

Together, the theorems entail that a bound PAST is connected through a chain of c-commanding PAST's to a PAST in an extensional context. And this chain must not be interrupted by an intervening PRES. Similarly, a bound PRES is connected by an uninterrupted PRES-chain to a PRES in an extensional context if we disregard the unclear status of PRES in noun complements. A similar rule is stated in Ogihara's (1989) system, where the configuration is stipulated. Here, it is a consequence of the tense constraints.

3.4. The Upper Limit Constraint

Recall Abusch's formulation of the constraint:

\[ \ldots \text{the now of an epistemic alternative is an upper limit for the reference of tenses.}[\ldots] \text{the local evaluation time is an upper limit for the reference of tenses.} \]

For tenses in extensional contexts, the local evaluation time is $u$, for tenses in intensional contexts, it is denoted by the bound variable $t_0$. By convention, the local evaluation time is expressed by $t_0$ in both cases. Therefore, the constraint can be reformulated like this:

[ULC] The Upper Limit Constraint:

Every tense has to fulfill the constraint that $t_i$ is not after $t_0$, where $i$ is the reference index of the tense.

As it stands, the constraint does not exclude a "forward-shifted" reading for the sentence (3-9a):

(3-9)   a. John believed that Bill was asleep    [H, 28]
In Abusch’s system it is possible to analyse this as a de re belief which John has about a particular time in the past. One would expect that this time is not later than the time of the believing. The ULC, however, doesn't imply this, as we will see. A de re predicate takes a structured proposition as argument, here a time and a property of times (cf. (Cresswell & von Stechow 1982)). We follow Heim’s comments and localize that information into the verb by giving it an additional time argument for the res-time, whereas the sentential object is a property of times. The LF of the sentence is something like this:

\[(3-9) \quad b. \text{John } PAST_1 \text{ believed } PAST_2 \lambda \lambda_0 [\text{Bill } t_3 \text{ was asleep}]]\]

res-movement

The PAST of the embedded sentence may be brought to the res position by a process called res movement. If we adhere to Chomsky's GB theory, such a movement is not possible because it violates the theta criterion, a consequence which can be avoided if we choose the structured proposition approach. For the purposes of this paper, we ignore this problem. The translation into the logical language is (3-9c), and the meaning of de re belief is given below.

**Meaning for de re belief**

\[
\| \text{believe} \| (t_{res})(R)(t)(x)(w) \text{ is defined only if } c \text{ supplies a suitable time-concept } f_c \text{ such that } f_c(w,t) = t_{res}.
\]

Where defined, \( \| \text{believe} \| (t_{res})(R)(t)(x)(w) = 1 \text{ iff } R(f_c(w',t'))(t')(w') = 1 \text{ for all } w' \text{ and } t' \text{ compatible with } x's \text{ beliefs in } w \text{ at } t. \)

As to the logical type of the symbol, since de dicto believe is of type \( \langle \langle i, st \rangle, \langle i, e, st \rangle \rangle \), the de re symbol is of type \( \langle i, \langle \langle i, i, st \rangle, \langle i, e, st \rangle \rangle \rangle \).

\[(3-9) \quad c. \quad \text{believe}(t_2; \pi_2)(\lambda t, \lambda t_0[be - asleep(t_2)(Bill)])(t_1; \pi_1)(John)(w_0)\]

where \( \pi_2 = R^{res}(t_2, t_0) \land R^{res} = <, \)

and \( \pi_1 = R^{believe}(t_1, t_0) \land R^{believe} = <\)

To interpret the formula (3-9c), a suitable time concept might be the function which assigns to any \( w \) and \( t \) the last time \( t^* \) before \( t \) such that John sees Bill in \( w \) at \( t^* \). If \( t_2 \) denotes that time, the formula expresses the proposition that John
believes that Bill was asleep the last time when he met John. This restriction makes sure that we have to choose the correct time for the free variable $t_2$, which occurs in the res position.

As we said, the formula does not exclude that we have a "forward-shifted" de re reading; i.e., the res $g(t_2)$ might denote a time which is later than the time $g(t_1)$ of the believing, the evaluation time. We could exclude it by an additional constraint which stipulates that the res-time is not later than the event-time, where the event-time is the time argument proper.

**[ULC-res]** ULC for res-time

Let $i$ be the reference index of a tense in res-position, and let $j$ be the reference index of the ordinary time argument (the evaluation time index). Then, the tense in res position has the additional constraint that $t_i$ is not after $t_j$.

Abusch (personal communication) tells me that she has in mind such an implementation. But (Heim 1994) has arguments against it, as we will see. Thus, let us accept the constraint as tentative only. Applying this to the formula (3-9c), we have to replace the presupposition $\pi_2$ for the res time with the more informative presupposition:

\[(3-9) \quad d. \pi_2^* = R_{\text{res}}^t(t_2, t_0) \land R_{\text{res}}^e = <\land \neg t_2 > t_1\]

ULC-res bars forward-shifting, but it does not exclude the possibility that the res-time PAST2 overlaps with the evaluation time PAST1. This is a welcome result because the sentence can be used to express a simultaneous de re reading. All depends on the time concept which connects the evaluation time/reference time with the res-time.

The ULC rules out a "forward-shifted" reading for sentence (3-10a), provided it could have an LF like (3-10b), as assumed by Heim in her discussion of the example.
(3-10) a. *When Mary was in her twenties, she thought she was unhappy on her 40th birthday

b. Mary PAST\textsubscript{1} believed λ\textsubscript{0}[her 40th birthday \ λ\textsubscript{2}[she PAST\textsubscript{2} was unhappy]]

(I formalize 'thought' as 'believe')

For reasons that will become clear later, I don’t think this is a correct analysis. But let us accept it for the sake of the argument. Abbreviating "her 40th birthday in w" as "b(w)", we can translate the LF into the formula (3-10c).

(3-10) c.

\[
\begin{align*}
\text{believe}\left(\lambda t_0 \lambda w_0 \lambda t_2 [be - unhappy(t_2; \pi_2)(she)(w_0)](b(w_0))(t_1; \pi_1)(she)(w_0)\right) \\
\pi_1 = R^{\text{believe}}(t_2, t_0) \land R^{\text{believe}} = < \\
\pi_2 = R^{be}(t_2, t_0) \land \left[ R^{\text{believe}} = < \lor R^{be} = < \right] \\
\end{align*}
\]

By \textit{\textbf{\lambda}}-conversion, this is equivalent to (3-10d):

(3-10) d. \[
\begin{align*}
\text{believe}\left(\lambda t_0 \lambda w_0 [be - unhappy(b(w_0); \pi_2^*)(she)(w_0)](t_1; \pi_1)(she)(w_0)\right) \\
\pi_1 = R^{\text{believe}}(t_2, t_0) \land R^{\text{believe}} = < \\
\pi_2^* = R^{be}(b(w_0), t_0) \land \left[ R^{\text{believe}} = < \lor R^{be} = < \right] \\
\end{align*}
\]

If we interpret this with respect to an assignment \textit{g} for which \textit{g}(R^{be}) = <, we obtain an "anteriority" reading: the birthday is before the subjective now. The sentence has that reading. Therefore, this consequence is welcome. But what about a \textit{g} such that \textit{g}(R^{be}) = \neg<? The presuppositions contained in the formula don't rule out such a possibility. On the other hand, sentence (3-10a) doesn't have such a reading, and the ULC correctly excludes it. If we apply the constraint to the formula (3-10c), we obtain the presupposition \pi_2^* = R^{be}(t_2, t_0) \land \left[ R^{\text{believe}} = < \lor R^{be} = < \right] \land \neg t_2 > t_0 instead of \pi_2. As a consequence, (3-10d) has to be replaced by (3-10e):
The second presupposition excludes the possibility that the birthday is after the subjective now, i.e., the bound variable $t_0$. This solves our problem: if $g(R^{be}) = \neg<$, the birthday must overlap the subjective now, as an inspection of $\pi_2$ immediately reveals.

The argument presupposes that framesetters like on her 40th birthday can bind a tense. I don't think that this is correct. Framesetters will be analysed in section 6. I will defend the view that only intensional predicates can bind tenses. If this is correct, ULC is redundant for bound tenses. Note by the way that an analysis along the lines indicated has to face another problem: we have to bar the following LF:

\begin{equation}
(3-10) \quad \text{f. She PAST$_1$ believed } \lambda_0[\text{her 40th birthday } \lambda_0[\text{she PAST$_0$ was unhappy}]]
\end{equation}

If we translate this into our logical language, we obtain the formula (3-10g):

\begin{equation}
(3-10) \quad \text{g. } \text{believe} \left[ \lambda t_0 \lambda w_0 \left[ \text{be - unhappy} \left( b(w_0); \pi_2 \right) \left( \text{she} \right) (w_0) \right] \left( t_1; \pi_1 \right) (\text{she}) (w_0) \right] \\
\pi_2 \# = \mathcal{R}^{be} \left( b(w_0), b(w_0) \right) \wedge \left[ \mathcal{R}^{be} = < \lor \mathcal{R}^{be} = <= \right] \wedge \neg b(w_0) > b(w_0)
\end{equation}

The presupposition $\pi_2 \#$ has become almost trivial. If we let $g(R^{be})$ be $\neg<$, the content of belief has become absolute with respect to the subjective now. To be sure, $b(w_0)$ varies with different $w_0$'s, but this is not at issue. I think the sentence doesn't have the absolute reading. The analysis proposed in the following sections will not run into problems of this sort, anyway.

To summarize the discussion: ULC-reference is presumably not necessary, ULC-res seems to be needed. Notice, however, that ULC-res speaks about tenses in res-positions. Contrary to what Abusch seems to believe, it doesn't hold for times expressions in res-positions in general as I have argued in (von Stechow 1995). I cannot go into this in this paper, however.
3.5. Future

Abusch holds the view that there is no future tense in English. But there must, of course, be a way to represent posteriority. How is this possible without violating the ULC? Consider example (3-10a). We use the raising verb \textit{will/would} for the representation of posteriority. For related analyses, \textit{vide} (Ogihara 1989, von Stechow 1991).

(3-11) a. John will cry \[H, 24\]
b. PRES\textsubscript{1} will \(\lambda_0[\text{John INF}_0 \text{ cry}]\) \[H, 25\]

Following (Heim 1994), we assume that bar infinitives have a projection which carries a time argument.\footnote{The reader may wonder how this fits with the previous assumption that the \(t_0\) is located in \textit{to} as in:}

(i) a. John believed Bill to be asleep \[H, 12\]
b. John PAST\textsubscript{1} believed \(\lambda_0[\text{Bill to}_0 \text{ be asleep}]\)

A more unified approach would perhaps be to assume that the tense variable is always located in the bar infinitive, whereas \textit{to} is semantically empty; i.e., an infinitival like \textit{to cry} has the LF:

(ii) \[
\begin{array}{c}
\text{NP} \\
\text{PRO} \\
\text{IP} \\
\end{array}
\begin{array}{c}
\text{INF-P} \\
\text{INF} \\
\text{VP} \\
\end{array}
\begin{array}{c}
t_0 \\
\text{cry} \\
\end{array}
\begin{array}{c}
to \\
\end{array}
\]

In English, the infinitive ending is morphologically invisible, but compare the German translation:

(iii) \[
\begin{array}{c}
zu \text{ wein-en.} \\
to \text{ cry-INF} \\
\end{array}
\]
The translation of (3-11b) into the logical language is (3-11c), which is equivalent to (3-11d):

\begin{align*}
(3-11) & \quad c. \ WOLL(\lambda t_0[\text{cry}(t_0)(\text{John})(w_0)])(t_1; \pi_1), \ \pi_1 = t_1 \otimes t_0 \\
& \quad d. \ \exists t_0[t_0 > (t_1; \pi_1) \land \text{cry}(t_0)(\text{John})(w_0)], \ \pi_1 = t_1 \otimes t_0
\end{align*}

I have written the presupposition for PRES1 in a condensed form. More explicitly, it should be the statement \( R^{\text{WOLL}}(t_1, t_0) \land R^{\text{WOLL}} = \neg \langle \land \neg t_1 > t_0 \), which entails \( \pi_1 \).

The formulae don’t violate the ULC for the simple reason that the first argument of the posteriority relation > is not expressed by a tense. Rather it comes from WOLL, a quantifier over times.

In the last section I said that tenses can only be bound by intensional predicates. As far as the example is concerned, the analysis of WOLL is not yet incompatible with that view, because there is no tense in the scope of WOLL. But there might be one, and WOLL can bind it, because it binds the distinguished time variable. Such a case is illustrated by an example discussed in (Dowty 1982):

\begin{align*}
(3-12) & \quad a. \ \text{John will find a unicorn that is walking} \\
& \quad b. \ \text{PRES1 will } \lambda_0[\text{a unicorn that PRES0 is walking } \lambda_2[\text{John INF0 find } t_2]] \\
& \quad c. \ WOLL(\lambda t_0 \exists x[\text{unicorn}(x) \land \text{walking}(t_0)(x)(w_0)])(t_1; \pi_1), \ \pi_1 = t_1 \otimes t_0
\end{align*}

I haven’t written down the presupposition for the bound variable \( t_0 \) which translates the bound PRES0, because this presupposition is cancelled as we know from the discussion of bound tense. Concerning the time argument located in INF, the non-finite inflection, \textit{vide} footnote 3. A remark on the logical representation of nouns like \textit{unicorn} is called for. Usually, they are represented with a time and world argument as well. The matter is subtle, however, and I am not concerned with this issue in this paper.

I think the analysis given is more or less correct. It follows that WOLL can license bound tenses. This is the reason why Abusch claims that WOLL is a modal verb, hence has an intensional predicate. How could that be? By all standards, cf. e.g. (Kratzer 1978), modal verbs are quantifiers over worlds. Obviously, the meaning rule given doesn’t analyse WOLL as a quantifier of this sort. In order to justify her claim that WOLL is an intensional predicate, Abusch
says that it is a quantifier over branching future times, but she doesn't elaborate on the idea.

Mats Rooth (personal communication) proposes the following meaning rule to grasp the idea. Let the variables s, s', s'' range over world stages, which are partially ordered by the relation ≥. s' ≥ s means that s' is a possible future of s. Formulas are evaluated with respect to stages. The meaning rule for WOLL would then be something like this:

\[ \text{WOLL } \phi \text{ at stage } s = 1 \text{ iff } \forall s' \left( s' \geq s \rightarrow \exists s'' \left( [s \leq s'' \leq s' \lor s' \leq s''] \land \|\phi\|'''' = 1 \right) \right). \]

A bit of reflection reveals that the complicated definiens is equivalent to the simple formula

\[ \exists s'' \left( s \leq s'' \land \|\phi\|'''' = 1 \right). \]

Thus, the universal quantification over future stages is spurious under this account. There is an additional problem with this kind of formulation, hinted at in Heim's paper: it is not clear how "genuine" modality, i.e., quantification over different worlds, can be expressed in this approach. I am sure it can be done somehow, but I don't want to go into problems of this sort in this paper. This is the reason why I chose a more traditional framework with worlds and times as primitives. The best I can figure out at the moment is something like this:

**WOLL as a modal verb?**

WOLL is a symbol of type \( \langle \langle \text{is}, t \rangle, \langle \text{is}, t \rangle \rangle \).

\[ \|\text{WOLL}(P)(t)(w)\| = 1 \text{ iff for every } w', \text{ such that } w' \text{ and } w \text{ have the same future from } t \text{ on, there is a } t' : t' > t \text{ and } P(t')(w') = 1. \]

Worlds are conceived as world histories, of course. Obviously, this universal quantification over worlds with the same future as the evaluation world w is nothing but a tortuous way of speaking about the future of w. The rule is fully equivalent with the extensional rule given above. In other words, the modalization is spurious in the same way as the universal quantification in Rooth's rule. Nevertheless, the rule might grasp a conceptual reality. In what follows, I will work with the simpler extensional version of the rule, but I will assume that WOLL behaves like an intensional predicate.

Next, consider example (3-10a) again, which was discussed in the previous section. If we want a reading which expresses that the subject localizes
herself before her 40th birthday, we have to use a *would* + infinitive construction for the embedded clause:

(3-13) a. When Mary was in her twenties, she thought she would be unhappy on her 40th birthday [H,13b]

In order to get the interpretation right, we furthermore have to give a time argument to "on her 40th birthday" which may be thought as the subject of "on". "on" is interpreted as identity.

(3-13) b. Mary PAST1 thought λ₀[ PAST₀ would
    \[ λ₀ on₀ her 40th birthday [λ₀ she INF₀ be unhappy]]\]

c.  \[
    \text{think}
    \left(\lambda t₀\left[\text{WOLL}\left(\lambda t₀\left[t₀ = b(w₀) ∧ be – unhappy(t₀)(she)(w₀)\right]\right)\right]\right)(t₁; π₁)(Mary)(w₀)
    \]

    \[
    π₁ = t₁ o t₀
    \]

Again, the presupposition of the bound PAST is left away, because it is cancelled for the reasons given. In what follows, we will always represent embedded tenses as bound variables without presuppositions.

We handle Abusch’s standard example for the sequence of tenses in a similar way:

(3-14) a. He decided a week ago that in ten days he would say to his mother that they were having their last meal together [A, 43]

We give a somewhat truncated LF together with its interpretation.

(3-14) b. he PAST₁ decided
    λ₀[ PAST₀ would [λ₀ he INF₀ say
    \[ λ₀[ they PAST₀ were having lunch]]\]

c. decide\left(\lambda t₀\lambda w₀\left[\text{WOLL}\left(\lambda t₀\left[say(α)(t₀)(he)(w₀)\right]\right)\right]\right)(t₁; π₁)(he)(w₀),

    α = λ t₀ [having – lunch(t₀)(they)]

    \[
    π₁ = t₁ o t₀
    \]
Thus, the embedded PAST has a bound variable reading which makes it "simultaneous" with the time of the saying, which is in the future. The sentence has the same structure as Ogihara's (1989) example:

(3-15)  A week ago he said that in ten days he would buy a fish that was still alive

PAST under *would* and even under *will* can have a back-shifted reading relative to a future time. The following example is not yet a problem for Abusch's theory:

(3-16)  a. He will think that he was sick  

It is correctly predicted that the embedded PAST cannot have a bound variable reading expressing simultaneity. If it had, the PRES of the matrix would express the precedence relation, a contradiction to its tense constraint. But it has a relative back-shifted reading. To obtain it, Abusch would assume a de re interpretation in analogy to example (3-9). A reasonable LF is (3-16b), which has the interpretation (3-16c):

(3-16)  b. PRES1 will λ0[he INF0 think PAST2 λ3λ0[he t3 was sick]]

c. WOLL(λt0[think(t2; π2)(λt0[be − sick(t0)(he)])](t1; π1)(he)(w0)),

where π1 = t1 o t0 and π2 = t2 o t0

The de re strategy runs into difficulties, however, if there is relative anteriority without a predicate of attitude. Heim gives the following example:

(3-17)  I will charge you whatever time it took  

The analysis is a bit complicated because of the quantifier *whatever time*. Changing Ogihara's example slightly, we obtain a sentence which illustrates the same point:

(3-18)  a. In two years, he will buy a fish that was alive before

At present, the fish might not even be born. I see no obvious way how this kind of back-shifting from the future can be expressed by a de re strategy. An adequate
analysis requires a relative past, a topic we will be concerned with in the next section.

3.6. Relative tense

The analysis given so far has treated the relational information belonging to the meaning of a tense as a presupposition. We obtain relative tenses, if we locate this information in the content. Borrowing a term from chemistry, we may call such tenses isomeric: they are built from the same meaning atoms, but the atoms take a slightly different configuration.

Meaning of relative tenses

Let $\alpha$ be $\text{PRES}^\text{rel}$ or $\text{PAST}^\text{rel}$, symbols of type $t$. $\|\alpha^k\|^g = 1$, iff $g(R_k)(g(t_i), g(t_0)) = 1$ and $g$ satisfies the constraints of the node $k$. If the constraints are not fulfilled, $\|\alpha^k\|^g$ is not defined.

Let us apply this to the example (3-18a). We ignore the framesetter and build the LF (3-18b):

(3-18) b. PRES₁ will $\lambda_0[\text{a fish that } \exists_3[\text{PAST}^\text{rel} \_3 \& \_3 \text{ is alive}] \lambda_2[\text{he INF}_0 \text{ buy } t_2]]$

Some comments are in order. $\text{PAST}^\text{rel} \_3$ is an expression of type $t$, which expresses the statement that $t_3 < t_0$. The time argument of the verb is of type $i$. Therefore, the symbol cannot remain in situ but has to be "QR-ed". Usually, QR applies to nominals. We could convert $\text{PAST}^\text{rel} \_3$ into a nominal before QR-ing it. In this particular case, the nominal would express the information $\lambda P[t_3 < t_0 \& P(t_3)]$. We could $\lambda$-bind the trace and would have an ordinary QR-configuration. Such an LF is obviously equivalent to the notation used in (3-18b), which shows in a transparent way what is going on. I prefer to keep it.

Another point is worth mentioning: the second argument of a relative tense is always the distinguished time variable $t_0$, which happens to be bound. The first argument must be different, however. Otherwise, we would obtain a contradiction if the temporal relation is $<$, because we would have to consider the statement $t_0 < t_0$. For the same reason, we would obtain a triviality for $\neg<$.

The first argument of the relative tense has to be quantified, because an intensional
context doesn’t permit free tense, as we know. The translation of (3-18b) is straightforward:

\[
(3-18) \text{ c. } \text{WOLL} \left( \lambda t_0 \exists x \left[ \begin{array}{c}
fish(x) \land \exists t_3 [t_3 < t_0 \land \text{alive}(t_3)(x)(w_0)] \\
\land \text{buy}(x)(t_0)(\text{he})(w_0)
\end{array} \right] \right)(t_1; \pi_1)
\]

A relative PAST may occur under PAST, of course. For instance, the de dicto reading of (3-19a) has the LF (3-19b):

(3-19) a. Mary believed that Bill bought a car

b. Mary \text{PAST}_1 believed \lambda_0 \exists_2 [\text{PAST}^{\text{rel}}_2 \& \text{a car } \lambda_3 [\text{Bill } t_3 \text{ bought}]]

We cannot represent the embedded PAST as a bound variable PAST for aspectual reasons. The sentence is as bad as \textit{Mary believes that Bill buys a car} under that reading. Simple past forms of accomplishments have the perfective aspect, which excludes "simultaneity". For a discussion of the perfective in English, \textit{vide} (Klein 1994).

3.7. Two consequences of the Bound-PRES constraint

The first consequence is the so-called \textit{double access phenomenon} in English, which occurs whenever a PRES is in the scope of an intensional predicate whose tense is PAST. The phenomenon was noticed first in (Smith 1978). Abusch gives the following example:

(3-20) a. John thought that Mary is pregnant  \hspace{1cm} [A, 59]

The first thing to notice is that the tense constraint for PRES excludes an LF with bound PRES in the subordinate clause:

(3-20) b. *John \text{PAST}_1 thought \lambda_0[\text{that Mary } \text{PRES}_0 \text{ is pregnant}]

PRES$_0$ inherits < from PAST$_1$, if it is in an intensional position, a violation of the PRES-constraint. In order to avoid it, PRES$_0$ has to be moved into the res-position, which is an extensional position and is therefore not subject to the inheritance mechanism. It follows that a de re analysis is well-formed:
(3-20)  c. John $\text{PAST}_1$ thought $\text{PRES}_2 \lambda_3 \lambda_0 [\text{Mary } t_3 \text{ is pregnant}]$

This means that $t_1$ denotes a time before $u$ at which John has a belief about the time denoted by $t_2$, namely, that Mary is pregnant at that time. $t_2$ can be denoted by the present tense, because it lasts from the time of the believing up to now, and the meaning of PRES doesn't exclude that $g(t_2)$ starts in the past; the only thing that matters is that $g(t_2) \neg< g(t_0)$. I think this is a very strong point for Abusch's analysis, because the English sentence has exactly this meaning. I know of no viable alternative to this. Cf. (von Stechow 1995) for further discussion of this point.

Another interesting prediction of the Bound-PRES constraint is observed in (Heim 1994): The constraint predicts that we obtain transparent readings for object-opaque verbs only if the verb has a PAST and the object has a relative with a PRES:

(3-21)  a. John was looking for a man who lives next door  \[H, 54\]

Here, John must look for a particular man. The explanation is this. We analyse $\text{to look for}$ with Montague (say with his PTQ) as a verb that takes an NP-intension as object. If the object remains in situ, PRES gets bound, and we derive a violation of the Bound-PRES constraint. To obtain a consistent LF, we have to scope the object to a transparent position. The following two LFs illustrate the argument:

(3-21)  b. *John $\text{PAST}_2$ was looking for $\lambda_0 [\text{a man who } \text{PRES}_0 \text{ lives next door}]$

c. a man who $\text{PRES}_1 \text{ lives next door } \lambda_3 [\text{John } \text{PAST}_2 \text{ was looking for } t_3]$

According to Heim the facts of English seem to be as predicted.

4. Heim's interpretation of Abusch

4.1. Logical Form and semantics

We now turn to the interpretation given in (Heim 1994). We start with a comment on Heim's LFs. They look like ours with two differences. First, the morphological representation is abstract; i.e., stem and tense morphemes are
decomposed in the syntax. The second feature concerns tense: there are two predicate modifiers < and ¬< which have to do with past and present tense. Their distribution is, however, not trivially regulated in the sense that a past verb is modified by < and a present verb by ¬<. It will turn out that a bound variable PAST is the argument of a verb modified by ¬<. We will explain the details in a moment. To get an idea of the LFs, consider the representation of (4-1a), which is (4-1b):

\[(4-1)\quad a. \text{John believed that Bill was asleep} \quad [H, 28]
\]
\[b. \text{John PAST}_1 \langle\text{believe}\rangle \lambda_0[\text{Bill PAST}_0 \langle\text{be asleep}\rangle] \quad [H, 56]\]

As before, the λ-operator is located in COMP.

After this introduction, we are ready for the semantics. These are Heim's interpretation rules for the two tenses:

**Tenses (Heim)**

\[a. \|PAST\| = \lambda(i). \quad [H, p.19]\]
\[b. \|PRES\| \text{ is defined iff } \lambda(i) o \lambda(0); \text{ where defined, } \|PRES\| = \lambda(i). \quad [H, 64]\]

g is a variable assignment. o means "overlaps". Note that the semantics does not require that PAST\(_i\) refers to a past time. Thus, this relational information must be located somewhere else in the system. As the formulation of rule (b) shows, Heim works with partial interpretations whose definitions have to fulfill certain presuppositions. The semantics for the two tenses look extremely trivial and are almost alike. Hence the differences in meaning must be the impact of further conditions. The first of these is the special role played by the time variable t0. In fact, we have adopted Heim's conventions in the previous system and state them explicitly only now.

**The distinguished variable t\(_0\)**

For the assignment \(g_c\) coming from the context c:

\[a. \quad g_c(0) = g(t_0) = t_c.\]
\[b. \quad t_0 \text{ is the only variable that can be λ-bound from COMP}.\]
The idea behind these assumptions is that the unmodified variable assignment that the interpretation starts with is provided by the context. Hence, any free time variable $t_0$ denotes $t_c$, the time of utterance.

Verbs are interpreted exactly as before. The interpretation of the predicate modifiers $<$ and $\neg<$ is introduced in an exemplary way for the verb $cry$. It should be obvious how a general meaning rule could be abstracted from that (vide (Heim 1994), (48)).

**The predicate modifiers $<$ and $\neg<$**

(Heim 1994, p.19): "Suppose that the affixes $<$ and $\neg<$ may be freely affixed to any verb or other predicate at LF, but every predicate must receive one or the other affix."

a. $\llbracket< - cry \rrbracket(t)$ is defined only if $t < g(0)$. Where defined,
\[
\llbracket< - cry \rrbracket(t)(x)(w) = 1 \iff \llbracket cry \rrbracket(t)(x)(w) = 1. \quad [H, 46]
\]
b. $\llbracket\neg< - cry \rrbracket(t)$ is defined only if $t\neg< g(0)$. Where defined,
\[
\llbracket\neg< - cry \rrbracket(t)(x)(w) = 1 \iff \llbracket cry \rrbracket(t)(x)(w) = 1.
\]

Our formulation of the ULC has been taken from Heim. Since her system has a somewhat different architecture, we quote her formulation for convenience:

**ULC (Heim):**

\[
\llbracket T \rrbracket^{e,c}_{\alpha} \text{ is undefined if } \llbracket \alpha \rrbracket^{e,c} > g(0); \text{ where defined, } \llbracket T \rrbracket^{e,c}_{\alpha} = \llbracket \alpha \rrbracket^{e,c}. \quad [H, 18]
\]

The label $T$ refers to the category tense. To illustrate these rules, consider a semantic representation for *Bill cried*:

(4-2)  Bill PAST₁ $\prec$-cry

By the meaning of the $\prec$-prefix, this is defined with respect to the contextually given assignment $g_e$ only if $g_e(1) < g_e(0) = t_c$. Where this is so, (4-2) expresses the set of worlds $w$ such that Bill cries in $w$ at $g_e(1)$. (Recall that $cry$ is of type $\langle i, \langle e, \langle s, t \rangle \rangle \rangle$ and that the world argument is left unsatisfied. Therefore, the sentence expresses a set of worlds.) Obviously, these are satisfying truth conditions.
In order to obtain bound variable readings for tenses, Heim assumes the following syntactic well-formedness condition:

**The Tense Licensing Conditions (TLC)**

[H, 49]

a. Every occurrence of PAST must be "in the domain of" some predicate that is <-affixed.

b. No occurrence of PRES may be "in the domain" of any predicate that is <-affixed.

The notion "in the domain of" has to be defined in a rather complicated way:

**Domain of β:**

[H, 67]

α is in the domain of β iff

a. α is the event-time argument of β,
   or
b. the trace of α is the event-time argument of β,
   or
c. α is contained in an intensional argument of β.

To illustrate the TLC, consider the "simultaneous" reading for sentence (4-3a), which is represented by the LF (4-3b):

(4-3) a. John believed that Bill was asleep

[H, 28]

b. John PAST₁ <-believe λ₀[ Bill PAST₀ ¬<-be asleep]

[H, 56]

PAST₁ is in the domain of believe, because it is the event time argument of believe (cf. condition (a) of the definition). Since believe has the <-prefix, PAST₁ is licensed by condition (a) of the TLC. PAST₀ is in the domain of believe since it is contained in an intensional argument of that predicate, which illustrates condition (c) of the definition of "domain". As before, PAST₀ is licensed by condition (a) of the TLC. Since PAST₀ is nothing but the bound time variable t₀, a point to which we will return, the LF means that John believes at time g_c(I) the set of world-times ⟨w,t⟩ such that Bill is asleep in w at t, where g_c(I) has to precede the time of utterance on account of the presupposition created by the <-affix.
We need condition (b) of the definition if we want to obtain the back-shifted reading for sentence (4-3a), which is analysed by the de re method, as we know by now:

\[(4-4) \quad \text{John PAST}_1 \leftarrow \text{believe PAST}_2 \lambda \lambda \lambda_0 [\text{Bill t}_3 \leftarrow \text{be asleep}] \quad [H, 65]\]

Here, the trace of PAST$_2$, i.e., t$_3$, is the evaluation-time argument of a $\leftarrow$-affixed predicate. Hence, condition (b) of the domain definition applies, and PAST$_2$ is licensed.

In order to evaluate this LF with respect to the variable assignment $g_c$ delivered by the context c, we assume that $f_\epsilon$ is the contextually determined time concept which connects the time of the believing $g_c(1)$ with the res $g_c(2)$; in other words, $f_\epsilon(w_c, g(1)) = g(2)$. By the meaning of PAST, we know that both $g_c(1)$ and $g_c(2)$ are before $g(0)$. Since Heim doesn't assume (ULC-res), the res-time $g_c(2)$ might be after the event time $g_c(1)$ in her system, and she deliberately holds the view that this might happen. We will return to this point in a moment. Therefore, within certain limits, the LF might represent a "forward-shifted" reading.

On the other hand, the LF cannot represent a "simultaneous" reading under plausible assumptions. To be sure, $g_c(1)$ and $g_c(2)$ might overlap. But then the time concept $f_\epsilon$ must relate the two and will produce the effect that the complement is not defined anymore. Suppose the concept is identity: "the time at which I am". We have to spell this out as $f_\epsilon(w, t) = t$, for every $w$ and $t$. Applying the meaning rule for de re belief, we find out that (4-4) is true in the real world $w_c$ iff for every $w, t$ compatible with what John believes in $w_c$ at $g_c(1)$,

\[\lambda * t \lambda * t [\text{Bill t}_3 \leftarrow \text{be asleep}] \equiv_{\{c, \lambda, \lambda, 0\}} (f_\epsilon(w, t))(t)(w) = 1.\]

Here, $\lambda^*$ is the metalinguistic $\lambda$-operator. By the meaning of the $\leftarrow$-prefix, this is defined only if $f_\epsilon(w, t) < t$. But $f_\epsilon(w, t) = t$. Therefore, the simultaneous interpretation is not possible for this time concept.

Everything depends on the time concept, however. There might be concepts which do not exclude that the res-time is later than the evaluation/event time. Heim's example is this:

\[(4-5) \quad \text{When I woke up, I was convinced I had overslept. I wondered what happened at six when the alarm went off. I figured I probably turned}\]
it off in my sleep. Then I realized it was only five o'clock.

The relevant formalization of the underlined sentence could very well be the following LF, according to Heim (with an accommodation of the formalism to Heim's final version):

\[(4-6) \quad I \text{PAST}_1 \prec \text{figure PAST}_2 \lambda \lambda \lambda_0 [I \text{probably } t_3 \prec \text{turn it off in my sleep]}\]

\[g_c(1) = 5 \text{ o'clock}, \quad g_c(2) = 6 \text{ o'clock}\]

Heim (1994, p.14) comments on this:

Semantics does not constrain the objective location of the referent of the lower tense, but its subjective location in the mind of the subject.

Let us assume that the relevant time which connects the time of the figuring out \(\text{PAST}_1\) with the res \(\text{PAST}_2\) is something like "the nearest time which is 6 o'clock and at which this alarm clock rings". This relation applies to 5 o'clock and 6 o'clock (in that order). The content of the attitude can be described then as as "I probably turned the alarm clock off at the nearest time which is 6 o'clock and at which the alarm clock's ringing must be earlier than the "subjective now"."

If the example is well-chosen, it constitutes a counterexample against the ULC-res, but the point is subtle and I am not sure of it. In any case, ULC-res is an independent principle, and one could add it to Heim's theory as well, if it were correct.

### 4.2. Bound tense

We can show that Heim's system entails a result which is analogue to that obtained in section 3.3.

**Bound tense in (Heim 1994):**

Any licensed bound tense, i.e., bound PRES or PAST, reduces to a \(\lambda\)-bound time variable.
Proof (for concrete examples):

a. Consider bound PRES: \( \lambda_0[Bill \text{ PRES}_0 \rightarrow \neg \text{ be asleep}] \) = that \( f \), such that for any \( t \) and \( w \): \( f(t)(w) = 1 \) iff \( \neg \text{ be asleep} \[g[t/0]0\] \( \text{ PRES}_0 \[t/0\]) \)(Bill)(w) = 1, for any \( t \) and \( w \) for which \( f \) is defined.

\[ \text{ PRES}_0 \[t/0\] = g[t/0]0 = t, \] provided \( g[t/0]0 \circ g[t/0]0 \), which is so.

\( \neg \text{ be asleep} \[g[t/0]0\] \( \text{ PRES}_0 \[t/0\]) \) is defined if \( g[t/0]0 \neg \text{ PRES}_0 \[t/0\] \), i.e., if \( t \neg \rightarrow t \), which is a tautology.

It follows that the abstract \( \lambda_0[Bill \text{ PRES}_0 \rightarrow \neg \text{ be asleep}] \) means exactly the same as \( \lambda_0[Bill \ t_0 \rightarrow \neg \text{ be asleep}] \).

b. Consider bound PAST next:
This case is even more trivial, because PAST_0 is interpreted as t_0. Therefore,

\( \lambda_0[Bill \text{ PAST}_0 \rightarrow \neg \text{ be asleep}] = \lambda_0[Bill \ t_0 \rightarrow \neg \text{ be asleep}] \).

The abstract \( \lambda_0[Bill \text{ PAST}_0 \neg \text{ be asleep}] \) need not be considered because it has inconsistent presuppositions, because \( \lambda_0[Bill \text{ PAST}_0 \neg \text{ be asleep}] \) is that function \( f \) such that for any time \( t \): \( f(t) = \neg \text{ be asleep} \[g[t/0]0\] \( g[t/0]0 \)(Bill). The presupposition of the \(-\)affix is that \( g[t/0]0 < g[t/0]0 \), which is impossible.

c. Finally, consider cases like \( \lambda_1 \lambda_0[1 \ t_1 \neg \text{ turn it off}] \) or \( \lambda_1 \lambda_0[1 \ t_1 \text{ turn it off}] \), where \( t_1 \) is of the category T. \( t_1 \) is interpreted as a bound variable by the rules of interpretation.

4.3. Comparing the two interpretations

The systems are almost identical. I think my interpretation sticks more to the letter of Abusch’s paper. It works with the relation variables and implements Abusch’s inheritance mechanism. As far as the examples are concerned, the two interpretations yield the same truth conditions. There are some differences in localizing the presuppositions of tenses, however. In my system, tenses themselves bear the presuppositions. In Heim’s system, the presuppositions come from the \(-\) and the \(\neg\)-prefix.
My implementation might have the advantage that it accounts for relative tenses easily. We merely have to locate the relational information of a tense in the content instead of considering it a presupposition. I don't see how this is possible in Heim's account. Presumably, we have to introduce relational tenses independently from denotational ones. To be sure, relational tenses are different symbols in my interpretation as well, but there is a systematic connection between relational and denotational tenses, which we have labeled isomerism.

A technical difference between the two systems is that my presuppositions are restrictions on variable assignments, whereas Heim restricts the range of temporal predicates. The difference might seem subtle, but it might be crucial: when it comes to the analysis of framesetters, my implementation faces no problems, whereas I don't see how Heim's interpretation can deal with the facts.

5. Subjunctive forms

Subjunctive forms are a diagnostic for intensional contexts. They are evaluated with respect to the distinguished time variable t0 which is shiftable only in intensional contexts. This is the point to be established in this section, a point due to Abusch. The matter will become important when we turn to adverbs of quantification.

One alternative to the two approaches treated on the previous pages is the theory developed in (Ogihara 1989), which expunges tenses in contexts where they have a bound variable reading. A related way of looking at the facts is to assume that finite forms can be semantically tenseless under certain conditions. On page 23, Abusch writes:

the temporal parameter of might and ought is treated as an IL-evaluation time (IL refers to Richard Montague's Intensional Logic., A.v.S.)

Let me explain what Abusch's statement means.
(5-1)  a. I *ought* to study more [A, 39]
     b. *When he was in high school, John *ought* to study more [A, 40]
     c. When he was in high school, John *ought to have* studied more [A, 41]

For the matrix clause, the evaluation time is the utterance time $t_u$. Sentence (5-1b) requires an evaluation time in the past. Obviously, this is not possible. One has to use (5-1c) instead, where the auxiliary *have* shifts the evaluation time to a past time and *ought* is evaluated with respect to the new evaluation time. The examples therefore seem to show that *ought* is not a simple past form. Is it a simple present form then? Abusch argues that this cannot be the case either:

(5-2)  a. John *believed* that he *ought* to study more [A, 45]
     b. John *believes* that he *ought* to study more
     c. He will always be a student that *ought* to work harder [H, Fn. 22]

If *ought* were a present form, we would expect (5-2a) to be ungrammatical because the rules for the sequence of tenses require a past form in the subordinate sentence. In reality, we find no contrast between (5-2)(a) and (b). *ought* seems to behave like a non-finite, i.e., a tenseless form because it is licensed in contexts that admit bound variable readings. *Ought* can be embedded under intensional predicates and under WOLL (example (c)). This is exactly Abusch’s theory.

It is instructive to remember what Montagovian IL representations for the examples (5-1)(a) and (c) would be, where the analysis of the latter example omits the subordinate clause:

(5-3)  a. $OUGHT[^{^\text{study}(j)}]$ 
     b. $HAVE[^{^\text{OUGHT[^{^\text{study}(j)}]}}]$ 

$OUGHT$ is a deontic necessity. $HAVE$ expresses anteriority; i.e., we assume the following meaning rules, where both $OUGHT$ and $HAVE$ are of IL-type $\langle s,t,t \rangle$:

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Meaning of OUGHT and HAVE:

a. \[ OUGHT^{w,t}(p) = 1 \text{ iff for every } w^* \text{ in } B(w), p(w^*,t) = 1, \text{ where } p \]
   is of type \( \langle i, \langle s, t \rangle \rangle \) and \( B^c \) is a deontic background assigning a
   proposition to any world \( w \).\(^4\)

b. \[ HAVE^{w,t}(p) = 1 \text{ iff for some } t^*, t^* < t \text{ and } p(w,t^*) = 1. \]

To be sure, Montague would treat OUGHT and HAVE as logical symbols, but
this is a purely technical difference. If we translate the IL-formulas (5-2)(a) and
(b) into our extensional logical language, we find the combination \( \lambda t_0 \lambda w_0 \) everywhere where we had the intensor \( ^{\wedge} \). Furthermore, the implicit world and
time parameters \( w \) and \( t \), on which the evaluation function depends, become
explicit arguments of the predicates. Thus, we obtain:

(5-4) a. \[ OUGHT_{w_0 t_0} \left( \lambda t_0 \lambda w_0 \left[ \text{study}_{w_0 t_0}(j) \right] \right) \]
   \[ = OUGHT_{w_0 t_0} \left( \text{study}(j) \right) \]

b. \[ HAVE_{w_0 t_0} \left( \lambda t_0 \lambda w_0 \left[ OUGHT_{w_0 t_0} \left( \lambda t_0 \lambda w_0 \left[ \text{study}_{w_0 t_0}(j) \right] \right) \right] \right) \]
   \[ = HAVE_{w_0 t_0} \left( OUGHT(\text{study}(j)) \right) \]

For the extensional language, we have to assume that OUGHT and HAVE are of
Ty2-type \( \langle i, \langle s, t \rangle, t \rangle \rangle \). Apart from this modification, the meaning rules for the
symbols are the same.

We are now in a position to make Abusch's observation about the modals
in question precise.

[ETCM] The evaluation time constraint for "subjunctive" modals

The modals OUGHT, MIGHT, SHOULD and WOULD (and perhaps
others) require \( t_0 \) as their time argument.

This restriction is thought to apply to our formal language. A more appropriate
treatment should carry over all that into a direct interpretation of LF. WOULD is
the formalization of a particular meaning of would, viz. the one we find in the
consequent of counterfactual conditionals, for instance.

The condition stated is a purely syntactic one. If there is any semantic
reason why the condition should hold, it must have to do with the fact that the
forms in the list are old subjunctive forms. A non-bound tense shifts the

\(^4\)For this style of meaning rules, vide (Kratzer, 1978).
evaluation time to a time different from $t_0$. Thus, the ETCM means that the said modals are semantically tenseless.

The ETCM correctly predicts that the evaluation time of *might* must be $t_0$ in (5-5):

(5-5)  John married a woman who **might** become rich  

The ETCM does not prevent the subjunctive modals from occurring in the complement of an intensional predicate. Thus, (5-2)(a) to (c) are represented by the formulas (5-6)(a) to (c), respectively:

(5-6)  

a.  $\text{believe}(\text{OUGHT}(\text{study} - \text{more}(\text{he}))(t_1; \pi_1)(\text{John})(w_0)), \pi_1 = t_1 < t_0$

b.  $\text{believe}(\text{OUGHT}(\text{study} - \text{more}(\text{he}))(t_1; \pi_1)(\text{John})(w_0)), \pi_1 = t_1 \neg< t_0$

c.  $\text{WOLL} \left( \lambda t_0 \left[ \text{student}(\text{he})(t_0)(w_0) \wedge \text{OUGHT}(\text{work} - \text{harder}(\text{he}))(t_0)(w_0) \right] \right)(\text{he})(t_1)(w_0), \pi_1 = t_1 \neg< t_0$

In (5-6)(a) and (b), the "intensional arguments" time and world are left unsaturated. Thus, *OUGHT* certainly doesn't have a time argument different from $t_0$. The analysis of (5-6b) proceeds in an analogous way. As the formalization of (5-6c) shows, the time argument of *OUGHT* is a bound occurrence of $t_0$. The formalization is not quite accurate because it neglects the adverb *always*. An appropriate treatment will require a modified meaning rule for *WOLL*. We will take up this issue in section 6.

At first sight, this looks quite promising, but the facts are more complicated than the picture sketched so far suggests. A first remark concerns the syntax of *ought to have*. Our formalizations show that we have assumed that, in the logical representation, the two auxiliaries exhibit a different subordination relation than they show at the surface. In a language like German (or Italian) we don't find this misfit between syntax and logical form, as the following comparison shows:

(5-7)  

a.  Hans hätte$^1$ mehr arbeiten$^3$ müssen$^2$

b.  John ought$^1$ to have$^2$ studied$^3$ more

Increasing subordination is indicated by increasing numbers. The word order of the English construction is an idiosyncrasy which is certainly due to the fact that
English has lost the non-finite form of \textit{ought} and related modals. There is no way to have a compositional semantics for the construction.

The formalization (5-3b) presupposes that the complement of HAVE is an intensional predicate since it is an intension. This works for this particular example but creates problems for other examples.

(5-8)  
\begin{itemize}
\item a. John has bought a fish that is alive
\item b. Bill has been a student that ought to study more
\end{itemize}

Neither of the two licences a bound variable reading in the subordinate clause; in both cases the embedded predicates have to be evaluated with respect to the speech time $t_0$. Another problem is that \textit{has} — a present form — seems to licence a bound variable reading for embedded past:

(5-8)  
\begin{itemize}
\item c. John has bought a fish that was alive.
\end{itemize}

This may be interpreted as "contemporaneous". Thus, HAVE behaves like PAST with respect to the rules of consecutio. The fact has been recognized by (Ogihara 1989), who assigns the feature PAST both to the past tense and to HAVE, and the rule of tense deletion is formulated with respect to the feature. To be sure, HAVE expresses a relative PAST. In order to avoid inconsistency, it seems best to say that HAVE never licences a bound variable reading, i.e., doesn't qualify as an intensional predicate. We have already discussed that WOLL counts as an intensional predicate for several reasons.

Finally, consider the readings for (5-9a). I think we have three readings here: first, the possibility that John's bride become sick could have occurred at some time before the speech time, i.e., possibly after the wedding; second, this was a possibility simultaneously with the wedding; third, the possibility was real before the time of the wedding.

(5-9)  
\begin{itemize}
\item a. John married a woman who \underline{might} \underline{have} \underline{become} rich \hspace{1cm} [cf. A, 44]
\item b. Hans heiratete eine Frau, welche reich \underline{hätte} \underline{werden} könne
\end{itemize}

As in example (5-7), I have indicated increasing subordination by increasing hyperscripts. The German translation shows that the highest finite form is \textit{hätte}, i.e., a subjunctive past form of \textit{haben} "have". For the time being, let us represent this subjunctive form as HAVE\textsubscript{0}. It follows that the sequence of auxiliaries is represented as HAVE\textsubscript{0} MAY, where MAY stands for a non-finite version of
might. A possible LF for (5-9a) might then be (5-9c), and the translation into our formal language is (5-9d):

\[(5-9)\]
\[
c. \text{a woman who have}_0 \lambda_0[\text{inf}_0 \text{may}_0 \lambda_0[\text{inf}_0 \text{become-rich}]]^{}
\]
\[
\lambda_3[\text{John past}_2 \text{married} t_3^{}]
\]
\[
d. \exists x_3^{} \land \text{have}_0[\text{may}_0(\text{become} \rightarrow \text{rich}(x_3))](t_0^{})(w_0^{}) \land \text{marry}(x_3^{})(t_2^{}; \pi_2^{})(w_0^{}), \pi_2^{} = t_2^{} < t_0^{}
\]

This LF correctly expresses the independent reading; i.e., the evaluation time of the relative clause and that of the main clause are evaluated independently of each other. There is no way to express the contemporaneous or the anteriority reading. We certainly could scope the object to a position lower than the matrix tense, but PAST cannot bind the distinguished variable $t_0$. Thus, sticking to the assumptions of our system, we have to say that (5-9c) is the only possible LF for our sentence – modulo alphabetic variance – and it must be good enough to represent all three readings which our intuition has taken for granted.

It is tempting to generalize the observations made for subjunctive modals to cover subjunctive forms in general, at least for one use of the subjunctive mood. The idea is that subjunctive forms are semantically tenseless. To further support the idea, consider counterfactuals, which show that the verb in the antecedent is best considered as a subjunctive form without semantic tense.

\[(5-10)\]
\[
\text{If I were/ was not in Austin, I would be in Prague.}
\]

In most cases, simple indicative past forms have the same shape as their subjunctive counterparts. But the verb to be distinguishes between the two. The following contrast shows that the antecedent and the consequent of the conditional (5-11a) cannot contain a semantic tense:

\[(5-11)\]
\[
a. \text{If Bill listened to you, he would make less mistakes}
\]
\[
b. \ast \text{When Bill was in high school, he would make less mistakes if he listened to you}
\]

We have to keep the meaning of would constant, of course. In other words, we should reinterpret would as the "habitual" one. As in the previous section, we have to form periphrastic past forms to get the sentence right:
(5-12) When Bill was in high school, he would have made less mistakes if he had listened to you

On the other hand, (5-11a) can be embedded in intensional predicates, even if they are in the past.²

(5-13) Mary believed that Bill would make less mistakes if he listened to you

The pattern is exactly as for the examples (5-2a-c), and the analysis is therefore analogous. The only difference is that would and could have to be analysed as two-place auxiliaries. An appropriate LF for (5-11a) could be something like:

(5-14) $\lambda_0 [\text{if Bill listened}_0 \text{ to you}]$

$\lambda_0 [\text{would}_0 \lambda_0 [\text{he INF}_0 \text{ make less mistakes}]])$

We have to make sure that the two subjunctive forms are interpreted with respect to the distinguished variable $t_0$. would is a 2-place counterfactual modality taking propositions as arguments. I assume that the restriction can be an adjunct to the clause headed by would. If we assume a meaning rule for would in the style of Lewis(1973), would translates into $\mu$, whereas could is translated as $\diamond \rightarrow$.

What is the systematic connection between the subjunctive and semantic tense then? We say that the subjunctive morpheme selects a 0-tense. In other words, we propose the following analysis:

(5-15)

_____________________________
²Not everyone accepts (5-13). Mats Rooth considers the sentence slightly odd. The corresponding German construction is acceptable (Maria glaubte, daß Willi weniger Fehler machen würde, wenn er auf dich hören würde).
We follow (Zeller 1994) by assuming that the subjunctive projection is hierarchically higher than the TP, and it is lower than AgrP, of course. The specifier of the subjunctive projection is semantically empty.

As for the semantics, recall from section 3.1 that PRES₀ and PAST₀ must satisfy the constraint \( R(t₀, t₀) \). We know how this works for embedded forms. What about non-embedded subjunctive forms like the ones discussed in this section? We have no problem with the subjunctive present as in Everyone be silent: here, the temporal relation associated with the verb is \( \neg< \), and this relation satisfies the constraint. We have, however, a problem with non-embedded PAST₀. Due to the semantics for PAST, the only relation associated with the PAST node, and hence in the store, is \(< \). But the precedence relation cannot satisfy the constraint \( R(t₀, t₀) \). The only way to settle the issue seems to be stipulation:

**Nonembedded subjunctive tenses**

A non-embedded subjunctive tense node is associated with the temporal relation \( \neg< \).

The stipulation looks artificial, but I see no way of avoiding it in Abusch's system. In (Stechow 1995) I developed a somewhat simpler approach, but I had to introduce an ambiguity for tenses: there are deictic tenses and there are bound variable tenses, which are more accurately, tenses that are interpreted as the distinguished variable \( t₀ \). Subjunctive tenses are precisely the latter.

6. Framesetters and Temporal Adverbs of Quantification (TADVs-Q)
6.1. The MAX-operator and TADVs-Q

One the most difficult tasks for any theory of tense is to get the interplay between framesetting adverbs (e.g. *today*), temporal adverbs of quantification (TAD-Qs) and tenses right. This is the subject of Bäuerle's dissertation (Bäuerle 1979). Abusch doesn't address this problem. The analysis to be proposed can be summarized as follows: We introduce a MAX-operator which has two arguments, a frame time and a tense. It restricts the reference of the time variable to the maximal time of the frame time which is before \( t_0 \) or overlaps \( t_0 \) if the tense is PAST or PRES, respectively. We then explore the hypothesis that TADV-Qs quantify over subintervals of such times.

In (Stechow 1991) I held the view that tense restricts a TADV-Q. Here is an example for such a treatment:

(6-1) a. Today, Vashek barked always
    b. ALWAYS(today) \( \lambda_1[\text{PAST}^{rel}_{1,0}] \lambda_1[\text{Vasheek } t_1 \text{ barked}] \)
    c. \( \exists m \left[ \text{Partition of today}(m) \land \forall t(t \in m \land t < t_0 \rightarrow \text{bark}(\text{Vashek})(t)(w_0)) \right] \)

A partition of an interval is a set of mutually distinct subintervals of the interval such that the fusion of the elements of the set is the interval. We cannot quantify over subintervals simpliciter, because a complete barking takes a while. Therefore, Vashek cannot bark at every subinterval. (Quantification over the elements of a partition was first introduced in (Kratzer 1978), to my knowledge.

Never mind how the LF (6-1b) is obtained from the surface: there is a general problem with this analysis, which has been pointed out to me by Irene Heim.

She gave me an example of the following kind:

(6-2) a. Today, Vashek barks always
    b. NOT: \( \exists m \left[ \text{Partition of today}(m) \land \forall t(t \in m \land t \not< t_0 \rightarrow \text{bark}(\text{Vashek})(t)(w_0)) \right] \)
    c. RATHER: \( \exists m \left[ \text{Partition of today}(m) \land \forall t(t \in m \rightarrow \text{bark}(\text{Vashek})(t)(w_0)) \right] \)

(6-2b) is what we would obtain if PRES did restrict the TAD-Q. The sentence, however, does not mean that. It rather means that Mary is funny at each interval of today and not only at the intervals which overlap the speech time. The correct
reading is expressed by the tenseless statement (6-2c). Thus, the present cannot
go with the restriction of the quantifier. It cannot go to the nucleus either. It
somehow disappears. It would be nice if the disappearance were the outcome of
the semantics of tense.

To solve the problem a proposal made in (Kratzer 1978) may help. Kratzer
interprets "PAST(P)(t)" as "P is true of the maximal subinterval of t which is
before t₀", where t is the evaluation time. She assumes that PAST is a sentential
operator and t is the invisible time parameter of the intensional language. Using
this semantics, we can interpret (6-1a) as: "For every subinterval of today
intersected with the largest time before now we have it that Mary is funny at that
subinterval". We cannot carry over this idea to Abusch's framework directly,
because there a tense is not a sentential operator but rather an argument of a
predicate. Furthermore, our tenses are not exactly definite descriptions as
Kratzer's interpretation assumes but time variables denoting the reference time.

To implement the idea, let us introduce an operator which gives us the
maximal present or past subinterval of some time given.

**The MAX-operator**

MAX is a logical symbol introduced by the following rule. If α is
either PRES or PAST, then MAX(α, τ) is an expression of type i for
any time variable τ.

\[ \text{MAX}(\alpha, \tau) \text{ is defined only if } \|\alpha\| \subseteq g(\tau) \text{; where defined,} \]
\[ \|\text{MAX}(\alpha, \tau)\| = \text{the maximal time } t \text{ such that } t \subseteq g(\tau) \text{ and} \]
\[ t = \|\alpha\|^{[\tau]} \].

j is the **referential index** of the tense α, and τ is the **frame variable**, because it
is bound by a framesetter.

Now we are able to give a useful LF for (6-1a):

(6-3) a. Today λ₁[\text{MAX}(\text{PRES}_2,1) \lambda_3[\text{always}(3) \text{ Vashek barks}]]

#For technical reasons, I have changed the logical type of the predicate: the
subject and the world argument are saturated earlier than the time argument. The
meaning of always is given by the following rule:
Meaning of *always*

*always* is a symbol of type \( \langle i, \langle i, t, t \rangle \rangle \) \( \|always\| (t) (P) = 1 \) iff there is a partition \( m \) of \( t \) such that for every \( t' \): if \( t' \in m \) : if \( t' \subseteq t \), then \( P(t') = 1 \), for any time \( t \) and set of times \( P \).

The formula which translates the LF is this:

(6-3) \( b. \lambda t_1 [\lambda t_3 [\text{always}(t_1) (\text{barks}(\text{Vashek}) (w_0)) (\text{MAX}(t_2 ; \pi_2 , t_1)) (\text{today})] \]

\( \pi_2 = t_2 \cup t_0 \)

Eliminating the two lambda operators by conversion, we obtain the equivalent formula:

(6-3) \( c. \text{always}(\text{MAX}(t_2 ; \pi_2 , \text{today})) (\text{bark}(\text{Vashek}) (w_0)), \pi_2 = t_2 \cup t_0 \)

Recall that I have changed the type of the verb; i.e., one place predicates are of type \( \langle e, \langle s, \langle i, t \rangle \rangle \rangle \). Therefore, the subformula \( \text{bark}(\text{Vashek}) (w_0) \) is of type \( \langle i, t \rangle \), i.e., denotes a set of times. With respect to some assignment \( g \), the formula is true if there is a partition of \( g(t_2) \), such that Vashek barks at any element thereof, where \( g(t_2) \) is the maximal time which is in today and overlaps \( g(t_0) \), which is the whole of today as intended.

The reader may check for himself that the sentence

(6-4) Today, Vashek barked always

is treated in a completely analogous way: We choose \( \text{PAST}_2 \) instead of \( \text{PRES}_2 \) and constrain its reference of the maximal time of today which is before \( t_0 \). Then, we quantify over subintervals thereof.

Before I go on, I would like to stress that the MAX-operator is a logical operation which has no impact on the inheritance mechanism of the theory.

(Bäuerle 1979) claims that each verb is modified by a TADV-Q. I see no necessity for this assumption. Consider (Partee's, 1973) celebrated example *I didn't turn the stove off*. The theory developed here can pick up a PAST and predicate the property "not I turn the stove off" thereof. Surely, Partee had something like this in mind when she created the example. It is interesting to consider the same sentence in combination with a framesetter:
(6-5)  a. Yesterday, I didn’t turn the stove off  
    b. Yesterday $\lambda I[\text{MAX(PAST}_2,1) \text{ not I turned-off the stove}]$

Since the negation of an accomplishment is a state and therefore has the subinterval property, the LF means that there is no time in yesterday at which I turned off the stove. I think this is the prominent meaning of the sentence. The formalization makes use of the MAX-functor but doesn’t contain a TADV-Q. To be sure, we could obtain the same effect by introducing an invisible universal TADV-Q $\forall(2)$, but this is simply not necessary, since the MAX-functor gives us the information needed.

Next, let us consider TADV-Qs under the future. Sentence (6-6a) should have the interpretation (6-6b):

(6-6)  a. Tomorrow, Vashek will always bark  
    b. $\exists m[\text{Partition of tomorrow}(m) \land \forall t(t \in m \rightarrow \text{bark(Vashek)}(t)(w_0))]$

The formula which represents the truth condition shows that it is not possible to interpret will/would as an existential quantifier as we did so far. The auxiliary will gives us the entire time in tomorrow which is after the present time. This will be the domain of quantification of the TADV-Q. The example further shows that tomorrow gives no frame for PRES, but rather for WOLL. In order to give a correct account for the truth conditions, we have to relativize WOLL (and HAVE) to framesetters as well. The revised meaning rules could be these:

**The temporal auxiliaries WOLL and HAVE (revision)**

WOLL and HAVE are of type $\langle ii, \langle it, t \rangle \rangle$.

a. $\|\text{WOLL}\|(t)(t')(P) = 1$ iff $P(t^*) = 1$, where $t^*$ is the maximal $t''$: $t'' \subseteq t'$ and $t'' > t$

b. $\|\text{HAVE}\|(t)(t')(P) = 1$ iff $P(t^*) = 1$, where $t^*$ is the maximal $t''$: $t'' \subseteq t'$ and $t'' < t$

In these rules, t is the event time variable and t' is the frame variable. It might turn out that these rules are not "modular" enough, for the maximality is built in the meaning, which is not a safe strategy. A safer way would be to say that WOLL and HAVE express $>$ and $<$ respectively. The rest is due to "logical glue". But things are complicated enough. Therefore, let it go. Now comes the logical form for (6-6a):
The formula which translates the LF is obtained from the literal translation by \(\lambda\)-conversion. It gives us the information that \(g(t_2) \circ g(t_0)\) and that Vashek barks at any member of some partition of \(t\), where \(t\) is the maximal subinterval of tomorrow which is after \(g(t_2)\).

Finally, consider the combination of WOLL and TADVs-Q in intensional contexts.

(6-7) a. Bill believed that Vashek would always bark
   \(\lambda_0[\text{PAST}_0 \text{ would}(2) (\lambda_0[\text{always}(0) \lambda_0[\text{INF}_0 \text{ Vashek bark}])]\)

Here, the frame index 2 should be contextually specified. All this looks quite satisfying, but there is at least one serious problem left, which we will address in the next section.

6.2. The Present puzzle

Consider the following sentence:

(6-8) a. Today, Vashek barks each time when the bell rings

The problem is that the present tense in the restricting clause \textit{when the bell rings} seems to be bound to have a bound variable reading, because the meaning of the sentence has to be paraphrased as (6-8b) and not as (6-8c):
(6-8)  
   b. For each t, t is a subinterval of the maximal interval in today which
       overlaps \( t_0 \), if the bell rings at t, Vashek barks at t.
   c. *For each t, t is a subinterval of the maximal interval in today which
       overlaps \( t_0 \), if t overlaps \( t_0 \) and the bell rings at t, Vashek barks at t.

The disturbing information which is provided by the PRES in the subordinate
clause is underlined in the paraphrase (b). In (Stechow 1995), I assumed that the
antecedent of a TADV-Q is a context that allows bound variable readings; in
other words, a quantifier like \( \text{always} \) behaves as if it were an intensional
predicate. I will first show that this speculation cannot be correct: the antecedent
of a TADV-Q is an extensional context which doesn't license bound variable
readings.

Suppose the antecedent of a TADV-Q were an intensional context. If this
and what we have said about subjunctive forms were correct, we should expect
modals like \( \text{might} \) and \( \text{ought} \) to have a bound reading in these contexts. This,
however, is not so.

(6-9)  
   a. *Gestern schlief Vashek immer, wenn er bellen müßte
      *Yesterday, Vashek was always asleep when he ought to bark
   b. *Otto war gestern niemals da, wenn er mir helfen könnte
      *Yesterday, Otto was never available when he might help me

I was not able to check the grammatical status of the English sentences. My
guess is that they are odd in the same way as their German counterparts are. In
other words, according to our assumptions, the LF for (6-9b) cannot be (6-9c).
Rather, it has to be (6-9d):

\[
(6-9)  \quad c. \quad \text{yesterday } \lambda_1[\text{MAX(PAST}_2,1)] \lambda_3[\text{NEVER}(3)]
\quad \lambda_0[\text{when}(0)] \lambda_0[\text{MIGHT}_0 \lambda_0[\text{INF}_0 \text{ he help me}]]
\quad \text{was Otto available}]
\quad d. \quad \text{yesterday } \lambda_1[\text{MAX(PAST}_2,1)] \lambda_3[\text{NEVER}(3)]
\quad \lambda_4[\text{when}(4)] \lambda_5[\text{MIGHT}_0 \lambda_0[\text{INF}_0 \text{ he help me}]]
\quad \text{was Otto available}]
\]

The meaning rules for \( \text{when} \) and \( \text{NEVER} \) are these:
Meaning of *when* and *NEVER*

a. *when* is a symbol of type \(\langle i, \langle it, t \rangle \rangle\). \(\mathcal{\| when \|}(t)(P) = 1\) iff \(P(t) = 1\).

b. *NEVER* is a symbol of type \(\langle i, \langle it, t \rangle \rangle\).

\[\mathcal{\| NEVER \|}(t)(P)(Q) = 1\] iff there is no \(t' : t' \subseteq t\) and \(P(t') = 1\) and \(Q(t') = 1\).

The difference between the two LFs (6-9c) and (6-9d) is that the quantifier shifts the intensional parameter in (c) but leaves it intact in (d). The second formula has a vacuous restriction and this is what we want, because it is the LF of an ungrammatical sentence.

Let us return to example (6-8a), which involves a present tense in the restriction of the TADV-Q. The analysis of the sentence is this:

\[(6-8) \quad d. \quad \text{today } \lambda_1[\text{MAX(PRES}_2,1) \lambda_3[\text{ALWAYS}(3)] \\
\quad \quad \quad \lambda_4[\text{when}(4) \lambda_5[\text{PRES}_5 \text{ the bell rings}]] \\
\quad \quad \quad \text{is Vashek asleep}]\]

To be sure, PRES\(5\) is a bound variable \(t_5\), but it carries the presupposition that \(t_5\) overlaps \(t_0\), and this is the Present puzzle.

Note that the puzzle does not arise when a past tense is in the antecedent:

\[(6-10) \quad a. \quad \text{Yesterday, Vashek was always asleep, when the bell rang} \\
\quad b. \quad \text{yesterday } \lambda_1[\text{MAX(PAST}_2,1) \lambda_3[\text{ALWAYS}(3)] \\
\quad \quad \quad \lambda_4[\text{when}(4) \lambda_5[\text{PAST}_5 \text{ the bell rang}]]] \\
\quad \quad \quad \text{was Vashek asleep}]\]

The PAST in the antecedent is redundant, as the reader may check. The problem reappears, however, when the antecedent contains a future, because the future is composed of PRES+WOLL. There is no way to represent (6-11) adequately:

\[(6-11) \quad \text{Tomorrow, Vashek will always be asleep, when the bell will ring}\]

In (Stechow 1991) I speculated that present forms may be tenseless because there is no such thing as a present morpheme in languages like English or German. (Zeller 1994) even argues that there is no such thing as a present at all. To take Zeller's line would be a radical departure from Abusch's assumption that there is a semantic present. Apart from that, there would be a problem in
technically executing Zeller's idea: a tenseless present form cannot simply be
interpreted as having the distinguished time variable \( t_0 \), for that would require
TADV-Qs to be able to bind that variable when it occurs in the restriction of the
TADV-Q. This is not possible as we know from the discussion of the subjunctive
examples. My conclusion is that the approach developed cannot solve the present
puzzle.

6.3. Event times come from aspects

The tacit presupposition of our account was that TADVs-Q quantify over
subintervals of the reference time, where the reference time is the time denoted
by the tense in question. Recall that we claimed at several occasions that a tense
can be bound by an intensional predicate only. We have seen that TADVs-Q are
extensional predicates. Therefore, they don't license bound tense. The analysis
given in the last section violated this assumption: TADVs-Q were supposed to
be able to bind tense in their restriction.

(Klein 1994) offers a more refined analysis which might solve the puzzle. His account is roughly this: tenses connect the speech time with the reference
time (Klein prefers the term assertion time). Then there is a second kind of
temporal relations which Klein calls aspects: they relate the reference time with
the event time (which Klein calls situation time). This account may be regarded
as a reconstruction of Reichenbach’s (1947) analysis of tense.

The idea to be worked out in this section is that TADVs-Q may quantify
over event times introduced by aspects. The relevant aspect for the analysis of
the problematic sentence (6-8a) is the Perfective. It states that the event time
is contained in the reference time. For the Imperfective the situation is the other
way around. In other words, we have the following two meaning rules:

Perfective and Imperfective

a. PERF\textsubscript{i}(j) is translated as \( t_i \subset t_j \).

b. IMPERF(ective) is translated as \( t_i \supset t_j \).

Let us apply this to a simple case first.
The meaning may be described as follows: the maximal time of today which precedes with the speech time contains an event time which is a bow-wow-interval with Vashek as agent. One would think that the quantifier $\exists_4$ is a TADV-Q. In a moment, we will realize that this is not so. This quantifier merely serves the purpose of closing the event-time variable existentially. It is logical glue.

The formula is the reduct of the explicit translation and means that the entire past of today is part of a rain-interval.

Similarly, we want to say that LF (6-14) means that the past of today is part of a time at which Vashek barks always:

The formula shows the crucial difference with respect to the earlier analysis: the TADV-Q quantifies over subintervals of event times, not over subintervals of tenses. And we know it cannot quantify over tenses. But a TADV-Q can quantify over event times. This consideration leads us to the solution of the present puzzle:

The formalization ignores when, because this word merely transports the event time. The LF means that today contains an event time such that any subinterval thereof is a bow-wow of Vashek provided it is a ringing of the bell. It is important
to realize that the second occurrence of PRES₂ is anaphorical. The first occurrence is maximalized and must denoted a maximal time of today which overlaps the speech time. This is the entire time of today. The second occurrence of PRES₂ picks up that time. Therefore, this time can serve as the frame for the PERF-variable t₆, which is quantified by the universal quantifier ALWAYS.

As it stands, the meaning is too weak, because the event-time should cover the entire day. One can obtain the intended result by maximalizing the event-time of the matrix clause by means of the operator max, which has the following definition:

Maximalizing event times

\[ \|\max(\tau;\pi)\|^e \text{ is defined if } \|\tau;\pi\|^e \text{ is defined; when defined,} \]
\[ \|\max(\tau;\pi)\|^e = \text{the maximal time } t \text{ such that } \|\tau;\pi\|^e[t\rangle \text{ is defined.} \]

Here, \( \tau \) is a time variable and \( \pi \) is the presupposition of the variable, i.e., an expression of type t. Using the operator, we can replace (6-15b) by (6-15c):

\[ (6-15) \quad \text{c. today } \lambda_1[\text{MAX}(\text{PRES}_2, 1)\lambda_3\exists t_4[\max \text{ PERF}_4(3)] \lambda_5[\text{ ALWAYS(5) } \lambda_6[\text{PRES}_2 \lambda_7[\text{PERF}_6(7) \text{ the bell rings}]] \text{ Vashek barks}]_COLORS] ] \]
\[ \quad \text{d. } \exists t_4 \left[ \lambda \lambda_7[\text{ring the bell}(w_0)(t_7; t_7 \subset t_2)] \right] \left[ \text{bark(Vashek)}(w_0) \right] \]

The formula translating the LF is obtained after several reductions have been applied. It expresses the intended meaning correctly, as the reader may check for himself. The crucial idea is that the tense goes together with the frame. The two aspects mentioned give us subintervals or superintervals of the frame. We quantify over these. Accordingly, the PRES₂ figures as free variable t₂, whose reference is constrained by the frame. We have to express the constraint only once, i.e., for the matrix tense.

All this looks rather complicated, but I don't know of a simpler way to analyze the facts. For convenience, I will indicate an explicit logical form for an IMPERFECTIVE + TADV-Q.
(6-16) a. Today, Vashek was always barking

b. 

The *always* in this formula is not relativized to the elements of a partition. I quantifies over each subinterval of the frame time.
6.4. Remarks on TADVs-Q in Heim's interpretation

In footnote 12 of her comments on Abusch, Heim writes:

A problem with this analysis [= her interpretation of Abusch] is that binding the tense to $\lambda_0$ leaves no room for adverbials. But in fact they can appear: John believed Bill to be asleep at that time/every time he called/. . . Perhaps what's at fault is the analysis of adverbials as binding tense; see note 6.

I don't want to give a full account of TADVs-Q for Heim's interpretation. I rather want to point out some of the difficulties her analysis has to face. It seems to me that Heim's theory will lose some of its elegance if the difficulties are overcome. The reason is that the complications mentioned in the last three sections cannot be avoided in this framework either.

An example of an analysis in which the TADV-Q binds the tense is the following:

(6-17)  a. John was in Paris at some time
        [H,5]
        b. some($C_1$) time $\lambda_2$ [John PAST$_2$ <-be-in-Paris]  [H,8]

I think this is very close to what (Abusch 1993) seems to have in mind, because she gives an example where the TADV-Q and the tense are coindexed, though no interpretation is provided. Heim says that the accommodation of presupposition of the <-prefix requires that $C_1$ is a variable over PAST-times, i.e., every t in g($C_1$) is before g(0). If g($C_1$) is big enough, (6-17b) can be read as: "there is a time t, t is before g(0) and John is in Paris at t".

Let me show what the limits of this procedure are. Consider the following example.

(6-18)  a. Today, Vashek was always asleep
        b. today $\lambda_2$[always($C_1$)(2) $\lambda_3$[Vashek PAST$_3$ <-be asleep]]

The semantics of this always is roughly the following: "always(X)(t)(P) iff for every t': if t' is a member of X and t' is a subinterval of t, then P(t')". If g($C_1$) includes every time of today which is before g(0), the LF gives us the intended reading.
We meet the first obstacles if we replace PAST by PRES:

(6-19) a. Today, Vashek is always asleep
    b. today \( \lambda_2[\text{always}(C_1)(2) \lambda_3[\text{Vashek } \text{PRES}_3 \neg<-\text{be asleep}]] \)

The LF is defined only if every time in \( g(C_1) \) overlaps the speech time. This means that the TADV-Q quantifies only over subintervals of today which overlap the speech time, an inadequacy.

As Heim's footnote suggests, the error might be the assumption that the TADV-Q binds the present. Suppose we were able to define an operator \( \text{maxi} \) such that

\[
\text{maxi}(\text{today})(\text{PRES}_2) = \text{MAX}_t[\text{t overlaps } g(0) \text{ and } \text{t is a subinterval of today}]
\]
\[
\text{maxi}(\text{today})(\text{PAST}_2) = \text{MAX}_t[\text{t is before } g(0) \text{ and } \text{t is a subinterval of today}]
\]

Then we could represent (6-18a) and (6-19a) as (6-18c) and (6-19c), respectively.

(6-18) c. today \( \lambda_1[\text{Vashek maxi}(1)(\text{PAST}_2) \text{ always}[<-\text{be asleep}]] \)
(6-19) c. today \( \lambda_1[\text{Vashek maxi}(1)(\text{PRES}_2) \text{ always}[\neg<-\text{be asleep}]] \)

The semantics of the adverb is roughly: "\( \text{always}(P)(t)(x) = 1 \) iff for every subinterval \( t' \) of \( t: P(t')(x) = 1 \)." If the maxi-operator had the properties indicated, these LFs would give us the correct meanings.

Now, I see no way to define such an operator. The reason is that the definiens of \( \text{maxi} \) makes use of the relational information "\( t \text{ overlaps } g(0) \)" and "\( t \text{ is before } g(0) \)", respectively. Recall, however, that Heim's semantics for tenses is purely denotational. To be sure, we can recover the relational information for \( \text{PRES}_2 \), because we have the presupposition that \( g(i) \text{ overlaps } g(0) \). But \( \text{PAST}_i \) doesn't have any such presupposition; therefore, the relational information is lost. To be sure, the information is contained in the \(<\)-prefix of the verb, and there might be a method to recover it from there. Even if there is a solution, it would complicate the theory, I guess. And one would have to deal with the more complicated examples which forced us to quantify over the event time variables introduced by aspect.
I hope it has become obvious that a number of questions are left open in Heim's theory. Since I see no straightforward answers for them, I leave the matter at this stage.

7. Postscript

Throughout this paper, I tried to follow Abusch as closely as possible. The system works for English, but it is complicated and might not be the best approach after all. Abusch's complicated semantics for the tenses are mainly motivated from what I take to be an idiosyncrasy of English, namely the double access phenomenon. In other languages, for instance, German, a present may very well be embedded in an intensional context and may express "simultaneity":

(7-1) Ich glaubte, daß Maria krank ist
     I believed that Mary sick is
     "I thought that Mary was sick"

Therefore, German PRES may very well be dominated by PAST, even if PRES is in an intensional context. It seems to follow that Abusch's tense rules are not general enough: they seem to be restricted to English.

Another point made in this paper is that one should always have in mind the combination of tenses with framesetters and TADVs-Q. These phenomena are a touchstone for any semantics of tenses. I was forced to enrich the theory by introducing aspects in the sense of (Klein 1994).

The theory developed here is complicated but, to a large extent, it works. One should not leave it at this stage; one should rather try to look for something simpler. Perhaps, one should say this: Tenses are ambiguous between variables carrying relational constraints and bound variables. In an extensional context, a tense always expresses a variable with constraints. In an intensional context, it usually expresses a bound variable, and it may express a relative tense, i.e., a relation between two bound time variables. In my understanding of Abusch, she made the valuable attempt to get the bound reading from the reading in extensional position, but that might be a mistake. For the reasons given, I don't see how this can work for German. My guess is that an approach which starts by assuming an ambiguity for tenses will turn out to be simpler. Of course, we have to say how bound tenses are distributed. In (Stechow 1995) I have tried to develop an approach along these lines, which is not entirely successful, but conceptually simpler.
References


