A PROBLEM FOR A COMPOSITIONAL TREATMENT OF \textit{de re} ATTITUDES*
ARNIM VON STECHOW & THOMAS EDE ZIMMERMANN

1. THE PLOT

Barbara Partee is reported to have said that she never was sure whether the complement of an attitude verb should be a sentence or a proposition, where proposition is to be understood in a pre-theoretical sense as sentence meaning. In this paper we take up the question by investigation a suggestion made by David Kaplan in (Kaplan, 1977), viz. that the complement of an attitude is in fact a proposition, which is described by some character. Our answer to Partee's question is that the complement of a \textit{de re} attitude is both a proposition and a sentence.

We will start be proposing a generalisation of Kaplan's theory of characters: both contexts and indices will be regarded as triples consisting of an individual, a world and a time. By doing so, we will be able to express many (but not all) \textit{de re} and \textit{de se} readings that have been discussed in the literature – e.g. in (Lewis, 1979) – without invoking a complicated syntax like e.g. Cresswell and von Stechow’s (1982) structured propositions.\footnote{The framework developed in the following is very similar to the proposals made in (Stechow, 1984), (Zimmermann, 1991: 205ff.), and (Haas-Spohn, 1995: 144f.), among others.}

We will show that an unrestricted version of Kaplan's proposal trivialises the semantics of attitudes. We will show that, according to it, whenever a subject x believes a true proposition p at some index, then x has to believe any other proposition q that is true at that index – a disastrous result for the theory of attitudes. To overcome the problem, we will have to restrict existential quantification over characters believed by the subject to suitable ones. The difficult question will be to say what suitability means precisely, and we will say that a suitable character must at least be expressible by a sentence of the subject's language and probably satisfy further constraints.

The paper is organised as follows. Section 2 gives an informal account of Kaplan’s theory of attitude reports. Section 3 introduces a formal language that is reasonably close to natural language and hence may be regarded as a formal substitute for English or Chinese. The section introduces Kaplan’s Prohibition against Monsters. Section 4 gives the formal analysis of Kaplan’s theory of attitudes. The sentence gives an explanation of what it means to have an attitude toward a character. Section 5 contains the formal semantics of attitude reports and shows how it is compatible with the usual syntax for complement clauses, i.e. no extra \textit{de re} syntax is needed. Section 6 shows that without some restriction on the very notion of a character, the theory predicts trivial, inadequate truth conditions for attitude reports. The section also contains some ideas as to what a more adequate, restricted notion of character should look like. The appendix contains the formal proof for the trivialisation of the semantics sketched in section 6.

* We have to thank Orin Percus, Uli Sauerland and Philippe Schlenker for critical and helpful comments. We want to apologize that many good points made in the comments have not been taken up in the revision for the simple reason that the paper would never have been finished in time if we had tried to do justice to all comments.
2. **KAPLAN'S ANALYSIS OF ATTITUDE REPORTS**

Before we start, we give a short overview of Kaplan's analysis of attitudes according to section XX of (Kaplan, 1977), which is entitled "Adding 'says'". Kaplan reconstructs the meaning of a sentence as a character, which is a function from contexts to propositions. Propositions are functions that map world-time pairs into truth-values.

Consider the famous scenario c₁ where Kaplan sees a man in the mirror with his pants on fire. The man happens to be Kaplan himself but he doesn't realize this. Kaplan thinks: "His pants are on fire". After a while he realizes for obvious reasons that the man in the mirror is he himself. He comes to think: "My pants are on fire." This is scenario c₂. The second scenario causes Kaplan to rush to the shower, the first doesn't. Since the object of the attitude causes the difference in reaction, the two objects of belief must be different. Kaplan concludes from this that the characters expressed by (2-1)(a) and (b) differ in cognitive significance even if they express exactly the same proposition at the two contexts described.

(2-1)  
   a. His pants are on fire.  
   b. My pants are on fire.

Let us refer to the characters expressed by (2-1)(a) and (2-1)(b) as ‘[[His pants are on fire]]’ and ‘[[My pants are on fire]]’, respectively. The two facts described can therefore be paraphrased in the following way:

(2-2)  
   a. Kaplan believesₐ in c₁ [[His pants are on fire]].  
   b. Kaplan believesₐ in c₂ [[My pants are on fire]].

We are using the subscript ‘ₐ’ to mark relations between a subject and a cognitively significant object of attitude. It is obvious then that a subject bears an attitude toward a character.

Kaplan's famous prohibition against monsters entails that neither English nor any other natural language has a word for the relation ‘believesₐ’. And the facts of English certainly support Kaplan's position because the sentence

(2-4)  
   Kaplan believesₐ that his pants are on fire.

doesn't describe the fact (b). We have to use the sentence

(2-4)  
   Kaplan believesₐ that his pants are on fire.

both for the report of fact (a) and of fact (b). We have used the subscript ‘ₙ’ (for ‘reporting’) in order to distinguish this relation from the former one. This time, the object of the relation is a proposition. Following the lead of section XX, reporting belief ought to receive the following analysis.

(2-5)  
   Kaplan believesₙ that his pants are on fire in world w at time t if and only if there is a character χ and a context c such that t(c) = t, w(c) = w, χ(c) = [[His pants are on fire]](c), and Kaplan believesₐ χ in w at t.

It is a consequence of this analysis that the information precisely toward which character the subject bears the attitude has been lost in the report. Kaplan's analysis has some extremely attractive consequences in so far as it is compatible with a very simple syntax. Linguists who have tried to give a syntax and semantics for attitudes have always offered some rather complicated syntax. Only very few researchers have discussed Kaplan's proposal of section XX
at all. Another attractive feature of the proposal is that it provides a unified account for the analysis of de re and de se attitudes.

The second point should be obvious from the meaning rule just given. A sufficient syntactic environment to make the semantics work is a tree of the form:

\[(2-6) \ [S \text{ Kaplan} \ [VP \text{ believes} \ S \text{ his pants are on fire}] \]

At a particular context, the embedded S has to express a proposition while the matrix S expresses a truth-value. Semanticists who have been concerned with the syntax of de re attitudes mostly have proposed a much more complicated logical form. The reason for the syntactic complication is that these semanticists have taken up a proposal by David Lewis, according to which the object of a de re attitude is a structured proposition. For the example considered, the structure would consist of the res Kaplan and the property of having one’s pants on fire. Lewis’ truth-condition would be roughly this one:

\[(2-7) \text{ Kaplan believes the structured proposition } \langle \text{Kaplan, } \lambda x. x \text{s pants are on fire}\rangle \text{ in } w \text{ at } t \text{ iff there is a suitable relation } R \text{ which Kaplan uniquely bears to Kaplan and Kaplan believes in } w \text{ at } t \text{ the proposition that the pants of the unique thing related to Kaplan by } R \text{ are on fire.}

We are using the subscript s for distinguishing this structuring belief relation from the two other belief relations that have been introduced. When R is the relation \( \lambda x. x \text{ sees, we have a de re reading. When R is identity, then we have a de se reading. To get the logical syntax right, we must keep the res Kaplan and the predicate of having one’s pants on fire apart. (Cresswell and von Stechow, 1982) achieve that by assuming a sort of movement rule which brings the res to the right edge of the sentence, thereby creating a } \lambda -\text{abstract. Following modern practice, we will move to the left edge. The res and the } \lambda -\text{abstract are, however, not composed by means of functional application. The meaning of that tells us that the two (or more) form an ordered pair. Thus the logical syntax for the example is something like the following:}

\[(2-8) \ [S \text{ Kaplan} \ [VP \text{ believes} \ cp \text{that } S \text{ he } \lambda_1 [S \text{1's pants are on fire}] \] \]

Linguists could object that 'res movement', i.e., the movement of he from the specifier position to the left boundary of the sentence violates well-established locality restrictions and is not attested elsewhere in syntax. This is not quite true because focussing particles can be analysed by the same kind of long movement too. In any case, none of the other proposals for the syntax of de re reports appears to be as simple as the one that fits Kaplan's theory. Thus it would be genuine progress in our understanding of logical form, if Kaplan's theory did work. As we will see, there are non-trivial obstacles.

Kaplan does not offer an analysis of what it means to believe a character. In the next
section, we will offer one, following proposals of (Haas-Spohn, 1995).

3. **Natural Language Semantics in Kaplan's Style**

In this section we will make precise Kaplan's semantics within a \(\lambda\)-categorial language in the style of (Cresswell, 1973). For convenience, we will use syntactic categories like S, NP and VP, but the only thing that really matters are the logical types, which determine the semantic types of the expressions of the language.

We will assume the primitive types \(e\) (individuals), \(i\) (times), \(s\) (worlds), \(t\) (truth-values) and \(c\) (contexts). The first four are commonly accepted types. The type formation rules are the following: (a) every primitive type is a type; (b) if \(a\) and \(b\) are types, then \((ab)\) is a type; (c) nothing else is a type.

Outer brackets will be omitted. Semantic domains \(D_a\) are defined in the familiar way, i.e. \(D_e, D_i, D_s\) and \(D_t\) are the individuals, the time intervals, the worlds, and the truth-values respectively, and \(D_c = D_e \times D_w \times D_t\). \(D_{ab}\) is the set of (possibly partial) functions with arguments in \(D_a\) and values in \(D_b\). Whenever something is in \(D_a\), we will say that this thing is of type \(a\).

For Kaplan, meanings of natural language expressions are characters, which are functions from contexts into intensions. There are reasons for treating contexts as primitives, entities that might be thought as situations located in a world at a time and a place (cf. (Lewis, 1980), Section 6). It proves, however, to be technically more convenient to follow ‘The Formal System’ [= Section XVIII] of (Kaplan, 1977)\(^8\) and to identify contexts with triples \(<x,w,t>\), where \(x\) is the agent or speaker of \(c\) (= \(a(c)\)), \(w\) is the world of \(c\) (= \(w(c)\)), and \(t\) is the time of \(c\) (= \(t(c)\)). Thus the set of all contexts \(C\) comes out as a subset of \(D_c\).\(^9\)

Intensions are functions from indices to extensions. There are reasons for treating indices as world–time-pairs, i.e. contexts without speakers. It proves, however, to be technically more convenient to follow Stalnaker (1978) and have extensions and intensions depend on the same sets. In order for the parameters of an index to allowed to shift independently, it is natural to take this set to be all of \(D_c\) rather than \(C\) alone. Hence characters correspond to be ‘square’ matrices i.e. (infinite) tables with a row for each triple in \(D_c\), representing each (‘horizontal’) intension by the extensions it determines for the indices in the columns. So the diagonal of a character thus conceived will always be an intension itself\(^10\) and it can serve as a subject-dependent object of attitude along the lines of (Lewis, 1979). Apart from technical convenience, filling up the index with contextual parameters allows for a principled division between context and index by the criterion of bindability: whenever a contextual parameter can be abstracted from, or bound, it is part of the index; otherwise it belongs to the context. In particular, the extension of the

---

\(^8\) An individual in a world at a time can be identified with a context because it gives us any kind of information a context could yield. The individual is located at a particular place in that world at that time, the content of *here*. There might be a particular audience addressed in \(w\) at \(t\) by the individual, the content of *you*. The individual might share certain assumptions with other participants of the conversation, Stalnaker's common ground, and so on. All this is determined by the triple containing speaker, world, and time.

\(^9\) Thanks to Philippe Schlenker for pointing out an error in an earlier formulation.

\(^10\) This is the main technical advantage of square characters. See Section 2.3 of (Zimmermann, 1991) for more on the relation between various ways of forming characters and diagonals.
logophoric first person pronoun (in a language like Amharic) relating to the subjective agent in indirect speech, would depend on the agent parameter of the index rather than that of the context. Since we are only interested in English, where the first person is clearly a matter of context, we will leave these matters open in this short note.\footnote{Thanks are due to Philippe Schlenker (p.c.) for pointing out the availability of such an analysis of logophoricity in the present framework. The same strategy of ‘indexicalizing’ apparent contextual parameters may also be applied to explain away the quantified contexts of (Partee, 1989). The bindability criterion of teasing apart context and index goes back to (Lewis, 1980) and turns Kaplan’s Monster Prohibition from an empirical hypothesis into an \textit{a priori} principle; cf. (Zimmermann, 1991: 167). Given enough examples of the (Partee, 1989) kind, it will also trivialize the two-dimensional framework!}

To sum up, intensions are entities in $D_c^a$, for any type $a$. We will call these things $a$-intensions. Kaplanian characters are the entities in $D_{c(ca)}^a$, for any type $a$. We will call such semantic objects $a$-characters. A $t$-character $\chi$ is \textit{true at a context} $c$ iff $\chi(c)(c) = 1$. A $t$-character is \textit{a priori} if it is true at every context. A $t$-character $\chi$ is \textit{analytic} if it is a priori and, for every context $c$, $\chi(c)$ is the necessary proposition, i.e. [the characteristic function of] the set $D_c$.

For our discussion we will assume that the expressions of natural language belong to a $\lambda$-categorial language in the style of (Cresswell, 1973). The syntactical categories can be indexed with the logical types but, for convenience, we will use some common symbols like S, NP and V, VP etc. in addition. Both the constants and the variables of the language are indexed with types. We can form $\lambda$-abstracts by means of the familiar formation rules.

\begin{enumerate}
\item Syntax for our mini-language
  \begin{enumerate}
  \item $\alpha$ is an expression of type $a$ if it is a constant or a variable of form $\beta_a$.
  \item If $\beta$ is of type $a$ and $\alpha$ is (i) of type $ab$, or (ii) of type $(ca)b$, or (iii) of type $(c(ca))b$, then $(\alpha\beta)$ is an expression of type $b$.
  \item If $\alpha$ is of type $b$ and $x$ is a variable of type $a$, then $(\lambda x \alpha)$ is an expression of type $ab$.
  \end{enumerate}
\end{enumerate}

We are now in a position to state the most important restriction for the interpretation of natural languages that is due to (Kaplan, 1977), viz. the prohibition against monsters, which says that semantic operations, i.e., functions that map two or more meanings to another meaning, can be intensional at most. In other words, there is no principle of semantic composition $f(\chi_1,\ldots,\chi_n)$ that genuinely depends on the argument characters $\chi_1,\ldots,\chi_n$. We can state the principle in the following positive form:

\begin{enumerate}
\item Prohibition against monsters
  \begin{enumerate}
  \item Let $f$ be any principle of semantic composition that is used for the interpretation of a natural language, $f$ is an $n$-place function defined for characters and giving a character as result. Then there is a function $g$ defined on intensions such that $f(\chi_1,\ldots,\chi_n)(c) = g(\chi_1(c),\ldots,\chi_n(c))$ for any characters $\chi_1,\ldots,\chi_n$ and context $c$.
  \end{enumerate}
\end{enumerate}

The negative formulation would be: "There is no composition principle $f$ which cannot be redefined as a function $g$ of the form stated; such a composition principle would be a \textit{monster} in Kaplan’s sense. If the prohibition (3-2) is indeed universal, it follows that verbs of attitudes must be intensional functors. They cannot be monsters.

Here is interpretation for our mini language that makes the three kinds of relevant
principles of composition precise. We define a pair of functions $\llbracket \ldots \rrbracket^g$, where $\llbracket \ldots \rrbracket$ is the interpretation function proper and $g$ is a variable assignment.

(3-3)  

| a. | When $\alpha_a$ is a constant, then $\llbracket \alpha_a \rrbracket^g$ is a character of type $\alpha$ specified by the lexicon. |
| b. | When $\xi_a$ is a variable, then $\llbracket \xi_a \rrbracket^g = \lambda c \in D_c. \lambda j \in D_c. g(\xi_a)$ |
| c. | **Extensional functional application (EFA)**  
When $\alpha$ is of type $ab$ and $\beta$ is of type $a$, then $\llbracket (\alpha \beta) \rrbracket^g = \lambda c \in D_c. \lambda j \in D_c. \llbracket \beta \rrbracket^g(c)(j)(\llbracket \gamma \rrbracket^g(c)(j))$ |
| cii. | **Intensional functional application (IFA)**: When $\alpha$ has the form $(\beta \gamma)$, $\beta$ of type $(ca)b$ and $\gamma$ of type $a$, then $\llbracket (\beta \gamma) \rrbracket^g = \lambda c \in D_c. \lambda j \in D_c. \llbracket \beta \rrbracket^g(c)(j)(\lambda k \in D_c. \llbracket \gamma \rrbracket^g(c)(k))$ |
| ciii. | **Monstrous functional application (MFA)**: When $\alpha$ has the form $(\beta \gamma)$, $\beta$ of $(ca)b$ and $\gamma$ of type $a$, then $\llbracket (\beta \gamma) \rrbracket^g = \lambda c \in D_c. \lambda j \in D_c. \lambda u \in D_u. \llbracket \beta \rrbracket^g[(3^{[a]})^g](c)(j)$ |
| d. | **Abstraction**: When $\alpha$ has the form $(\alpha \beta)$, then $\llbracket \alpha \beta \rrbracket^g = \lambda c \in D_c. \lambda j \in D_c. \lambda u \in D_u. \llbracket \beta \rrbracket^g[(3^{[b]})^g](c)(j)$ |

It is clear that the EFA and IFA are legitimate principles of composition, whereas MFA is ruled out by Kaplan’s constraint.

Here is a simple example of the evaluation of a sentence of the kind discussed in (Kaplan, 1977), where the tree gives the LF for the surface structure:

(3-4)  

We assume the following lexical entries:

(3-5)  

| a. | $\llbracket \text{PRES}_{(ct)} \rrbracket = \lambda c \in D_c. \lambda j \in D_c. \lambda P \in D_c. P(j[t(j)/t(c)])$. |
| b. | $\llbracket \text{I} \rrbracket = \lambda c \in D_c. \lambda j \in D_c. a(c)$. |
| c. | $\llbracket \text{located}_{(ct)} \rrbracket = \lambda c \in D_c. \lambda j \in D_c. \lambda x \in D_e. \lambda y \in D_e. y \text{ is located at place } x \text{ in } w(j) \text{ at } t(j)$. |
| d. | $\llbracket \text{here} \rrbracket = \lambda c \in D_c. \lambda j \in D_c. \lambda y \in D_e. \text{ the place of } c \text{ (= p(c))}$. |

The evaluation of the LF given in (3-4) requires IFA at one step of the evaluation. All the other

---

12 The idea to formulate the different kinds of functional applications in the meta-language is inspired by chapter 12 of (Heim and Kratzer, 1998).
13 We are using the conventions introduced in chapter 3 of (Heim and Kratzer, 1998): if ‘$P$’ denotes a function of some type $at$, $P(j)$ is short for ‘the $y \in D_t$: $y = P(j)$’, for any index $j$. $j[t(j)/t(c)]$ is like $j$ with the possible difference that $t(j) = t(c)$.
steps require EFA. So the tree obeys Kaplan's prohibition against monsters. Here is the
calculation of the tree's truth-condition:

For any context \(c\) and index \(j\):

\[
\lbrack \text{PRES}_{(cjt)} \text{I located here} \rbrack (c)(j) = 1
\]

equivalent (by IFA) \(\lbrack \text{PRES}_{(cjt)} \rbrack (c)(j) = 1\)

iff (by the meaning of \(\text{PRES}_{(cjt)}\)) \(\lbrack \text{I located here} \rbrack (c)(k) = 1\), where \(k = (w(j), t(c), a(j))\)

iff (by \(2 \times \text{EFA}\)) \(\lbrack \text{located} \rbrack (c)(k) (\lbrack \text{here} \rbrack (c)(k)) (\lbrack \text{I} \rbrack (c)(k)) = 1\)

iff (by meaning of \(\text{located}\)) \(\lbrack \text{I} \rbrack (c)(k)\) is located at \(\lbrack \text{here} \rbrack (c)(k)\) in \(w(k)\) at \(t(k)\)

iff (by meaning of \(\text{I}\) and \(\text{here}\)) \(a(c)\) is located at \(p(c)\) in \(w(k)\) at \(t(k)\)

iff (by definition of \(k\)) \(a(c)\) is located at \(p(c)\) in \(w(j)\) at \(t(c)\)

Obviously this character is \textit{a priori}, i.e., true at every context. Given that the content of the LF
is a contingent proposition at every context, the tree provides an example for a synthetic \textit{a priori} statement, precisely as Kaplan wants to have it.

Now, what would be a monster? The best known one is the diagonal operator, which makes of every character a diagonal intension. Here is a version of a diagonal operator that is
used in (Haas-Spohn, 1995) for the analysis of attitude reports:

\[\delta I am here\]

\[\lambda c. \lambda j. \lbrack \delta \rbrack (c)(j)(\lambda d. \lambda k. \lbrack I am here \rbrack (d)(k)) (MFA!)
\]

Clearly these operators are monsters and their composition with other expressions requires
MFA. As an example, consider the diagonalisation of the LF given in (3-7):

\[\delta I am here\]

\[\lambda c. \lambda j. \lbrack \delta \rbrack (c)(j)(\lambda d. \lambda k. \lbrack I am here \rbrack (d)(k)) (MFA!)
\]

If verbs of attitudes did subcategorise diagonalised characters, they would be monsters. Many authors have claimed that they are monsters, but Kaplan says that they are not. Let us see what his analysis is.

4. \hspace{1cm} \textbf{ATTITUDES}

This section gives a formal reconstruction of Kaplan's theory of attitudes sketched in chapter XX
of (Kaplan, 1977). We must carefully distinguish between a particular attitude and sentences
reporting this attitude. According to Kaplan, the object of an attitude is a character, the object of an attitude report is a proposition. We start with the analysis of attitudes themselves.

Consider the context c where Kaplan sees himself in the mirror and thinks: “My pants are on fire”. Kaplan would describe this as: “Kaplan believes in w(c) at t(c) the character χ, where χ = λcλj[the pants of a(c) are on fire in w(j) at time(c)].”

A sentence which reports this belief is the following one:

(4-1) Kaplan believes that his pants are on fire.

What does it mean to bear an attitude towards a character? We follow (Haas-Spohn, 1995) in assuming that this means that the subject of the attitude self-ascribes the diagonal property. 14 Let us call the belief relation that relates a subject and a character 'believes_a'. The relation has the following structure:

(4-2) Subject x believes_a character χ in world w at time t iff x self-ascribes χ in w at t.

Self-ascription can be analysed in the standard way by means of doxastic accessibility, which is not a quantifier over worlds but over ‘centred worlds’, i.e., indices (or contexts):

(4-3) For any j, k ∈ D_c: k ∈ Dox_j iff a(k) has every property in w(k) at t(k) that a(j) believes to have in w(j) at t(j).

It is natural to assume that Dox_j ⊆ C, i.e. that the subject does not attribute contradictory properties to the context she takes herself to be in. A meaning rule for the relation believes_a can now be formulated as follows:

(4-4) Monstrous believes_a is of type (c(ct))(et). [[believes_a]] =

λc ∈ D_c. λj ∈ D_c. λχ ∈ D_c(χ) ky ∈ D_c. ∀k ∈ C. k ∈ Dox_j[|a(j)/y|] → (δ(\(\chi(c)))(\chi)(k) = 1]

Let us see which truth-condition is predicted for the following sentence, if this word belonged to English:

(4-5) Kaplan believes my pants are on fire.

[[PRES Kaplan believes_a my pants are on fire]](c)(j) = 1

iff (by IFA)

[[PRES]](c)(j)λj.[[Kaplan believes_a PRES my pants are on fire]](c)(j)) = 1

iff (by meaning of PRES)

[[Kaplan believes_a PRES my pants are on fire]](c)(k) = 1, where k = j[t(j)/t(c)]

iff (by EFA)

14 Haas-Spohn uses a somewhat different diagonal operator. If χ is a sentence character, δχ = λc ∈ C. χ(c)(<w(c),t(c)>) = 1. We don't follow this proposal for technical reasons: we want to stay as close as possible to Kaplan's suggestion, that the 'content' should always be an intension. It is obvious that there is a one-to-one correspondence between Haas-Spohn's set of contexts and a property of individuals. Our proposal spells this out. A related proposal is made in (Stechow, 1984). For extensive discussion of different methods of diagonalisation, see (Zimmermann, 1991: 174ff ).
\[ \text{believes}_a \text{ PRES my pants are on fire} \equiv (\text{Kaplan}) = 1 \]

iff (by \text{MFA} + meaning of \text{Kaplan})

\[ \text{believes}_a \equiv (\lambda \cdot \lambda \cdot \lambda \cdot \text{PRES my pants are on fire})(\text{Kaplan}) = 1 \]

iff (by the meaning of \text{believes}_a)

\[ \forall l \in C. [l \in \text{Dox}(\text{Kaplan})] \rightarrow (\delta)(l) \rightarrow (\text{PRES my pants are on fire})(j)(j))(l) = 1 \]

iff (by the meaning of \delta)

\[ \forall l \in C. [l \in \text{Dox}(\text{Kaplan})] \rightarrow (\lambda j \in C. [\text{PRES my pants are on fire}])(j)(j))(l) = 1 \]

iff (by \text{\lambda-conversion})

\[ \forall l \in C. [l \in \text{Dox}(\text{Kaplan})] \rightarrow (\text{PRES my pants are on fire})(l) = 1 \]

iff (\text{IFA}, \text{EFA} + lexical meanings)

\[ \forall l \in C. [l \in \text{Dox}(\text{Kaplan})] \rightarrow a(l)'s \text{ pants are in fire on w(l) at t(l)} \]

iff (by definition of \text{k})

\[ \forall l \in C. [l \in \text{Dox}(\text{Kaplan}, w(j), t(c))] \rightarrow a(l)'s \text{ pants are on fire in w(l) at t(l)} \]

This can be paraphrased as: For all Kaplan believes in the index world w(j) at the time of the context t(c) he might be someone who is situated in a world and a time where his pants are on fire.

Clearly, the English sentence (4-1) doesn't mean this, and according to Kaplan, no natural language can have attitude predicates that have that meaning.\(^{15}\) The truth-condition would be satisfactory for direct discourse but not for indirect. The prohibition against monsters entails that the English verb \text{believes} cannot mean \text{\text{believes}_a} and the facts observed for English certainly support Kaplan's position.

5. **ATTITUDE REPORTS**

We will now make precise Kaplan's semantics for attitude reports sketched in section 2. Let us mark the verb \text{believe} of the object language with the subscript \text{r}. \text{believe}_r is an intensional functor whose argument is an egocentric proposition.

\(^{15}\) We take this to be an empirical claim; see however fn. 13 above for an alternative point of view. The claim has been challenged by a number of scholars, e.g., (Israel and Perry, 1996) and (Schlenker, 1999). (Stechow, 2003) argues that the semantics given by Schlenker can be reformulated within an intensional framework. Recently, Andrew Nevins informs us that Zazaki, a Kurdean language, has genuine monstrous attitude verbs. The sentence "John said he was there at that time" is spelled out as "John said that I was here now". It would seem then that Zazaki has a word \text{say}_a, which has a semantics similar to \text{believe}_a introduced in the text. Lela Samushia (p.c.) reports similar observations concerning indirect speech in Svan, spoken in North West Georgia.
(5-1) *Intensional believes*$_r$ is of type (ct)(et).$^{16}$

\[
[[\text{believes}_r]] = \lambda c \in D_c. \lambda j \in D_j. \lambda P \in D_{ct}. \lambda y \in D_e. \exists \chi \in D_{ct(1)} \exists j' \in D_c. j' = j[a(j)/y] \land \chi(j') = P \land [[\text{believes}_a]](c)(j)(\chi(y)).
\]

\[j[a(j)/y] = <y,w(j),t(j)>\] is like \(j\) with the possible exception that \(a(c) = y\); \(j[a(j)/y]\) plays the role of context for the subject of the attitude, but it need not be a context proper. Note that the meaning rule for *believes*$_r$ contains monstrous belief in the *definiens*, but the functor is purely intensional as evidenced by the logical type of the verb. An LF for the sentence (5-2) Kaplan believes that his pants are on fire.
is given by the following tree:

\[(5-3)\]

\[
\begin{array}{c}
TP \\
S
\end{array}
\begin{array}{c}
V \quad \lambda 1 \\
\lambda I
\end{array}
\begin{array}{c}
\lambda \exists \chi \exists j' \\
\ast
\end{array}
\begin{array}{c}
V_{ct(1)} \quad \text{believes}_r \\
\ast
\end{array}
\begin{array}{c}
\text{PRES} \\
\lambda 1 \\
\lambda I
\end{array}
\begin{array}{c}
Kaplan \\
\lambda I
\end{array}
\begin{array}{c}
\text{his pants are on fire} \\
\ast
\end{array}
\begin{array}{c}
\text{PRES} \\
\lambda 1
\end{array}
\begin{array}{c}
PRES \\
\lambda 1
\end{array}
\end{array}
\]

The subject Kaplan is QR-ed and thereby binds the possessive pronoun in the subordinate clause. The composition of believe$_r$ with its complement requires IFA, and nowhere do we need MFA in order to interpret the tree. The reader is invited to perform the necessary semantic computations and to convince herself that for a given pair \((c,j) \in D_c \times D_c\) we obtain the following result:

\[(5-4)\] $[[(5-3)]](c)(j) = 1$ iff $\exists j' \exists \chi[j' = j[a(j)/Kaplan] \land \chi(j')] = \lambda k \in D_c. [\text{Kaplan’s pants are on fire in } w(k) \text{ at } t(c)] \land [[\text{believes}_a]](c)(j)(\chi(Kaplan))].$

---

$^{16}$This rule has been improved thanks to an observation by Chris Tancredi.
iff \( \exists j' \exists \chi \[ j' = j[a(j)/Kaplan] \& \chi(j') = \lambda k \in D_c. [\text{Kaplan's pants are on fire in } w(k) \text{ at } t(c)] \]
\& \forall l \in D_c. [l \in \text{Dox}_c(\chi(j[a(j)/y]) \& \chi(j) = \lambda k \in D_c. [\text{Kaplan's pants are on fire in } w(k) \text{ at } t(c)] \& \forall l \in D_c. [l \in \text{Dox}_c(\chi(j[a(j)/y]) \& \chi(l) = \lambda k \in D_c. [\text{Kaplan's pants are on fire in } w(k) \text{ at } t(c)]]]}

The truth-condition makes it clear that the sentence doesn't tell us which character (or egocentric proposition) Kaplan actually believes. This information is lost. It might be the character expressed by "My pants are on fire", in which case Kaplan self-believes the property of having his pants on fire. Or it might be the character "The pants of the guy in the mirror are on fire", in which case Kaplan self-ascribes the property of seeing a man in the mirror that has burning pants. Precisely this loss of information makes it possible to have \textit{believes} as an intensional functor.

If you study the truth-condition, you see why the definition of \([\text{believes}_r]\) has to involve a quantification over the index that replaces the actual speaker by the subject of the sentence. Suppose we didn't do that but omitted the condition "\( \exists j' \in D_c. j' = j[a(j)/y] \ldots \)". In that case the subject Kaplan could not believe the character 'my pants are on fire' if the agent of \( j \) happens to be a different person, say Barbara. With respect to the unchanged index \( c \) the character would express the proposition 'Barbara's pants are on fire'. But we want the character to express the proposition 'Kaplans pants are on fire'. With respect to the changed index \( j[j(c)/Kaplan] \), this requirement is fulfilled.

Note that our semantics accounts for descriptions of attitudes \textit{de se} of the sort that have been discussed in (Lewis, 1979). As an example consider the case of mad Heimson:

(5-5) Heimson believes that he is David Hume.

The character \textit{believed}, by Heimson is perhaps 'I am David Hume'. In that case Heimson self-ascribes the property of being identical with David Hume. Similar considerations apply to the famous example of the two gods Jehovah and Zeus, who know every true proposition but fail to know who is who. So Kaplan’s theory seems to fare very well. It is compatible with a straightforward syntax and it seems to cover all the relevant examples of attitude that can be covered by competing theories.

6. KAPLAN TRIVIALISED?

If we adopt Kaplan's semantics for attitude reports in the above form, it turns out that the truth-conditions arising from operators like \textit{believes}, are far too weak. In fact, they amount to a material conditional between the subject’s sense of reality and the truth of the embedded clause. As a case in point, the truth condition given in (5-4) above boils down to:

(6-1) \([5-3]\) \( c(j) = 1 \) iff
\[ j[a(j)/Kaplan] \subseteq \text{Dox}_c(\chi(j[a(j)/y]) \& \chi(l) = \lambda k \in D_c. [\text{Kaplan's pants are on fire in } w(j) \text{ at } t(c)]]\]

Of course this is disastrous; it means that (5-3) is true in a context \( c \) if either Kaplan has some erroneous belief about the situation he is in, or his pants are on fire in that situation. In particular, Kaplan’s burning pants alone would make the attitude report (5-3) come out true. More generally, the semantics entails that Kaplan believes every proposition that is true at index\( j[j(c)/Kaplan]! \)

How come? The reason why (5-4) boils down to (6-1) is rather simple. According to
(5-4), the report (5-2) is true in a given context if one can find a character \( \chi \) meeting two conditions: (A1) the ‘horizontal’ proposition determined by \( \chi \) at the index of belief \( j[a(j)/\text{Kaplan}] \) must be the proposition \( \varphi \) that Kaplan’s pants are on fire (at the time of utterance); and (A2) the ‘diagonal’ proposition of \( \chi \), i.e. [the characteristic function of] the set of points \( k \) such that \( \chi(k)(k) = 1 \), must be true of all of Kaplan’s doxastic alternatives (at a certain point). Such \( \chi \) are all too easily found, especially if (as we may assume for ease of exposition) Kaplan’s pants are on fire in the world of the belief index at the time of utterance. In order to see this, one may conceive of t-characters \( \chi \) as infinite matrices where the columns represent indices and each row \( k \) represents the course of truth values of the (horizontal) proposition \( \chi(k) \) in the indices represented by the columns. Then, in order to find some \( \chi \) verifying (A1) and (A2) one may start out with an empty matrix and first fill in [the course of values of] the proposition \( \varphi \) in line \( j[a(j)/\text{Kaplan}] \); this can be done because \textit{tabula rasa est}. And, unless we revise any truth values during the construction, this will guarantee that \( \chi(j[a(j)/\text{Kaplan}]) = \varphi \), as required by (A1). We may then proceed and fill in 1s in the diagonal points \( (k,k) \), whenever \( k \) is any of Kaplan’s doxastic alternatives. Again this can be done, because after the first step all diagonal cells but one are still empty – and the one that is not is precisely the point \( j[a(j)/\text{Kaplan}] \), where the first step already put in a 1, due to our assumption that Kaplan’s pants are on fire in \( w(j) \) at \( t(c) \). Hence, whatever we do with the remaining cells of the matrix, the result of filling them up with truth values will meet condition (A2), as well as (A1).\(^{17}\)

Unfortunately, we cannot offer a foolproof way out of the embarrassment. Let us first discuss two (and a half) conceivable strategies of de-trivializing the above Kaplanian treatment of attitude reports. In order to motivate them, we will take a brief look at Kaplan’s approach to indirect discourse, which, as was already mentioned, is largely adequate and, in particular, is not prone to the above reasoning. Here is Kaplan’s original sketch of an account of reports of the form ‘\( x \) indirect-discourse –verb that … at \( t \)’:\(^{18}\)

(6-2) \( \exists c, C \ [c \text{ is a context } \wedge C \text{ is a character } \wedge x \text{ is the agent of } c \wedge x \text{ direct-discourse-verb } C \text{ at the time } t \text{ of } c \wedge \text{the content of } C \text{ in } c \text{ is that … } ] \)

Let us, for specificity, adopt the following analysis of the direct discourse verb \textit{say}:\(^ {19} \)

(6-3) \( \text{Monstrous } \textit{say}_a \text{ is of type } (c(ct))(et). \ [\textit{say}_a] = \lambda c \in D_c. \lambda j \in D_c. \lambda \chi \in D_{c(ct)}. \lambda y \in D_c. \exists S \ [S \text{ is a sentence in the language } L \text{ spoken by } y \text{ at } j \text{ & } \chi \text{ is the character of } S \text{ in } L \text{ & } x \text{ utters } S \text{ in } j] \)

\(^{17}\) A more detailed proof of the equivalence (6-1), and one that does not rely on the truth of \( \varphi \), can be found in the appendix.

\(^{18}\) (Kaplan, 1977: 554), Section XX [= (Kaplan, 1989), 554]. According to (6-2), an indirect speech report reports the exact content of the reported speech act. There may be a reading of English \textit{say} for which this kind of exactness is adequate. However, normally the indirect report is understood as specifying a contextual consequence of the original content, i.e. a proposition it implies, given some background knowledge. In this respect, then, (6-2) is not fully adequate, as was pointed out to us by Philippe Schlenker (crediting David Kaplan).

\(^{19}\) It may be noted that there is something impredicative about (6-3): although part of the definition of a character (of English), it quantifies over characters (of arbitrary languages, including English).
Using (6-2) and (6-3), the truth conditions of the indirect speech report (6-4), then, come out as in (6-5):

(6-4) Barbara says that David’s pants are on fire.

\[(6-5) \quad \text{[[} (6-4) \text{]]} (c)(j) = 1 \text{ iff } \exists \chi [ \chi(j(a(j)/\text{Partee})) = \lambda k \in D_c. [\text{Kaplan’s pants are on fire in } w(k) \text{ at } t(c)] \& \exists S [ S \text{ is a sentence in the language } L \text{ spoken by Barbara at } j \& \chi \text{ is the character of } S \text{ in } L \& \text{Barbara utters } S \text{ in } j]]\]

Like (5-4), (6-5) has the truth of the report depend on the existence of a character that (horizontally) determines the proposition \( \varphi \) expressed by the embedded clause. However, two differences strike the eye when comparing the Kaplanian treatments of attitude reports and indirect discourse:

- whereas the characters \( \chi \) satisfying the belief report (5-4) are arbitrary functions of type \((c(ct))\), (6-5) contains a restriction to the effect that \( \chi \) is expressible in the language spoken by the subject;
- whereas the (monstrous) attitude (4-4) underlying the belief report (5-3) only depends on the diagonals of the characters quantified over, the full characters are relevant for the (monstrous) direct discourse verb.

Given that Kaplan’s interpretation of indirect speech appears to be adequate, it is thus tempting to try and adapt it to the interpretation of attitude reports, thereby keeping one (or both) of the two above features that distinguish it from the approach of the previous section. The first possibility, then, would be to impose a restriction on the characters to which a subject can stand in a propositional attitude. The second option would be to treat attitudes as relations between subjects and characters that are not reducible to attitudes to the diagonals of these characters. (And, since the two strategies appear to be independent of each other, they may also be combined.) Note that both repair strategies, which we will briefly consider in turn, necessitate a revision of the analysis of attitudes per se. In other words, in order to exploit the analogy between indirect discourse and attitude reports, one would first have to tighten the analogy between discourse and attitudes.

Let us first discuss, and discard, the second possibility. If belief, or any related attitude, is a relation \( B \) between a subject and a character without being reducible to an attitude toward a corresponding diagonal intension, then there would have to be individuals \( x \) and characters \( \chi \) and \( \chi’ \) standing in \( B \) to \( \chi \) but not to \( \chi’ \) even though \( \delta(\chi) = \delta(\chi’) \). It is hard to see how this could be possible. The diagonal points are those that correspond to possible realizations of \( \chi \) (i.e. utterances of an expression that means \( \chi \)) and \( x’ \)’s standing in \( B \) to \( \chi \) therefore ought to amount to \( x’ \)’s believing \( \delta(\chi) \) in the obvious sense that the contexts in \( \delta(\chi) \) contain all those situations which, for all \( x \) believes (in the situation considered) [s]he may be in. For if \( \chi \) is the meaning of a sentence \( S \) of \( x’ \)’s language, then it appears that, ideally, \( x’ \) would be inclined to assent to \( S \) just under these circumstances, which is a strong indication of (de dicto) belief. Now, if \( \chi’ \) and \( \chi \) agree on their diagonals, then, by the same token, \( x \) would stand in the relation \( B \) to \( \chi’ \) just in

\[20\] This is, of course, Kripke’s (1979) disquotational principle.
case x stands in B to \( \chi \); for if \( S' \) expresses \( \chi' \), then x would be inclined to assent to \( S' \) once \( s \) he assents to \( S \), given that \( \delta(\chi') \) covers the situations being in which x does not exclude. It thus seems that, at least as far as belief and related attitudes to characters are concerned, the diagonal is all that matters. The second of the above differences between our account of attitude reports and the Kaplanian treatment of indirect speech therefore appears to be indispensable.

Let us now turn to the first of the above differences between indirect speech and attitude reports. If objects of attitudes are restricted in a way similar to the characters in (6-3), we need a modification of (4-4) along the following lines:

\[
\begin{align*}
(6-6) \quad \text{Monstrous believes}_a \text{ is of type } (c(ct))(et) \text{ [Revision One].} \\
\llbracket \text{believes}_a \rrbracket &= \lambda c \in D_c. \lambda j \in D_c. \lambda \chi \in D_{c(ct)}. \lambda y \in D_e. \\
& \quad (\Phi(\chi) \& \forall k \in D_e. [k \in \text{Dox}_j(a(j)y) \rightarrow [\llbracket \delta \rrbracket(c)(j)(\chi)(k) = 1])
\end{align*}
\]

In (6-6) we have merely added an unspecified (and underlined) condition \( \Phi \) on the object of belief \( \chi \). Of course the question is: what is \( \Phi \)? An answer is suggested by the above considerations on the role of the diagonal. Closer inspection of the argument reveals that we identified characters with meanings of the subject’s language. However, given standard cardinality assumptions on time and logical space, there are many more t-characters than sentences in the subject’s language. But if belief boils down to an attitude to the meaning of a sentence in the subject’s language, then \( \Phi \) ought to impose precisely this expressibility condition and we obtain:

\[
\begin{align*}
(6-7) \quad \text{Monstrous believes}_a \text{ is of type } (c(ct))(et) \text{ [Revision Two].} \\
\llbracket \text{believes}_a \rrbracket &= \lambda c \in D_c. \lambda j \in D_c. \lambda \chi \in D_{c(ct)}. \lambda y \in D_e. \exists S \text{ } S \text{ is a y’s language } L \text{ at } j \& \chi \text{ is the character of } S \text{ in } L \& \forall k \in D_e. [k \in \text{Dox}_j(a(j)y) \rightarrow [\llbracket \delta \rrbracket(c)(j)(\chi)(k) = 1])
\end{align*}
\]

If we now have our belief report construal (5-1) rest on this re-interpretation of \( \text{believes}_a \), then the above reasoning to trivialisation appears less cogent. For even though there may be many characters that determine the same proposition as the embedded clause, the ones whose diagonals cover the subject’s doxastic perspective need not be expressible in his or her language – or so it seemed to us until Philippe Schlenker (p.c.) pointed out that the truth condition of the belief report according to (6-7) is satisfied as long as the material implication between its ‘actualized’ and its ‘naked’ complement is expressible in the subject’s language.\(^{21}\)

Hence, continuing the above example, if Kaplan’s English contains (6-8) – where actually must be taken in the ‘technical’ sense expounded in (Kaplan, 1977) and adapted in (6-9), then the report (5-3) comes out true according to (6-7) as long as Kaplan’s pants are on fire:

\[
(6-8) \quad \text{It is either not so that Kaplan’s pants are actually on fire, or else Kaplan’s pants are on fire.}
\]

\[
(6-9) \quad \llbracket \text{actually} \rrbracket = \lambda c \in D_c. \lambda j \in D_c. \lambda \phi \in D_c. \phi(c) = 1
\]

To verify Schlenker’s observation, consider an arbitrary context \( c \) in which Kaplan’s pants are on fire and his English contains (6-8). Since, at any diagonal point, (6-8) boils down to a tertium non datur, its character \( \chi_S \) is a priori; a fortiori, \( \llbracket \delta \rrbracket(c)(j)(\chi_S)(k) = 1 \), for any \( k \in \) 21 Recently, Philippe Schlenker (crediting Isidora Stojanovic) pointed out to us that this observation has already been made in (Crimmins, 1998: 27, fn.20).
Dox_c(a(c)/Kaplan]. Hence, given its expressibility in his English, Kaplan stands to χ_8 in the relation [believes_p] (c(c), as defined in (6-7). However, since his pants are on fire in c, the left disjunct in (6-8) expresses a contradiction in c, and so: χ_8(c) = [Kaplan’s pants are on fire] (c) = [Kaplan’s pants are on fire] (c[a(c)/Kaplan]), which means that [believes_p] (c) (c) holds between Kaplan and [[Kaplan’s pants are on fire]](c). It follows that (5-3) is true in c.

Like the previous trivialization, Schlenker’s argument crucially depends on the fact that existential quantification over characters turns out to be surprisingly weak – there are far too many characters, even among the expressible ones. Hence a more restrictive version of (6-7) may still be viable. For instance, as Uli Sauerland (p.c.) suggested, one may try to add a ‘vividness’ restriction on the character argument of [believes_p], thus making a connection to Kaplan’s (1969)’s classical but non-compositional de re account, according to which descriptions denoting res are subject to a corresponding condition. While in the case at hand, some variety of vividness may be used to block the above reasoning – maybe (6-8) is too clumsy for Kaplan to entertain as a thought, or whatever – we are not positive that the danger of trivialization is going to be cast out by this restriction alone. But we will leave it at that, reserving any further explorations into these matters for future research.

One problem the present approach cannot solve is due to a conspiracy of coarse-grainedness, rigid designation, and the prohibition against monsters. As Ulli Haas-Spohn (p.c) pointed out, given that identity statements between names always express necessary propositions, embedding them under believe again trivializes our truth conditions: since practically any subject stands in the (direct) belief relation to the character expressed by some tautology of his or her language, the above treatment predicts that a subject that may be truthfully reported to believe that Ruth Rendell is Barbara Vine (which is true) may also be truthfully reported to believe that Müller-Thurgau is Riesling Sylvaner (which is also true). Similarly, as Philippe Schlenker (p.c.) remarked, someone who may be truthfully reported to believe that Müller-Thurgau is not Riesling Sylvaner stands in the direct belief relation to any character (expressible in his or her language) that expresses a contradiction and would therefore, according to our analysis, have to believe that Ruth Rendell is not Barbara Vine.22 However, since the argument turns on a substitution of intensionally equivalent clauses, it can only be escaped by giving up one of the above-mentioned three central tenets of the Kaplanian framework – propositions as sets, names as rigid designators, and no monsters – or by resorting to pragmatics. Again, we leave the final word on this matter for future research.

If anything like (6-7) is correct, then the truth of a belief report does not depend on the subject’s doxastic perspective alone but also on his or her ability to link this perspective to a (broadly) linguistic expression. For, according to (6-7), to believe something, i.e. to satisfy a given belief report, does not just amount to excluding certain possible situations but also involves the ability of putting this exclusion into words. In a way, then, our answer to Barbara Partee’s above-mentioned question of whether the objects of attitudes should be sentences or propositions is: both.

22 Versions of this argument have appeared in the literature; cf. (Schlenker, 2003), where Henk Zeevat (p.c) and (Zimmermann, 1991) are credited; the latter (ibid.: 227) credits (Stechow, 1984). Given the occasion, we refrain from making an obvious remark.
7. APPENDIX

We will show that the truth condition (A) of (5-4) (for some point of reference (c,j)) is equivalent to the material conditional (B):

(A) \( \exists k' \in D_c. [\text{Kaplan's pants are on fire in } w(k) \text{ at } t(c)] \land \forall l \in D_c. [l \in \text{Dox}_w(Kaplan,w(j),t(c)) \rightarrow \chi(l)(l) = 1] \)

(B) \[ j[c'/\text{Kaplan}] \in \text{Dox}_w(Kaplan,w(j),t(c)) \rightarrow \text{Kaplan's pants are on fire in } w(j) \text{ at } t(c) \]

The following abbreviations will help to render the formal proof more readable:

\( \varphi := \lambda k' \in D_c. [\text{Kaplan's pants are on fire in } w(k) \text{ at } t(c)] \)
\( \alpha := \lambda l \in D_c. l \in \text{Dox}_w(Kaplan,w(j),t(c)) \)
\( c^* := j[a(j)/\text{Kaplan}] \)

Hence \( \varphi \) is the proposition expressed by the embedded clause, \( \alpha \) is the strongest proposition believed by Kaplan, and \( c^* \) is the relevant belief index. Given this notation, (A) and (B) can be rewritten as:

(a) \[ \exists k' \chi(c^*) = \varphi \land \forall l \in D_c. [\alpha(l) = 1 \rightarrow \chi(l)(l) = 1] \]

(b) \[ [\alpha(c^*) = 1 \rightarrow \varphi(c) = 1] \]

Note that, since \( w(j) = w(c^*) \), (b) is the same as:

(b*) \[ [\alpha(c^*) = 1 \rightarrow \varphi(c^*) = 1] \]

We show that (a) \( \iff \) (b):

\( \implies \): Let \( \chi_0 \) satisfy the conditions in (a), i.e.:

(a1) \[ \chi_0(c^*) = \varphi \]

(a2) \[ \forall l \in D_c. [\alpha(l) = 1 \rightarrow \chi_0(l)(l) = 1] \]

Hence if \( \alpha(c^*) = 1, \chi_0(c^*)(c^*) = 1 \), by (a2). But then, by (a1): \( 1 = \chi_0(c^*)(c^*) = \varphi(c^*) \), which was to be shown.

\( \impliedby \): We assume that (b) is the case and distinguish two cases:

CASE 1: \( \alpha(c^*) = 0 \).

Then put (for arbitrary \( d \in D_c \)):

\( \chi_{1}(d) = \begin{cases} 
\varphi, & \text{if } d = c^* \\
\alpha, & \text{if } d \neq c^* 
\end{cases} \)

In particular:

(a1) \[ \chi_1(c^*) = \varphi \]

Moreover, if \( \alpha(l) = 1 \), then \( \chi_1(l)(l) = 1 \iff \alpha(l) = 1 \). In other words:

(a2) \[ \forall l \in D_c. [\alpha(l) = 1 \rightarrow \chi_1(l)(l) = 1] \]

Hence \( \chi_1 \) testifies (a).

CASE 2: \( \alpha(c^*) = 1 \).

Then, by (b*), we know that:

(b*) \( \varphi(c^*) = 1 \).

We may now put (for arbitrary \( d \in D_c \)):

\( \chi_2(d) = \begin{cases} 
\varphi, & \text{if } d = c^* \\
\alpha, & \text{if } d \neq c^* 
\end{cases} \)

In particular:

(a1) \[ \chi_2(c^*) = \varphi \]

Now suppose that \( \alpha(l) = 1 \). It remains to be shown that:
(a2⁺) \( \chi_2(l)(l) = 1 \).

If \( l = c^* \), then \( \chi_2(l)(l) = 1 \) iff \( \varphi(l) = 1 \), which is the case by \((b^+)^* \) above. Otherwise, \( \chi_2(l)(l) = 1 \) iff \( \alpha(l) = 1 \), which is what we have assumed. Hence \((a2^+)\) holds and \( \chi_2 \) testifies \((a)\).

**REFERENCES**

Russell, Bertrand. 1912. The Problems of Philosophy.


