# Finite-State Machines and Regular Languages

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## Some useful tasks involving language

- Find all phone numbers in a text, e.g., occurrences such as

  *When you call (614) 292-8833, you reach the fax machine.*

- Find multiple adjacent occurrences of the same word in a text, as in

  *I read the the book.*

- Determine the language of the following utterance: French or Polish?

  *Czy pasazer jadacy do Warszawy moze jechac przez Londyn?*



## More useful tasks involving language

- Look up the following words in a dictionary:

  *laughs, became, unidentifiable, Thatcherization*

- Determine the part-of-speech of words like the following, even if you can't find them in the dictionary:

  *conurbation, cadence, disproportionality, lyricism, parlance*

⇒ Such tasks can be addressed using so-called finite-state machines.

⇒ How can such machines be specified?



## Regular expressions

- A regular expression is a description of a set of strings, i.e., a language.
- They can be used to search for occurrences of these strings
- A variety of unix tools (grep, sed), editors (emacs), and programming languages (perl, python) incorporate regular expressions.
- Just like any other formalism, regular expressions as such have no linguistic contents, but they can be used to refer to linguistic units.

## The syntax of regular expressions (1)

Regular expressions consist of

- strings of characters: `c`, `A100`, `natural language`, `30 years!`
- disjunction:
  - ordinary disjunction: `devoured|ate`, `famil(y|ies)`
  - character classes: `[Tt]he`, `bec[oa]me`
  - ranges: `[A-Z]` (a capital letter)
- negation: `[^a]` (any symbol but a)
  `[^A-Z0-9]` (not an uppercase letter or number)



## The syntax of regular expressions (2)

- counters
  - optionality: `?`
    `colou?r`
  - any number of occurrences: `*` (Kleene star)
    `[0-9]* years`
  - at least one occurrence: `+`
    `[0-9]+ dollars`
- wildcard for any character: `.`
  `beg.n` for any character in between `beg` and `n`



## The syntax of regular expressions (3)

Operator precedence, from highest to lowest:

parentheses `()`

counters `* + ?`

character sequences

disjunction `|`

Note: The various unix tools and languages differ w.r.t. the exact syntax of the regular expressions they allow.



## Regular languages

How can the class of regular languages which is specified by regular expressions be characterized?

Let $\Sigma$ be the set of all symbols of the language, the alphabet, then:

1. $\{\}$ is a regular language
2. $\forall a \in \Sigma$: $\{a\}$ is a regular language
3. If $L_1$ and $L_2$ are regular languages, so are:
   (a) the concatenation of $L_1$ and $L_2$: $L_1 \cdot L_2 = \{xy | x \in L_1, y \in L_2\}$
   (b) the union of $L_1$ and $L_2$: $L_1 \cup L_2$
   (c) the Kleene closure of L: $L^* = L_0 \cup L_1 \cup L_2 \cup ...$ where $L_i$ is the language of all strings of length $i$.

## Properties of regular languages

The regular languages are closed under ($L_1$ and $L_2$ regular languages):

- concatenation: $L_1 \cdot L_2$
  set of strings with beginning in $L_1$ and continuation in $L_2$
- Kleene closure: $L_1^*$
  set of repeated concatenation of a string in $L_1$
- union: $L_1 \cup L_2$
  set of strings in $L_1$ or in $L_2$
- complementation: $\Sigma^* - L_1$
  set of all possible strings that are not in $L_1$
- difference: $L_1 - L_2$
  set of strings which are in $L_1$ but not in $L_2$



- intersection: $L_1 \cap L_2$
  set of strings in both $L_1$ and $L_2$
- reversal: $L_1^R$
  set of the reversal of all strings in $L_1$

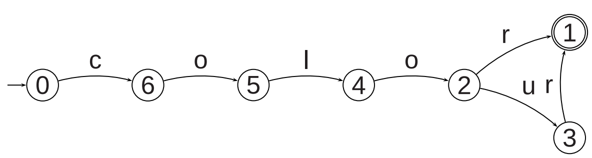


## Finite state machines

Finite state machines (or automata) (FSM, FSA) recognize or generate regular languages, exactly those specified by regular expressions.

Example:

- Regular expression: `colou?r`
- Finite state machine:






## Defining finite state automata

A **finite state automaton** is a quintuple $(Q, \Sigma, E, S, F)$ with

- $Q$ a finite set of states
- $\Sigma$ a finite set of symbols, the alphabet
- $S \subseteq Q$ the set of start states
- $F \subseteq Q$ the set of final states
- $E$ a set of edges $Q \times (\Sigma \cup \{\epsilon\}) \times Q$

  The **transition function** $d$ can be defined as

  $d(q, a) = \{q' \in Q | \exists (q, a, q') \in E\}$

## Language accepted by an FSA

The extended set of edges $\hat{E} \subseteq Q \times \Sigma^* \times Q$ is the smallest set such that

- $\forall (q, \sigma, q') \in E : \quad (q, \sigma, q') \in \hat{E}$
- $\forall (q_0, \sigma_1, q_1), (q_1, \sigma_2, q_2) \in \hat{E} : \quad (q_0, \sigma_1\sigma_2, q_2) \in \hat{E}$

The **language L(A) of a finite state automaton A** is defined as

$L(A) = \{w | q_s \in S, q_f \in F, (q_s, w, q_f) \in \hat{E}\}$



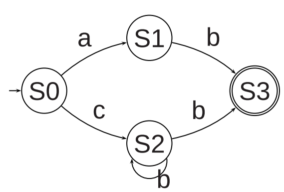
## Finite state transition networks (FSTN)

Finite state transition networks are graphical descriptions of finite state machines:

- nodes represent the states
  - start states are marked with a short arrow
  - final states are indicated by a double circle
- arcs represent the transitions



## Example for a finite state transition network




Regular expression specifying the language generated or accepted by the corresponding FSM: `ab|cb+`



## Finite state transition tables

Finite state transition tables are an alternative, textual way of describing finite state machines:

- the rows represent the states
  - start states are marked with a dot after their name
  - final states with a colon
- the columns represent the alphabet
- the fields in the table encode the transitions

## The example specified as finite state transition table

| | a | b | c | d |
|---|---|---|---|---|
| S0. | S1 | | S2 | |
| S1 | | S3: | | |
| S2 | | S2,S3: | | |
| S3: | | | | |



## Some properties of finite state machines

- Recognition problem can be solved in linear time (independent of the size of the automaton).
- There is an algorithm to transform each automaton into a unique equivalent automaton with the least number of states.



## Deterministic Finite State Automata

A finite state automaton is deterministic iff it has

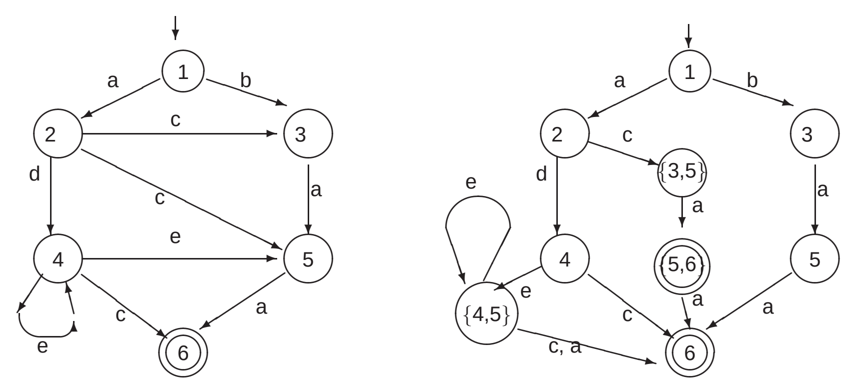
- no $\epsilon$ transitions and
- for each state and each symbol there is at most one applicable transition.

Every non-deterministic automaton can be transformed into a deterministic one:

- Define new states representing a disjunction of old states for each non-determinacy which arises.
- Define arcs for these states corresponding to each transition which is defined in the non-deterministic automaton for one of the disjuncts in the new state names.



## Example: Determinization of FSA



## From Automata to Transducers

Needed: mechanism to keep track of path taken

A **finite state transducer** is a 6-tuple $(Q, \Sigma_1, \Sigma_2, E, S, F)$ with

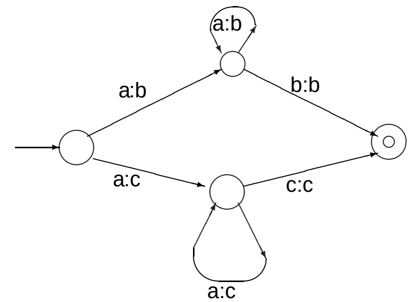
- $Q$ a finite set of states
- $\Sigma_1$ a finite set of symbols, the input alphabet
- $\Sigma_2$ a finite set of symbols, the output alphabet
- $S \subseteq Q$ the set of start states
- $F \subseteq Q$ the set of final states
- $E$ a set of edges $Q \times (\Sigma_1 \cup \{\epsilon\}) \times Q \times (\Sigma_2 \cup \{\epsilon\})$



## Transducers and determinization

A finite state transducer understood as consuming an input and producing an output cannot generally be determinized.

Example:






## Summary

- Notations for characterizing regular languages:
  - Regular expressions
  - Finite state transition networks
  - Finite state transition tables
- Finite state machines and regular languages: Definitions and some properties
- Finite state transducers



## Reading assignment 2

- Chapter 1 “Finite State Techniques” of course notes
- Chapter 2 “Regular expressions and automata” of Jurafsky and Martin (2000)