Sprache und Spieltheorie IV
Evolution von Farbkategorien

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Cognitive semantics

Gärdenfors (2000):

- meanings are arranged in **conceptual spaces**
- conceptual space has geometrical structure
- dimensions are founded in perception/cognition

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A subset C of a conceptual space is said to be *convex* if, for all points x and y in C, all points between x and y are also in C.

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Criterion P

A *natural property* is a convex region of a domain in a conceptual space.

Examples

- spatial dimensions: *above, below, in front of, behind, left, right, over, under, between ...*
- temporal dimension: *early, late, now, in 2005, after, ...*
- sensual dimenstions: *loud, faint, salty, light, dark, ...*
- abstract dimensions: *cheap, expensive, important, ...*

The naming game

- two players:
  - **S**ender
  - **R**eceiver
- infinite set of **M**eanings, arranged in a finite metrical space
  distance is measured by function $d : M^2 \mapsto R$
- finite set of **F**orms
- sequential game:
  1. nature picks out $m \in M$ according to some probability distribution p and reveals m to S
  2. S maps m to a form f and reveals f to R
  3. R maps f to a meaning m'

The naming game

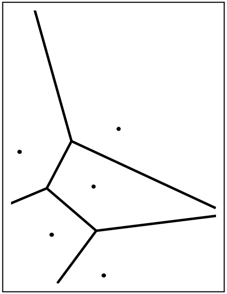
- **Goal:**
  - optimal communication
  - both want to minimize the distance between m and m'
- **Strategies:**
  - speaker: mapping S from M to F
  - hearer: mapping R from F to M
- **Average utility:** (identical for both players)

$$u(S,R) = \sum_m p_m \times \exp(-d(m, R(S(m)))^2)$$

vulgo: average similarity between speaker's meaning and hearer's meaning

Voronoi tesselations

- suppose R is given and known to the speaker: which speaker strategy would be the best response to it?
  - every form f has a "prototypical" interpretation: $R(f)$
  - for every meaning m: S's best choice is to choose the f that minimizes the distance between m and $R(f)$
  - optimal S thus induces a **partition** of the meaning space
  - Voronoi tesselation, induced by the range of R



Voronoi tesselation

Okabe et al. (1992) prove the following lemma (quoted from Gärdenfors 2000):

Lemma

The Voronoi tessellation based on a Euclidean metric always results in a partioning of the space into convex regions.

ESSs of the naming game

- best response of R to a given speaker strategy S not as easy to characterize
- general formula

$$R(f) = \arg\max_m \sum_{m' \in S^{-1}(f)} p_{m'} \times \exp(-d(m, m')^2)$$

- such a hearer strategy always exists
- linguistic interpretation: R maps every form f to the **prototype** of the property $S^{-1}(f)$

ESSs of the naming game

Lemma

In every ESS $\langle S, R\rangle$ *of the naming game, the partition that is induced by* S^{-1} *on* M *is the Voronoi tesselation induced by* $R[F]$*.*

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Lemma

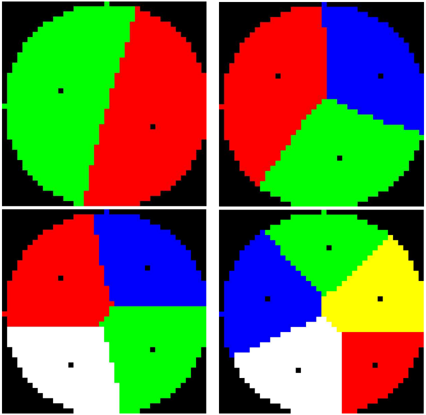
In every ESS $\langle S, R\rangle$ *of the naming game, the partition that is induced by* S^{-1} *on* M *is the Voronoi tesselation induced by* $R[F]$.

Theorem

For every form f, $S^{-1}(f)$ *is a convex region of* M.

Simulations

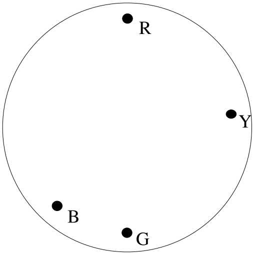
- two-dimensional circular meaning space
- discrete approximation
- uniform distribution over meanings
- initial stratgies are randomized
- update rule according to (discrete time version of) replicator dynamics



A toy example

- suppose
  - circular two-dimensional meaning space
  - four meanings are highly frequent
  - all other meanings are negligibly rare
- let's call the frequent meanings Red, Green, Blue and Yellow

$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$



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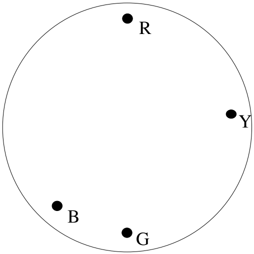
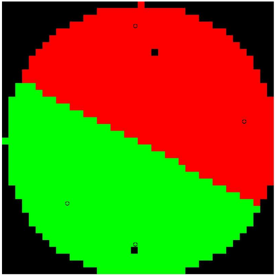


$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$

Yes, I made this up without empirical justification.

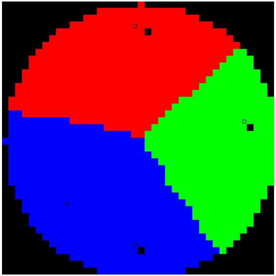
Two forms

- suppose there are just two forms
- only one Strict Nash equilibrium (up to permuation of the forms)
- induces the partition {**Red, Blue**}/{**Yellow, Green**}



Three forms

- if there are three forms
- two Strict Nash equilibria (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green, Blue} and {Green}/{Blue}/{Red, Yellow}
- only the former is **stochastically stable** (resistent against random noise)



Four forms

- if there are four forms
- one Strict Nash equilibrium (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green}/{Blue}