Mathematical and computational models of language evolution

Gerhard Jäger

Institute of Linguistics, Tübingen University

DGfS Summer School

August 20, 2013
The evolution of vowel spaces

- micro-variation in the inventory of vowels between languages: every language is different
- however, very strong tendencies:
  - most languages have five vowels
  - (almost) every language has [a], [i] and [u] like vowels
  - most vowel inventories are peripheral and symmetric etc.

- proposal (see for instance de Boer 2001):

  Vowel inventories must be evolutionarily stable!
What is a vowel?

Articulation

- speech sound
- voiced
- no constriction of the vowel tract
- vowel quality depends on
  - position of tongue
  - gesture of the lips
  - ...

Gerhard Jäger (UTübingen)

Language Evolution

8-20-2013
What is a vowel?

**Acoustics**
- periodic sonic wave

**Figure:** Amplitude of the vowel /u/
What is a vowel?

Acoustics

- spectral analysis:

**Figure:** Spectrogram of /a/-/e/-/i/-/o/-/u/
What is a vowel?

**Acoustics**
- Vowel is superposition of discrete harmonic waves:
  - Fundamental frequency
  - Formants

**Figure:** first five formants of /a-e-i-o-u/
What is a vowel?

**Acoustics**

- first two formants are crucial for identification of vowels

![Graph showing vowel spaces](image-url)
What is a vowel?

Acoustics

- more realistic picture:
Universal tendencies of vowel inventories

- comparison of vowel inventories in hundreds of languages reveals
  - virtually all languages use the vowels [a], [i], [u]
  - almost all vowels in all languages are peripheral
  - vowel inventories tend to be symmetrical
  - ...

Liljencrants and Lindblom (1972)

- vowel systems tend to maximize perceptual distance between vowels
- can be modeled as minimizing potential energy of a vowel system
- energy is proportional to sum of inverse squared distances
- fairly good typological predictions
Survey of 500+ vowel inventories

<table>
<thead>
<tr>
<th>number of vowels</th>
<th>vowel systems and their frequency of occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>4</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>5</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>6</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>7</td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td>8</td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>9</td>
<td><img src="image7" alt="Diagram" /></td>
</tr>
</tbody>
</table>

(from Schwartz et al. 1997, based on the UCLA Phonetic Segment Inventory Database)
Communication via the vowel space

Game theoretic model
- Signaling game
- types: between 3 and 9 vowel categories
- signals: each point within the two-dimensional (F1/F2) vowel space
The evolution of vowel spaces

Communication via the vowel space

One round of an evolutionary signaling game

- nature picks a vowel category \( v_S \) and shows it to \( S \)
- \( S \) picks a point \( p_{\text{intend}} \) in the vowel space
- a normally distributed random variable is added to \( p_{\text{intend}} \), yielding \( p_{\text{prod}} \)
- another normally distributed random variable is added to \( p_{\text{prod}} \), yielding \( p_{\text{perc}} \)
- \( R \) observes \( p_{\text{perc}} \) and picks a vowel category \( v_R \)
- if \( v_S = v_R \), both players score a point
Exemplar dynamics

- empiricist view on language processing/language structure
- popular in functional linguistics (esp. phonology and morphology) and in computational linguistics (aka. “memory-based”)

Basic idea

- large amounts of previously encountered instances ("exemplars") of linguemes are stored in memory
- very detailed representation of exemplars
- little abstract categorization
- similarity metric between exemplars
- new linguemes are processed in a similarity-based way
Exemplar dynamics: implementation

**Sender**
- chooses $p_{intend}$ at random from multiset $\{p|\langle v_S, p \rangle \in \text{memory}\}$
- if communication succeeds ($v_S = v_R$), oldest item in memory is replaced with $\langle v_S, p_{prod} \rangle$
- otherwise memory remains unchanged

**Receiver**
- $v_H$ is picked such that $\min\{d(p_{perc}, p)|\langle v_H, p \rangle \in \text{memory}\}$ is minimized
- if communication succeeds ($v_S = v_R$), oldest item in memory is replaced by $\langle v_R, p_{perc} \rangle$
- otherwise memory remains unchanged
Simulations

Setup
- population of 20 agents
- each agent has a memory of 4000 previous observations per vowel category (initialized with random values)
- 300k iterations of the signaling game
- sender and receiver are picked at random

Inspired by much more sophisticated simulations by Bart de Boer.
Simulation results

- black dots display average sender strategy for each agent and vowel category)
- colored dots display receiver strategies (colors represent vowel categories)
The evolution of vowel spaces

In detail

Gerhard Jäger (UTübingen)
The evolution of vowel spaces

In detail

![Diagram of vowel spaces with different stages of development.](image)
The evolution of vowel spaces

In detail

| 5 | 97 | 3 |   |   |

Gerhard Jäger (UTübingen)
In detail

The evolution of vowel spaces
The evolution of vowel spaces

In detail

Gerhard Jäger (UTübingen)
In detail
In detail

The evolution of vowel spaces

Gerhard Jäger (UTübingen)
Evaluation

- more than half of the typologically dominant patterns correspond to (experimentally determined) ESSs (150 out of 264 in the database)
- five out of seven ESSs correspond to empirically attested vowel systems
- even the two outliers look natural (symmetric systems with peripheral prototypes)
The evolution of vowel spaces

Theoretical considerations

ESS under replicator dynamics: strict Nash equilibria

- sender strategy: mapping from vowel categories to points in the vowel space
- receiver strategy: categorization of points
Voronoi tesselations

- suppose receiver strategy $R$ is given and known to the sender: which sender strategy would be the best response to it?
  - every signal $p$ has a “prototypical” interpretation: $R(p)$
  - for every vowel category $v$: S’s best choice is to choose the $p$ that minimizes the distance between $p$ and $R(p)$
  - optimal $S$ thus induces a partition of the meaning space
- Voronoi tesselation, induced by the range of $R$
Open question

- numeric calculation of the ESSs for the human vowel space
- Exemplar Dynamics is similar but not identical to replicator dynamics
- conjecture: as the variance of the random variables goes to 0, the attractor states of the exemplar dynamics converges towards SNEs
