Lecture Notes in Semantics
A Gentle Introduction to a Logically Grounded Analysis of Meaning

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*The original lecture notes were written in German by the first author and can be downloaded from www2.sfs.uni-tuebingen.de/sternefeld/Downloads/Ede_Semantik1_WS00-01.pdf. The present text has been modified, shortened, extended, and translated into English by the second author. For ease of comparison I sometimes added German translations in brackets. Style and exposition could further be improved, still awaiting the help of a native speaker of English. The pointing finger ☞ that occasionally accompanies proper names or technical terms is a request to look up the highlighted keyword in wikipedia (preferably the German version, which is much better than the English one), which will provide for invaluable background information that should not be ignored by any serious student of semantics.
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1 Literal Meaning (Wörtliche Bedeutung)

The subject of **semantics** is the systematic study of the **meaning** of linguistic expressions like morphemes, words, phrases, sentences or even texts.

Before we can start, however, we will have to narrow down the object of our investigation. The reason for this is that not everything that can be said about our understanding of words or utterances is relevant for a theory of **meaning**. Rather, we will only be interested in that part of “meaning” of a linguistic item that is associated with it by virtue of certain **linguistic conventions** of a specific type — this is what we will be calling the **literal meaning** of an expressions.

1.1 Hidden Sense (Verborgener Sinn)

Humans understand utterances automatically, immediately, effortlessly, and without explicitly thinking about meaning or about what they are doing when understanding language. Rarely are we forced to consciously reflect on meaning in a systematic way; sometimes such a situation arises when being concerned with the “interpretation” of literary texts, e.g. **poems** or **lyrics**. Here is a case in point:

(1)  And in-

<table>
<thead>
<tr>
<th>Schwerer Päonienduft</th>
<th>The heavy scent of peonies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Von fern</td>
<td>From far away</td>
</tr>
<tr>
<td>Le Ta</td>
<td>Le Ta</td>
</tr>
<tr>
<td>Gatte und Kind</td>
<td>Spouse and child</td>
</tr>
<tr>
<td>Verlassen</td>
<td>lonesome</td>
</tr>
<tr>
<td>Wenn der Schwan ruft</td>
<td>When the swan calls</td>
</tr>
<tr>
<td>Tusche von Meisterhand</td>
<td>A print by a master</td>
</tr>
<tr>
<td>Im Schnee</td>
<td>In the snow</td>
</tr>
<tr>
<td>Mädchen</td>
<td>Girl</td>
</tr>
<tr>
<td>Deiner Geburt</td>
<td>Of your birth</td>
</tr>
<tr>
<td>Erinnern</td>
<td>Remembering</td>
</tr>
<tr>
<td>Schriftzeichen im Sand</td>
<td>Writing in the sand</td>
</tr>
</tbody>
</table>

Indeed, an obvious question concerning these lines raise is: What do they mean? We clearly have to **interpret** the above lines in order to make sense out of them (and we implicitly assume that **some** sense **can** be made out of all this.)

The term “interpretation”, understood in this way, means that we unearth some **hidden meaning** that is not at all obvious to anyone confronted with (1). Given that Le Ta has no obvious meaning (its not an expression we could find in any
dictionary), how are we to interpret it? Is it a proper name? Perhaps, after all, this seems plausible. But what about the other terms that do have a plain meaning? What about the connections between these words? How does all this make sense, and, if it does, what is the hidden meaning of these words that seem to contribute to an additional sense yet to be discovered? This is the kind of question literary criticism is concerned with.

The above poem is is taken from Klaus Döhme: Leda & Variationen (Trier 1978). There we also find the following contribution to an “interpretation” of the poem:

(2) The redundancy-purged denseness of the Middle Chinese bi-stanza Ritor-nello (I Shing Min) with the classical AXXXA rhyme scheme endows this archetypal mythotope preeminently with its lyricalalness par excellence [Thanks to Janina Rado for the translation.]

(“In der redundanzfeindlichen Dichte des mittelchinesischen Doppelstrophen-Ritonells (I Shing Min) mit dem klassischen Reimschema AXXXXA gewinnt jenes archetypische Mythotop katexochen seine Lyrizität par excellence.”)

Whether this comment improves our understanding of the poem is doubtful: the commentary in (2) is at least as difficult to understand as the poem itself. At any rate, there is no need to bother about these questions too much here: both the poem and its interpretation are spoofs! The intention is precisely to create some feeling of hidden meaning, although the author only mimics a certain style that pretends to be deep and meaningful (see Klaus Döhme: Leda & Variationen, Trier 1978) for details).¹

Fortunately, in semantics we are not interested in hidden meaning, but only in the ostensible, primary meaning which is what the poem literally says. But even this is not easy to find out in the case at hand. One problem is syntactic:

¹One of the most famous real-life examples of a poem crying out for additional sense creation is the following one by William Carlos Williams (1923):

(i) so much depends
    upon
    a red wheel
    barrow
    glazed with rain
    water
    beside the white
    chickens

You’ll find thousands of internet pages in search of a hidden meaning (☞ The Red Wheelbarrow).
How can we identify the sentences or phrases in (1), and what shall we make out of incomplete sentences and incomplete phrases? Can we be sure that (3) is a sentence of (2):

(3) Deiner Geburt erinnern Schriftzeichen im Sand.

And can this be interpreted as an old-fashioned way of saying the same as (4)?

(4) An deines Geburt erinnern Schriftzeichen im Sand of your birth remind characters in the sand

If this interpretation is correct, everyone can understand its literal meaning, namely that there are signs in the sand that are reminiscent of your birth. That’s all. Nonetheless many details may still remain unclear: What does “your/deine” refer to (the reader of the poem?), how does this sentence relate to the meaning of the whole (and to the intentions of the author), are the signs scratched into the sand or are they mere shadows? All this does not belong to the literal meaning of the sentence.

The general point to be illustrated here is that lyrics or poems seem to bear some surplus meaning not contained in the literal meaning of the words. This extra sense is the topic of literary studies, which is in search of meaning behind the scene — which might be interesting enough, but fortunately it’s not what we are doing in semantics. Semanticists are primarily concerned with aspects of the literal meaning of words, phrases, and sentences: There are some signs or characters, there is some sand, there is an addressee referred to by “your” etc. Although literal meaning can be quite unhelpful in the context of poetry, this does not bother us in semantics. In semantics, we aim low and are content with dealing with the obvious only.

Now, compared with the discovery of hidden meaning, the description of literal meaning seems to be a thoroughly boring enterprise that does not deserve any scientific occupation. Given that anyone can understand literal meaning in an effortless way, why should scientists care for (literal) meaning? Is it worth the effort to study something that is grasped by anyone without the least effort?

The answer is that, although understanding utterances proceeds automatically and effortlessly, we still have no explanation for why and how this is possible at all. To mention an analogy from human perception: When hearing a noise we can often identify the direction of its source. However, how this can be achieved by the human organism is quite far from trivial and has not been understood until recently (for more information, google “Räumliches Hören”, Ackern, Lindenberg). This kind of ignorance also holds for almost any aspect of human cognition: we have no idea how exactly the mind or brain works, and it’s only recently that
aspects of the working of human perception have been explained by reference to
certain neuro-physiological mechanisms.

Consequently, there is something to be explained if we want to understand
why and how humans can succeed in understanding phrases and sentences, in
particular sentences they might have never heard before. This is one of the central
topics in linguistic theorizing. In fact, recall that in syntax one of the basic issues
was “recursiveness”, namely:

(5) **Foundational research question in syntax:**
How comes that we can, at least in principle, decide for arbitrarily long
sentences (which we might have never heard before), whether or not they
are syntactically well-formed?

Now in semantics we may ask the parallel question:

(6) **Foundational research question in semantics:**
Given our restriction on literal meaning, how come that we can understand
arbitrarily long sentences we have never encountered before, and, in par-
ticular, that we can tell whether or not they make sense (are semantically
well-formed)?

1.2 Irony and Implicature (Ironie und Implikatur)

Before we can embark on such an endeavor, let us explain more precisely our
understanding of literal meaning. Suppose Fritz is leaving the Mensa and meets
his friend Uwe who is asking about the quality of the meal. Then Fritz says:

(7) **Das Steak war wie immer zart und saftig**
the steak was as always tender and juicy

Now, according to the literal meaning, the quality of the food should have been
excellent. But this is not the intended message: rather, Fritz wants to convey that
the steak was as it always is, namely neither tender nor juicy. And Uwe, his friend,
easily understands the message conveyed by (7). How does that happen?

As a prerequisite for such an understanding it is absolutely necessary for Uwe
to first understand the literal meaning. Knowing his friend and the usual quality
of the food in the Mensa very well (Geschmacksverirrung), and having no evidence
for a sudden lapse of taste on Uwe’s part, he also knows that the literal meaning
cannot possibly be the intended meaning. Besides, Uwe might detect a waggish
expression on Fritzen’s face. He therefore legitimately concludes that the utter-
ance is meant in an ironic manner (☞Irony). And this implies that the conveyed
meaning is exactly the opposite of the literal meaning. In order for this to work
properly it is necessary for the literal meaning to come first: only on the basis of
an understanding of the literal meaning is it possible to understand the utterance
as saying the opposite of the literal meaning.

NB: In classical rhetoric, irony is always defined as expressing the opposite
of the literal meaning. In ordinary language, however, the term *irony* is used in
a much broader sense. Suppose Fritz continues his description of the menu by
saying:

(8) Auch der Nachtisch war nicht giftig
    Also the dessert was not poisonous

Although this utterance can be called ironic, the term *irony* in its traditional nar-
row sense is not adequate in this case, because Fritz does not want to say the
exact opposite of (8), namely, that the dessert was poisonous. Nor does he want
to convey the literal meaning, namely that the quality of the dessert is such that
one is not in danger of being poisoned. Rather, what he wants to say is something
like:

(9) The quality of the dessert cannot be categorized as much better than not
    poisonous.

Which implies that it is very bad.

In linguistics, this is called an instance of an *implicature*. An implicature is
something that goes beyond the literal meaning, but cannot contradict the literal
meaning. The above implicature is of a special type; it is called *scalar* (germ.
*skalare Implikatur*) because the conveyed meaning involves a scale of grades; in
this case a scale that characterizes the edibility of food, ranging from deathly to
three Michelin stars. “not poisonous” seems to range somewhere in the lowest
range of the scale.

What we see from these examples is that the literal meaning often does not
suffice to really understand an utterance; it must be augmented in some way
or other. How this is done is explained in pragmatics, which is concerned with
systematic aspects of the use of linguistic forms.
Within semantics we stick to the literal meaning, which is, as we have seen, a prerequisite for a full understanding of an utterance.

1.3 The Way You Say It (Der Ton macht die Musik)

We have seen above that the intended effects of an utterance may go far beyond its literal meaning:

- Numerous texts (in particular literary ones) exhibit a hidden meaning that reveals itself only to an educated person
- Rhetorical effects like irony, exaggeration or scalar implicatures can reverse, augment or modify the literal meaning
- A certain choice of words or a stylistic register can express the speaker's attitude, over and above the literal content of the word

As an example for the last point, imagine that the manager of the Studentenwerk is interviewed by a journalist from a student’s journal, the Campus Courier. The junior editor was supposed to ask something like:

(11) Planen Sie tatsächlich eine Anhebung der Essenspreise?
Are you really planning to raise the meal prices?

Now, what the journalist actually utters is this:

(12) Willst Du allen Ernstes für den Fraß noch mehr Zaster verlangen?
(Fraß = coll. food; Zaster = coll. money)
Are you serious about demanding even more dough for your grub?

There are of course several features of (12) that render this utterance inappropriate (can you describe them?). But thinking about the literal meaning of (12) will reveal that by and large its relevant content is in fact the same as that of (11).

That is, both questions "mean" more or less the same. But in what sense of more or less? This again is a topic that is dealt with in pragmatics. It's not what you say, it's the way you say it that is relevant for pragmatics. From the viewpoint of linguistic semantics we may say that the literal meaning of both sentences is almost identical, and that small differences can be neglected. Nonetheless the expressions used in (11) and (12) have different connotations. Although we may refer to the same kind of thing with two different expression, the connotations of these expressions may differ (cf. connotation (usage) for a definition; dt.: Konnotation im Sinne von Nebenbedeutung.)
1.4 Figures of Speech (Sprachliche Bilder)

Another case of non-literal meaning is exemplified by so-called metaphors.

HOMEWORK: Browse the internet for definitions of the term „metaphor“. What is the difference between metaphoric, ironic, and idiomatic use of expressions? Consider the meaning of the adjectives in (13). Is the use of the adjectives metaphoric, ironic, or idiomatic? Should the extra meaning of these expressions be listed in a good dictionary of German? (And are they indeed in yours?)

(13) schreiende Farben (jazzy colours), purzelnde Preise (falling prices), schlagende Argumente (telling arguments)

(For a more thorough discussion, see the German version of this text.)

1.5 Difficult Sentences (Schwierige Sätze)

Above we claimed the understanding of the literal meaning to proceed automatically, unconsciously and effortlessly, similar to other acoustic, visual or sensual perception—but unlike the understanding of hidden sense. However, although in practice this seems to be true, we might come across sentences whose meaning is difficult to decipher, even when considering only the literary meaning of the words they contain. Consider eg. run-on sentences like:

(14) Die Frau, deren Schwester, deren Sohn, dessen Freundin in Frankreich studiert, nach Australien ausgewandert ist, in Italien lebt, wohnt nebenan.
    (the woman, whose sister, whose girl friend studies in France, emigrated to Australia, resides in Italy, lives next door)

After a while, having parsed its syntactic structure, we may find out that it means the same as:

(15) Die in Italien lebende Schwester der Frau nebenan hat einen Sohn, dessen Freundin in Frankreich studiert und der selbst nach Australien ausgewandert ist.

This sentence is not much longer than the original one, so the problem is not length. Rather it is the kind of construction that makes the sentence incomprehensible (without a tedious linguistic analysis).²

²In the above case, one might say that the problem is not really one of semantics, but that the complexity already arises with the syntactic parsing of (14), leading to a kind of recursive self-embedding structure that is difficult to parse syntactically. On the other hand, since syntactic
Apart from constructional complexities as exemplified in (14), there might be other reasons that make it difficult to grasp the literal meaning of a sentence. The American classical scholar (Altphilologe) Moses Hadas once started a book review with the following sentence:

(16) **This book fills a much-needed gap**

Dieses Buch füllt eine bitter benötigte Lücke

That a book fills a gap is normally understood as something positive and it is this positive expectation that drives our interpretation of the sentence. Moreover, the expression *much needed* is normally understood as something positive as well, except that — in this particular case — *much needed* is not attributed to the book but to a gap, that is, to the non-existence of the book. So the literal meaning of the sentence is that we do not need the book but the gap. In fact, the review is totally devastating. (Other memorable and facetious quotes of Moses Hadas include: “I have read your book and much like it.” and “Thank you for sending me a copy of your book. I'll waste no time reading it.”)

A more complex case of semantic processing difficulty is exemplified by:

(17) **No head injury is too trivial to ignore**

Keine Kopfverletzung ist zu trivial um ignoriert zu werden

At first hearing this sentence seems to say that we shouldn't trifle with brain injuries. But in reality, analysing the literal meaning, we may discover that the proposition made is very cynical, namely that any brain injury should be ignored! In order to see this, compare (17) with:

(18) **No beverage is too cold to drink**

Kein Getränk ist zu kalt, um getrunken zu werden

parsing also normally proceeds unconsciously and quickly, the memory overload that may cause the difficulty in (14) might not only involve syntax, but probably semantics as well. The two go hand in hand, and a priori it is not clear whether the difficulty should be located in syntax or in semantics. To make a point in favor of semantics, one might consider the following self-embedding structures:

(i) a. The woman [ the man [ the host knew —] brought — ] left early
b. The woman [ someone [ I knew —] brought — ] left early

In (b), we replaced *the man* with *someone*, and *the host* with *I*. Intuitively, (i-b) is easier to parse than (i-a) although the two sentences presumably have identical syntactic structures. The difference could then somehow be related to semantics. This implies that semantics does play a role in calculating the complexity of (i), although perhaps syntax and prosody may still be involved as additional factors that may influence the comprehensability of the construction.
Now, a beverage that is too cold to drink is one that should not be drunk, and accordingly, (18) says that

(19) Any beverage — as cold as it may be — can be drunk

But now, by analogy, (17) means the same as:

(20) Any head injury — as harmless as it may be — can be ignored

The message that should be conveyed by the original sentence seems to be that even harmless injuries have to be taken particularly seriously and should not be ignored. But thinking about it and taking into account the analogy between (19) and (20), you will find out that this is just the opposite of the literal meaning!³

A nice example I heard in an interview after a soccer game Germany vs. Japan (the qualification for the world championship 2006 where Germany lost) is this:

(21) Wir haben in der zweiten Halbzeit die letzte Konsequenz zu wenig vermissten lassen

In the second half-time we didn’t miss the ultimate determination enough

It’s easy to locate the mistake (a slip of the tongue), but it’s hard to calculate the literal meaning!

So what we have learned in this section is that the normal, unconscious understanding of such sentences might go wrong in various way: We may be mistaken about the literal meaning of a sentence. This proves that the literal meaning can

³In the blog http://semantics-online.org/2004/01/no-head-injury-is-too-trivial-to-ignore, Mark Liberman adds on to this a naturally occurring example he found:

(i) I challenge anyone to refute that the company is not the most efficient producer in North America.

Mark asks “Is this a case where the force of the sentence is logically the same with or without the extra not? Or did Mr. Duffy just get confused?”

I would certainly lean towards the latter explanation. But it’s quite well-known that it is hard not to be confused. The coolest case I know is [(17)] I believe it was brought into the literature by Wason and Reich:


It was supposedly found on the wall of a London hospital. Actually, a Google search suggests that the ultimate source of the quote is Hippocrates (460–377 BC). By the way, a number of the Google hits seem to come from sites run by injury lawyers. Also by the way, the full quote appears to be “No head injury is too severe to despair of, nor too trivial to ignore”, which is even more mind-boggling, at least for my poor little brain.
be detected and analysed in a systematic way without recourse to mere intuition (which, as we have seen, can be misleading); there is something systematic in the way meaning is built up that needs to be analysed and explained. This is what semanticists do. They try to build up meaning in a systematic fashion so that the exact content reveals itself in a way that is predicted by the semantic theory, not by intuition alone.  

2 Lexical Semantics (Lexikalische Semantik)

Linguistic expressions, as long as they may be, always consists of (structured sequences of) single words. Therefore it seems natural to start off an investigation of literal meaning with the study of word meaning (as opposed to the meaning of phrases or sentences). Using linguistic terminology, the entirety of words of a specific language is called a lexicon, therefore the investigation of word meaning is often called «lexical semantics». That's the topic of this section.

Let us start with a simple, but potentially confusing question:

2.1 What's in a word? (Was ist eigentlich ein Wort?)

You might remember from the syntax/morphology section (and in particular the discussion of compounding) that this question is far from trivial, and there is no general answer to it. This can be illustrated by the fact that speakers of German and English may have different intuitions about whether a string X + Y (like linguistics department) is to be analysed as one word or as two. Speakers of German normally seem to have firmer intuitions about compounds X+Y, because compounds are written without a blank between X and Y. This is not always the case in English, therefore speakers of English have less firm intuitions about words. However, the discussion of German orthography has revealed that there may very well be borderline cases (eg. we may ask ourselves: are fallen+lassen, Rad+fahren or liegen+bleiben one word or two?).

Another issue related to the question of wordhood is this: Is a single sequence of phonemes the realization of one or two words? Consider the following examples:

(1)  
   a. German: /razen/, written as Rasen; English: meadow  
   b. German: /razen/; written as rasen; English: rage

Perhaps the beginner should not be too optimistic in expecting to be able to analyse these sentences semantically (by the end of the course). The semantics of the above quoted sentences is difficult and even remains so for the trained semanticist. It is only by specializing in certain kinds of constructions that one may be able to analyze them, perhaps at the end of your academic studies in semantics.
Now, if a word consists only of a sequence of phonemes, (a) and (b) illustrate the same word. But this of course is absurd! Clearly, (a) and (b) contain different words. Although the pronunciation is identical, we (fortunately) still have two different spellings, and this is clearly indicative for two different words (which, as it happens, also belong to two different syntactic categories: Rasen is a noun, and rasen a verb). The same difference in syntactic category can be observed in the English examples in (2):

(2) a. light = not heavy vs. illumination
    b. rose = a flower vs. past tense of rise
    c. left = opposite of right vs. past tense of leave

But now consider the following German examples:

(3) a. German: Bank₁ (plural = Banken) = bank(ing house),
    German: Bank₂ (plural = Bänke) = bench
    b. German: Schloss = castle vs. lock

These words do not differ in syntactic category, but still in meaning. For each of these words you will find two lexical entries in your dictionary, and therefore we feel entitled to conclude that these are two different words. For (3-a) this becomes apparent by looking at the different plural forms of Bank. As illustrated in (3-b), however, it may happen that we do not find any grammatical difference between two words at all, except meaning. Thus, the two words Schloss₁ and Schloss₂ have the same syntactic category, the same gender (neutral), and the same inflection, though different meanings. In this case one often says that the word Schloss has two meanings. Saying so implies that there is only one word, whereas above we insisted that Schloss is not one word, but two. This is of course a pure matter of terminology. If we understand the term “word” as including meaning, then we have two words; if by “word” we understand only its form (with the exclusion of meaning), there is only one word. Unfortunate the difference is mostly neglected in everyday talk.

In these lectures we prefer to include meaning, so that difference in meaning suffices for there to be two words with one spelling. Different words with the same spelling are called homographs; different words with the same pronunciation are called Homophones. Note that homophones may differ in spelling:

(4) Homophones, also called Homonyms:
    a. four vs. for
    b. break vs. brake,
    c. … see the list in Wikipedia
And homographs may differ in pronunciation, cf.

(5) homographs, also called heteronyms or heterophones:

*desert* (to abandon; with stress on the second syllable) vs. *desert* (arid region; with stress on first syllable)

### 2.2 Ambiguity and Polysemy

The words discussed in the last section have one thing in common: they differ in meaning and thereby illustrate what is often called ambiguity. Quoting from *Ambiguity (Linguistic forms)*:

> Lexical ambiguity arises when context is insufficient to determine the sense of a single word that has more than one meaning. For example, the word “bank” has several distinct definitions, including “financial institution” and “edge of a river,” but if someone says “I deposited 100 dollar in the bank,” most people would not think you used a shovel to dig in the mud. The word “run” has 130 ambiguous definitions in some lexicons. “Biweekly” can mean “fortnightly” (once every two weeks - 26 times a year), OR “twice a week” (104 times a year).

Note that in this definition a single (sic!) word is assumed to have more than one meaning. Above, however, we argued that there are two words *bank*₁ and *bank*₂ which happen to have the same pronunciation. As noted above, this is a matter of terminology only; but it seems to me that our terminology is more precise. In linguistic texts, we use indices, eg. *bank*₁ and *bank*₂ as a sign to indicate ambiguity, but in normal speech the use of indeces is out of the question. Therefore, in simple texts, the less precise notion seems to be preferred.

Apart from this, there is yet another peculiarity in the quote above that might bother us: the assumption that ambiguity has to do with the context of an utterance seems to be misguided. Lexical ambiguity does not only arise when the

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5 Quoted from Wikipedia: “There is considerable confusion and contradiction in published sources about the distinction between homonyms, homographs, homophones and heteronyms.” See [Homonym](#) for details and discussion. As a rich source for homonyms in German we recommend: *Bilden Sie mal einen Satz mit … 555 Ergebnisse eines Dichterwettstreits*. Hrsg. von Robert Gernhardt und Klaus Cäsar Zehrer. Fischer Verlag 2007.

6 Interestingly, still further (and totally different) criteria have been used to define the notion word; see Di Sciullo and Williams (1987) for a thorough discussion. These different criteria also seem to play a role in the ongoing discussion of the “New German Orthography”; see Jacobs (2005).
context of use is insufficient to decide between different meanings: one can easily imagine that there is never any kind of misunderstanding in the use of bank\textsubscript{1} and bank\textsubscript{2}, so that in every single utterance of one of them it is clear (and unambiguously determined by the context!) which meaning is intended. Even then we would still say that the sequence written as bank is lexically ambiguous. The problem with the above quote is that it cannot serve as a definition of the term lexical ambiguity; rather it may serve as a kind of illustration: Of course, there might be contextually and referentially ambiguous cases like (6):

(6) Give me the glasses!

Imagine a particular situation with two wine glasses on a table and a pair of spectacles. Then, it might still be unclear whether glasses is the plural of (wine) glass, or whether we mean (eye)glasses, i.e., spectacles. If the ambiguity has not been resolved I would not know what to bring; but fortunately the circumstances allow for the disambiguation of an ambiguity.

The above example points to another difficulty. Translating the sentence (6) into German, we would have to decide between two terms: Gläser and Brille, the latter being the term for eyeglasses. Therefore one might be entitled to conclude that there is an ambiguity. However, without this criterion, we would be less sure. Indeed, there are archaic dialects of German that would permit for the same sort of use, so that Gläser could also mean Augengläser. Can we still say that there is a real ambiguity involved here? After all, as spectacles are also made out of glass, one might say that the term Gläser is not ambiguous, rather it is underspecified with respect to the kind of glasses that is intended.

So in many cases it holds that the different meanings are somehow related to each other, or are very similar, so that there there seems to be some vagueness involved. Therefore linguists have strived to develop criteria that ideally should decide whether two terms are ambiguous. We will only discuss one of them here.

(7) At the end of the day we had to deplore that John destroyed glasses and Bill too destroyed glasses

This sentence seems to be okay even in the case where glasses may have the two different interpretations:

(8) At the end of the day we had to deplore that John destroyed glasses\textsubscript{1} and Bill too destroyed glasses\textsubscript{2}

But now, we may ask whether we can conclude from (8) that

(9) Bill and John destroyed glasses

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Can (9) be used to express the same as (8)? This seems hardly possible, and the reason for this inability seems to be that *glasses* is indeed ambiguous\(^7\).

Another potential case of ambiguity is illustrated in (10):

\begin{align*}
(10) & \quad a. \quad \text{Er geht noch zur Schule (} = \text{the institution}) \\
& \quad \text{He still goes to school} \\
& b. \quad \text{Die Schule streikt heute (} = \text{all pupils, teachers etc.}) \\
& \quad \text{School is on strike today} \\
& c. \quad \text{Unsere Schule steht unter Denkmalschutz! (} = \text{the building}) \\
& \quad \text{Our school is classified as a historical monument} \\
& d. \quad \text{Schulen sollten von außen als solche erkennbar sein (} = \text{the building, but because of "als solche" at the same time also the institution}) \\
& \quad \text{Schools should be identifiable as such from the outside}
\end{align*}

These differences in meaning seem to be systematic, and it may well be that they are not listed in your dictionary; if being distinguished in a dictionary is a valid criterion, the differences we observe in (10) do not give rise to different words. Nonetheless, thinking of what the term expresses and of what kind of things we refer to with the expression in these sentences, it is quite obvious that some distinction in meaning is undeniable. Such systematic differences, arising as variants of one core meaning (= the institution), got a special name: the phenomenon is called ☞ polysemy. The difference between ambiguity and polysemy is this: ambiguities can be arbitrary, as with *bank\(_1\)* and *bank\(_2\)*, whereas polysemy is something systematic that can be observed with a whole range of expression (*Schule/school, Krankenhaus/hospital, Kirche/church* etc.).

Polysemy is often contrasted with homophony. Both require identical pronunciation, but whereas in homophonous pairs the different meanings are not related to one another, polysemous pairs require a close semantic relationship between the meanings of the words, ideally of the sort exemplified in (10). Here are some more examples where the semantic relation between the two meanings is of the more opaque sort:

\begin{align*}
(11) & \quad a. \quad \text{bright: *shining* or *intelligent*} \\
& b. \quad \text{to glare: *to shine intensely* or *to stare angrily*} \\
& c. \quad \text{a deposit: *minerals in the earth* or *money in the bank* or *a pledge* or} \\
& \quad \ldots
\end{align*}

For the linguistic layman this kind of relationship between words seems to be the most interesting aspect of semantics, giving rise to endless debates and histori-

\(^7\)Unfortunately, most ambiguity tests are unreliable; cf. Sadock and Zwicky (1975) for further discussion.
cal speculations about the nature of the similarity. Since we are not concerned with diachronic linguistics and etymology, we will refrain from any discussion of polysemy.

### 2.3 Sense Relations (Sinnrelationen)

It is often implied by the literature that semantic theories should account for certain kinds of intuitive judgments of native speakers of a particular language. These judgments and the corresponding intuitions can be of several sorts. One is **semantic** (as opposed to **syntactic** well-formedness), the other is the language user's ability to assess certain systematic aspects of the meaning of words to which we will return. Starting with the first, the reader will agree that the following sentences are strange:

(12) a. Der Koch **singt** ein Gewürz
    the cook **sings** a **spice**
    
    b. Die Gabel **bezweifelt** das
    the fork **doubts** it

The meaning of (12-a) is unclear because one can sing only songs or texts. Somehow the verb and the object do not fit together. In (12-b) there is a mismatch between the verb and the subject. The reason for the awkwardness of these (syntactically well-formed) sentences is that they violate certain semantic well-formedness conditions that accompany the verbs. These conditions are called **selectional constraints** (Selektionsbeschränkungen) of the verb: to doubt/bezweifeln requires the subject to be human, and the object of to sing/singen has to be something like a song. Native speakers who have learned the meaning of the verbs are clearly able to activate intuitions of this sort, ie. intuitions about selectional constraints. All selectional constraints are part of the meaning of particular lexical items.

Another semantic skill of speakers (and hearers) is their ability to make claims about the meaning of two words in comparison. This is exemplified in:

(13) **Postwertzeichen** bedeutet das selbe wie **Briefmarke**
    postage stamp means the same as postage stamp

(14) **glauben** heißt nicht das selbe wie **wissen**
    believe means not the same as know
    to believe does not mean to know

---

8As it turns out, many ambiguities evolved from polysemies; eg. the German example *Schloss* (lock/castle) has started off with a single basic meaning corresponding to lock; the castle reading then evolved from a building that locks the way out of a valley.
precipitation is a more general term than drizzle

dog and cat are with-each-other incompatible

These sentences are statements about so-called sense relations. (13) states that two words have the same meaning, they are synonymous. The sense relation expressed is synonymy (☞Synonymie). Synonymy between simple lexical items seem to be very rare in natural language. It is sometimes said that “true synonyms”, i.e. those whose connotations do not differ too much, are extremely rare (the phenomenon has been dubbed “Synonymenflucht”). This has been explained by an economy principle to the effect that language does not contain redundant material in the lexicon. Given the host of synonym pairs in (certain) closed categories (obschon, obzwar, obgleich), this claim needs some qualification; but it does seem that fully synonymous content words are a rare species. Also, we observe that many new terms have been coined that were initially intended to replace old ones with the same meaning: Compare Fahrkarte vs. Fahrausweis; Schaffner vs. Fahrdienstleiter; Mülleimer vs. Wertstoffbehälter; Toilette vs. WC-Center, Hotel vs. Beherbergungsbetrieb, etc. This has been called semantic environmental pollution (semantische Umweltverschmutzung). Note that most of these terms are compounds. It therefore remains true that there are hardly any two synonymous simplex (uncompounded) content words.

The next sentence (14) states a non-identity of meaning; this can also be called a sense relation, albeit normally a very uninformative one. (15) is more interesting. It says that one notion includes the other, or in other words, it logically implies the other. The more general including term is called a ☞hyperonym (Oberbegriff), the more special included term is called a ☞hyponym (Unterbegriff). If a notion A is a hyperonym of B, then B is a hyponym of A. The relation of inclusion is called hyponomy. The reverse relation of being included is called ☞hyperonomy/Hyperonomie.

Finally considering (16), assume that the utterance is not meant to report something about the biology of cats and dogs; rather, one wants to say that the notions exclude each other. That is, if something is a cat it cannot be a dog, and vice versa. The relevant sense relation is incompatibility.

Apart from these typical relations there are a number of other relations between words, like the one illustrated in (17):

(17) a. John kills Bill
    b. Bill dies
Here one would say that *kill* means something like (or is almost synonymous to) *cause to die*. Thus, dying is a sort of causal consequence of killing; hence the semantic relation is *causation* (Verursachung, Kausativierung).

### 2.4 Semantic Networks

As a result of establishing more and more sense relations, linguists have proposed that all sense relations that hold between words should be organized in a kind of *semantic network* (*semantisches Netz*). Networks consist of nodes labelled with lexical items and connected by semantic relations. These relations may contain all sorts of relevant information about the meaning of a lexical item; the most primitive networks represent sense relations only. Here is an example from an electronic data base called GermanNet:

(18) wandeln (*transform*)  
verändern (*change*)

schließen  öffnen  sich öffnen (*shut, open (transitive), open (intransitive)*)  

zumachen  aufmachen  aufgehen (*shut, make open, come loose*)

aufstoßen *push open*  
aufbrechen *break open*  
aufsperrn *unbar*

It is the task of lexical semantics to describe the network of a given language.

HOMEWORK: Discuss which sense relations (including causation) are represented by the arrows in (18).

One of the potentially interesting things about networks is that they may have gaps. This is illustrated in the following letter:
(Translation: I noticed that the German language lacks a word. When you aren’t hungry any more, you are full (satiated); but when you are not thirsty any more, you are … I’d like to ask you to introduce the term “schmöll” into your dictionaries.

Yours faithfully
Werner Schmöll)

The remarkable thing is not the fact that we do not have words for particular (kinds of) things. This is quite normal. E.g. we do not have a word for blond girls that were born on April 1st. Although we could of course invent a notion like first-april-girl, this is not a single word but an ad hoc compound. The problem is rather that thirsty lacks an antonym, a word that is incompatible and expresses the opposite. Thus, the opposite of black is white, the opposite of slow is fast, etc. This relation is another sense relation; cf. ☞Antonym, ☞Opposite (semantics))

9Robert Gernhard, Welt im Spiegel 1975. The question is whether we really need such a notion. After all, you can booze till you drop. On the other hand we do have the notion “abgefüllt” (filled) in German, though it has a broader meaning than schmöll. (Bottles can be filled, but they cannot be schmölled.) The German speaking readers should consult ☞Sitt.
Any parsimonious description of a semantic network will take advantage of the fact that we can reduce some sense relations to others. A particularly useful method for doing so is to describe sense relations between words by analyzing synonymous paraphrases. For example, what is the sense relation between brother and sister? One way of approaching the problem is by using the synonymous expressions male sibling and female sibling. Since male and female are incompatible (in fact even antonyms), we can automatically infer that brother and sister are also incompatible.

This implies a definition of sister as female sister, and we might now go on and describe (or define) siblings as people having the same parents. Continuing in this fashion one might try to find more and more primitive basic notions (and relations) that can be used to express (or define) large parts of the dictionary, which in turn helps us to find semantic relations between individual lexical items. For example, the part-hole relation is an important one holding between all sorts of things; this further sense relation is called meronymy. For example, finger is a meronym of hand, since a finger is part of a hand.

Going on this way we will eventually arrive at lexical items that cannot be decomposed any further. The atomic primitives we arrive at at the end of such a procedure have been called semantic markers. At the end of the day, all sorts of relations between items in such a web of semantic markers can be said to express sense relations. In particular, sense relations do not only hold between single words but also between complex expressions:

(19) Stute (engl. mare)
weibliches Pferd
Pferd weiblichen Geschlechts

(20) schwarzes Turnierpferd männlichen Geschlechts
schwarzer Hengst (sire)
Säugetier (mammal)

It is clear that any complete semantic theory must give an account of these relations. However, this cannot be achieved by simply describing the meaning of words alone. What we need in addition is a way to describe combinations of meaning that make up a phrase and ultimately a sentence.

Many linguists in the 1960s (notably Jerold Katz, Paul Postal, and Jerry Fodor) hoped to be able to reduce large portions of the vocabulary by applying such a compositional analysis and assigning a set of semantic markers to each lexical item. Also, computational linguists were inclined to adopt such a model, which has later been called markerese semantics. As it turned out, however, this method of doing semantics is way too primitive to account for other aspects of
meaning.

What we need in addition is a way to describe combinations of meaning that make up a phrase and ultimately a sentence. And as it turned out in the history of the discipline, it is not possible to develop a method of doing so by simply manipulating semantic primitives like markers. Rather, one might instead pursue the opposite strategy, starting with a comparison between the meaning of entire sentences and then find out more about the meanings of their parts and how they combine. This way of approaching the problem of word meaning turned out much more successful. The method is called clausal semantics, or logical semantics, and it is this kind of semantics we will be concerned with in the remainder of this text.¹⁰

3 Structural Ambiguity (Strukturelle Ambiguität)

3.1 Some Elementary Examples

We have seen that the same sequence of sounds or letters can express different meanings: words can be ambiguous. But ambiguity is not only found with words—as a phenomenon to be recorded in a dictionary. On the contrary: the more complex a linguistic expression, the more likely do we encounter ambiguity. And the most interesting aspect of this is: it also occurs in sentences not containing any ambiguous word.

Consider the following example:

(1) The parcel contained old socks and shirts

First, let us make sure that (1) does not contain an ambiguous word. Experience shows that ambiguities can easily be overlooked, but for the sake of the argument, let us assume that all words occurring in (1) are unambiguous. Now, still there is an ambiguity which can be revealed by the following question: Does (1) imply that all the shirts in the parcel are old? In one reading of the sentence, the answer is yes, in another possible reading the answer is no. In the latter reading, old modifies socks only, in the first-mentioned reading old modifies socks and shirts.

This ambiguity can be correlated with two different syntactic structures that can be associated with (1):

(2) The parcel contained . . .

¹⁰Proponents of this kind of theory have criticised markerese semantics for not being a semantics at all, because it does not deal with the relations between symbols and the world of non-symbols — that is, with purportedly “genuinely semantic” relations (cf. the criticism in Lewis (1972)) — a matter to which we return.
The important thing to note is that the bracketings in (a) and (b) differ. In the first structure, the adjective only modifies the first conjunct \textit{socks}, while in (b) it modifies the entire conjunct \textit{socks and shirts}. This syntactic ambiguity matched in semantics by two different meanings. Such ambiguities are called \textbf{structural ambiguities}.

The semantic ambiguity in (2) can also be described as a difference in \textbf{scope} (Skopus). In (2-a) we say that the adjective \textit{old} has scope over \textit{socks} only, whereas in (2-b) we say that the scope of \textit{old} is \textit{socks and shirts}. Given that the semantic ambiguity is a consequence of syntactic structure, we also say that it is \textit{induced} by a syntactic ambiguity, resulting from the different order in which we apply certain semantic rules that accompany the syntactic structures. (We will give a precise account of these rules in section 5.5.)

One can observe the same kind of ambiguity in arithmetic expressions like $9 - 5 + 3$: the ambiguity is usually resolved by using brackets: $9 - (5 + 3) (= 1)$ versus $(9 - 5) + 3 (= 7)$. And so it is natural language. Different ways of structuring may induce different semantic results. The reason for this is that we apply semantic operations in a different order: By first applying addition in $9 - 5 + 3$, this gives addition scope over subtraction (the result being 1). By first applying subtraction, subtraction has scope over addition (the result being 7). In mathematics such a kind of ambiguity is of course intolerable, therefore the use of the brackets adds syntactic structure that helps to disambiguate the expression. Strictly speaking, $9 - 5 + 3$ is not be well-formed, because in mathematics, all formulas should be unambiguous. By using brackets, the syntax of the expression tells us which operation to apply first, so it encodes the scope relation by correlating them with different syntactic structures.

Here is another famous example from natural language:

\begin{itemize}
  \item[(3)] John observed a man with binoculars
\end{itemize}

This sentence allows the two readings:

\begin{itemize}
  \item[(4)] a. John used binoculars to observe a man
  \item b. John observed a man who had binoculars with him
\end{itemize}

The ambiguity seems to go hand in hand with different syntactic structures:
In (a.) the prepositional phrase *with binoculars* is attached to a VP and therefore modifies the event of observing, in (b) the PP is part of a DP and therefore modifies the common noun *man*. This way, different meanings arise.

As we have seen in section (2.) it is sometimes difficult to decide whether a word is really ambiguous. The same applies to more complex expressions: most often, ambiguities are easy to detect. When we hear a sentence, we will most often think it is unambiguous, because one of its readings is pragmatically more salient than the other. This is because we naturally intend to interpret sentences as being *true* in a given situation or context of utterance, so that an alternative reading making the utterance false is not taken into consideration. Seeing only one interpretation, we are mentally blocked to see the alternative, as is often the case with optical ambiguities like the famous duckrabbit (Hasen-Enten-Kopf; ☞Kippfigur):

(6) ![Duckrabbit](image)

As linguists, however, we should nonetheless be capable of identifying different readings of sentences; in practice, this is achieved by using unambiguous **paraphrases**. Such paraphrases have already been used in our discussion of (3) (*John observed a man with binoculars*) when paraphrasing the two meanings in (4). Paraphrases must be unambiguous, such a “paraphrase” of (6) must, so to speak, denote either a duck 🦆 or a rabbit 🐰. Given the two pictures, we can “see” that the duckrabbit can be interpreted either as a duck or as a rabbit.

Returning to the linguistic example, we first ask ourselves whether any situa-
tion described by (4-a) can also be reported by using (3) (restricting ourselves to the literal meaning of the sentences). The answer should be YES, and the same must hold for (4-b). We now “see” that (3) can be interpreted either as (4-a) or as (4-b).

Looking at the disambiguating pictures, it is obvious that they denote different objects. It is not so obvious, however, that the paraphrases really have different meanings. In order to test this, we rely on a principle that has also been called “the most certain principle” in semantics (cf. Bäuerle and Cresswell (1989)):

(7) If a sentence A is true and another sentence B is false in the same situation, then A and B differ in meaning.

(7) is an axiom in our theory of meaning; we’ll come back to this connection between meaning and truth and falsity on many other occasions in what follows.

We now apply this principle to the case at hand: it is obvious that when only John has binoculars, one of the paraphrases is true and the other is false; likewise when only the man has binoculars, the previously true sentence now turns false, and the formerly false sentence becomes true. In such a case, the method of paraphrases can be used as a water-proof method for identifying ambiguities. We will rely on this method in other cases as well.

Summarizing so far, one might be tempted to say: “If a sentence may both be true and false in the same situation it is ambiguous.” This is a little bit of a simplification, but as slogan, it will do. The only apparent problem with it is parallel to the one we already discussed in the last section: is it only one sentence that is ambiguous, or do we have to assume two different sentences with different structures? Again this is a matter of terminology: if we abstract away from structure, we only have one sentence, if not, we have two. In this introduction we prefer to include structure in our notion of a sentence.

Before going on with another example of an ambiguity, let us return to our previous example old socks and shirts. What are possible paraphrases? Consider

(8) a. shirts and old socks
    b. old socks and old shirts

It is obvious that these paraphrases convey the same meanings as the original expression, thereby displaying the ambiguity. In order to apply the ambiguity test we face a problem: Only sentences can be true or false. Therefore we first have to put the relevant expressions in a sentence:

(9) The old socks and shirts cost 10 Euros
    a. The shirts and old socks cost 10 Euros
b. The old socks and old shirts cost 10 Euros

Assume that old shirts are cheaper than new ones. Then it is easy to imagine a situation A with old and new shirts in which (a) is true. Given our assumption about the difference between old and new shirts (b) must be false (the price must be less than 10 Euro). And conversely in a situation B where (9-b) is true, (9-a) is false (the price must exceed 10 Euro). This establishes the ambiguity.

As a further illustration of the method of paraphrases, let us discuss another ambiguity:

(10) Paul kennt Gertrude nicht, weil sie in Hamburg wohnt
    Paul doesn’t know Gertrude because she lives in Hamburg

The two different meanings are paraphrased in (11):

(11) a. The reason why Paul doesn’t know Gertrude is that Gertrude lives in Hamburg (and Hamburg might be too far away)
    b. The reason why Paul knows Gertrude is not because she lives in Hamburg (which is close enough, but he knows her for some other reason)

Note that the English translation in (10) is as ambiguous as the German sentence. So speakers of German and English should agree that (10) can be used in a situation described by (11-a), but it can also be used in a situation like that one in (11-b). The additional phrases in brackets are intended to disambiguate the situation. And the two situations are such that (10) can be both true and false, depending on which situation is chosen.

The next step is to ask how the ambiguity can be related to (or explained by) different possibilities for structuring the above sequence of words.

As the paraphrases reveal it is the negation expressed by doesn’t which seems to induce structural ambiguity: in (a) it is claimed that Paul doesn’t know Gertrude: here the verb know is in the immediate scope of negation. In (b), on the other hand, the scope of negation is the causal relation between the two subclauses, which is expressed by because. In German, this can be expressed unambiguously by putting the negation directly in front of because as shown in (12):

(12) Nicht weil sie in Hamburg wohnt, kennt Paul Gertrude (sondern aus einem anderen Grund)
    Not because she in Hamburg lives knows Paul Gertrude (but for some other reason)

In syntax, then, we would expect that in this case because is in the domain of
doesn’t, as is the case in the following structure:

(13)  Paul doesn’t

```
  VP
  /\       \CP
  know Gertrude because IP
      \       / she lives in H.
```

The domain of the negation in (13) is the entire VP; since because is part of that VP, in fact, the highest “operator” that connects the VP with the IP of because, because is in the immediate domain of the negation.

In the alternative reading, only know should be in the domain of negation; and the negation itself should be in the domain of the because-CP. Hence, the because-CP has to be attached higher up in the structure, as shown in (14):

(14)  IP

```
  Paul \       \ I’
    /\     /\ CP
   doesn’t know Gertrude because she lives in H.
```

As can easily be verified, the because-CP is not in the domain of the negative verb, hence it is not in the semantic scope of negation. We thus succeeded in detecting a structural ambiguity which ultimately, together with appropriate semantic rules (still to be explored), may help explain a semantic ambiguity not being related to any lexical ambiguity.

Note, however, that hitherto we only stated a parallelism between syntax and semantics, but the real work that ultimately explains why the sentences have different meanings has not yet been done. This should become clear by looking at a simple example where syntactic ambiguity alone does not suffice to induce semantic ambiguity: E.g. \( x + y - z \) is not semantically ambiguous between \( (x + y) - z \) and \( x + (y - z) \): the result is always the same number. It is the particular semantic rules combining the constituents in a tree that ultimately do the job, but no such rules have been stated yet. This is precisely what we are going to do in section 5.
3.2 Scope and Syntactic Domains

In the above examples we have demonstrated that scope relations correspond to syntactic domains: the syntactic ambiguity of *old socks and shirts* can be described by saying that in one reading only *socks* is in the domain of *old*, and in the other reading *socks and shirts* is in the domain of *old*. Corresponding to this syntactic difference, we say that in the first reading, *old* has scope only over *socks*, whereas it has scope over *socks and shirts* in the second reading.

What we did, then, was correlate semantic intuitions about scope with syntactic structure. The guiding principle we applied is the following:

\[(15) \text{ If } \alpha \text{ has scope over } \beta \text{ then } \beta \text{ is in the syntactic domain of } \alpha.\]

Now, the notion of a syntactic domain can be made more precise by assuming the following definition:

\[(16) \text{ If } \beta \text{ is in the syntactic domain of } \alpha \text{ if and only if either } \beta \text{ is a sister node of } \alpha \text{ or } \beta \text{ is contained in (= dominated by) a sister node of } \alpha.\]

Let’s apply these definitions to the structures discussed so far. Take the famous *green eggs and ham*. Assume that *green* only modifies *eggs*. Then *eggs* is in the scope of *green*, and since *eggs* is a sister node of *green* it is also in the syntactic domain of *green*. If *green* modifies the entire conjunct, then the conjunct is in the scope of *green*, and so is *eggs* and *ham*: Syntactically, the entire conjunct is in the syntactic domain of *green*, and so are the nouns *ham* and *eggs*.

Turning next to (10), our decision to adjoin the CP in (14) higher in the tree is guided by our semantic intuition that the negation should have scope over the verb alone, so that the verb is in the syntactic domain of the negation. On the other hand, in order to get the *because*-clause into the scope of the negation, it has to be in the syntactic domain of the negation, and therefore it has been attached lower in the tree, as shown in (13), repeated as (17):

\[(17) \text{ Paul doesn’t }\]

```
  VP
   /\   \\
  VP  CP
   /\    /\  \\
know Gertrude because she lives in H.
```

\[11\text{Those of you already familiar with syntactic theory might note that the notion of domain as defined above is the same as the relation of }\text{c-command (c-Kommando, k-Herrschaft).}\]
It seems, then that we have established a nice parallelism between syntax and semantics: scope seems to correlate with syntactic domains. However, it is important to note that this correlation only goes half way. We did NOT claim that if $\beta$ is in the domain of $\alpha$, then $\alpha$ is semantically in the scope of $\beta$. To see this, consider (17). We claimed that because is negated because it is in the syntactic domain of the negation doesn’t. But so is the verb know. Yet it makes little sense to say that the verb is negated as well. In fact, this is utterly wrong: When uttering (17) one claims that Paul does know Gertrude!

The same sort of problem can be observed in (18)

\[(18)\] John didn’t yet stop smoking cigars

The syntactic structure is something like

\[(19)\] John didn’t yet $[\text{vp stop } [\text{vp smoking cigars }]]$

Here again, both stop and smoking are in the scope (and the syntactic domain) of didn’t. Yet, it’s only the verb stop that is negated, because (19) says (or implies) that John is still smoking cigars!

(19) illustrates that the semantic notion of scope is still different from the syntactic notion of a domain. These differences (and also the similarities, cf. Heim and Kratzer (1998) chapter ??) can only be explained in a sophisticated semantic theory that goes far beyond of what can be achieved in an introductory course.

### 3.3 The Concept of Logical Form

Consider yet another ambiguity:

\[(20)\] Beide Studenten kamen nicht

Both students came not

This sentence is ambiguous, it can either mean

\[(21)\] Reading A: neither of the two students came

or it can mean

\[(22)\] Reading B: not both of the students came (one of them came).

The second reading requires support from intonation: a rise on beide and a fall on nicht. It is easy to see that if A is true, B is false, and if B is true, then A must be false.

Discussing this ambiguity in terms of scope, reading A is characterized by beide Studenten having semantic scope over nicht, whereas the reverse holds for
reading B. In syntactic structure, however, the negation is in the syntactic domain of *beide Studenten*, therefore it seems we only get reading A. Reading B is not what we see immediately in the structure. Therefore the existence of reading B seems to contradict the scope principle (15).

However, taking a closer look at the syntax of German will reveal that this contradiction is only apparent. Let us ask how the structure of (22) is generated. As we know from the Generative Grammar of German, the subject *beide Studenten* has been moved into the Specifier of C position (also called the pre-field position) by a syntactic movement rule (called *topicalization*). Hence, before movement we can assume a structure like

\[
(23) \quad \text{CP} \\
\quad \text{SpecC} \quad \text{C'} \\
\quad \text{C} \quad \text{VP} \\
\quad \text{nicht} \quad \text{VP} \\
\quad \text{DP} \quad \text{V} \\
\quad \text{beide Studenten} \quad \text{kamen}
\]

We then move the verb *kamen* into the C-Position, followed by movement of the DP *beide Studenten* into the SpecC-position:

\[
(24) \quad \text{CP} \\
\quad \text{DP}_j \quad \text{C'} \\
\quad \text{beide Studenten} \quad \text{C}_i \quad \text{VP} \\
\quad \text{kamen} \quad \text{nicht} \quad \text{VP} \\
\quad \text{DP} \quad \text{V} \\
\quad \text{t}_j \quad \text{t}_i
\]
Given this derivative structure we now see that we could indeed represent reading B, if only we are allowed to semantically interpret *beide Studenten* before moving into SpecC. This way, We thereby avoid a violation of the scope principle, because moving back the subject into the position of its trace, the DP “reconstructs” into the syntactic domain of the negation. In consequence, the negation can have scope over the subject, as desired.

The above explanation can be generalized: in fact, there are many more examples that allow for an ambiguity precisely because a certain movement has taken place. For example, most speakers would agree that

\[(25) \quad \text{jeden Schüler} \text{acci} \text{w} \text{ative lobte genau ein Lehrer} \text{nomi} \text{native} \text{lobte genau ein Lehrer} \text{lobte genau ein Lehrer} \]

\[(26) \quad \text{CP} \hspace{1cm} \text{C'} \hspace{1cm} \text{VP} \]

\[\text{DP}_j \hspace{2cm} \text{C}_i \hspace{2cm} \text{DP} \hspace{2cm} \text{V'} \]

\[\text{jeden Schüler} \hspace{2cm} \text{lobte} \hspace{2cm} \text{genau ein Lehrer} \hspace{2cm} \text{lobte} \hspace{2cm} \text{genau ein Lehrer} \hspace{2cm} \text{lobte} \hspace{2cm} \text{genau ein Lehrer} \hspace{2cm} \text{lobte} \hspace{2cm} \text{genau ein Lehrer} \]

has two readings:

\[(27) \quad \text{a. For every pupil there is exactly one teacher who praised him} \]
\[\text{b. There is exactly one teacher who praised every pupil} \]

In order to see how the meanings differ, consider first a situation with three teachers and six pupils. The relation of praising is represented by a line:

\[(28) \quad \text{teacher} \hspace{1cm} \text{pupil} \]

---

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In such a situation both (27-a) and (27-b) are true: every pupil is praised, and there is only one teacher who is praising. In consequence, (28) does not suffice to disambiguate the situation. But now consider (29):

(29) teacher pupil

In this situation (27-b) is still true because the additional teacher does not praise every student (but only one), so there is still exactly one teacher who does. On the other hand, (27-a) is false because there is one student who is praised by more than one teacher.

Next, consider (30):

(30) teacher pupil

In this setting (27-b) is false because no teacher praises all of the pupils. On the other hand, each pupil is praised by a teacher, and no one is praised by more than one teacher, hence (27-a) is true.

We have thus shown that the construction is ambiguous, and we related the ambiguity to movement. We would therefore expect that in a structure that does not involve movement, no ambiguity arises. And in fact, a subordinate sentence like (31) where no movement has taken place, is indeed felt as unambiguous, with exactly one teacher having wide scope over every pupil:

(31) ich glaube, dass genau ein Lehrer jeden Schüler lobte

   I believe that exactly one teacher every pupil praised
is unambiguous, having only the reading (30-b).

Given that phrasal movement can create ambiguity, one could generalize the scope principle by taking traces into account:

\[(32) \text{ If } \alpha \text{ has scope over } \beta, \text{ then } \beta \text{ or } \beta' \text{'s trace is in the domain of } \alpha.\]

Most linguists, however, would reject (32) as superfluous. They adhere to the scope principle in its original version. The reason is that they assume the existence of a level of syntactic representation potentially different from surface structure. At this level or representation, semantic scope is unambiguously represented in terms of syntactic domains and in accord with (13). But crucially, movement can be undone at this level. This undoing of movement is called reconstruction into the base position. Reconstruction of phrasal movement is optional: If it applies we get one reading, namely reading B in the case at hand, if it does not, we get reading A. It is clear that we do not need (32), because at the reconstructed level the trace has been replaced by the moved item and is in the domain of \( \alpha \).

The basic idea here is that we can read off semantic scope from a level of representation that does not necessarily coincide with the surface structure. This level has been called LOGICAL FORM (LF) (logische Form). In the case at hand, the LF for reading A does not differ from the surface structure in any relevant respect; here surface structure and LF coincide. In order to get reading B, however, we reconstructed into the base position. Hence the LF for this reading is different from the surface structure.

Apart from reconstructing phrasal movement there is also reconstruction of head movement. Consider:

\[(33) \text{ Paul schnarcht nicht} \]
\[
\text{Paul snores not}
\]
\[
\text{Paul doesn’t snore}
\]
The syntax of German dictates that the verb has been moved into the C-Position, hence it cannot be in the domain of negation. Yet it is undoubtedly the verb that is negated. The solution here is reconstruction again. As there is no alternative reading generated by the movement of V into C, it is assumed that all head movement is obligatorily reconstructed at the level of LF. So there is only one LF of (33), and this complies with the fact that (33) is unambiguous.\footnote{Reconstruction is only one of the operations that lead to a LF that differs from the surface. Other processes are discussed in the literature, but at present there is no agreement among linguists as to which operations should be allowed in the transition from surface structure to LF and how far LFs can depart from surface structure.}

**HOMEWORK:**

A) What are the two LFs of (20)?

B) Try to account for the ambiguity of:

\begin{verbatim}
(34) Genau 5 Bücher hat keiner gelesen
   Exactly 5 books has noone read
\end{verbatim}

Give two paraphrases; analyse the two readings of this sentence by describing two types of situations that make the paraphrases true and false.

### 3.4 More Ambiguities

Semanticists love to analyse ambiguities, and we as beginners can also profit enormously from this obsession. Why?

- The study of ambiguities may give you a rough idea of what semanticists are interested in and consequently, what semantics is about.

- Ambiguities can be revealed only by disambiguation, which forces the student of semantics to consciously reflect on meaning and how complex meanings emerge.

- Ambiguities also provide for a testing ground for theories: if a theory fails to predict a certain kind of construction to be ambiguous, it is in need of further elaboration or revision.\footnote{Ambiguities also tell us something about Logical Form and the relation between so-called overt syntax on the one hand (the surface syntax that forms the input to phonology) and logical syntax on the other (the structures called Logical Form, which serve as the input for semantics). However, not all semanticists accept Logical Forms that depart too much from the surface; hence the study of LF is not necessarily a silver bullet for the the study of semantics.}

Now, in order to get a feeling for the problems semanticists are interested in, here are some of further ambiguities for you to think about:
3.4.1 Discussion of Example (35)

Let us next turn to (35), repeated as

(35) Vor 20 Jahren waren die Professoren noch jünger
Ago 20 years were the profs PRTCL younger

(36) Gertrude sucht ein Buch für ihre Schwester
G. seeks a book for her sister

(37) Ich weiss was ich sage
I know what I say

(38) How many dogs did everyone feed?

(39) Hast Du Butter oder Margarine gekauft?
Did you butter or margarine buy

Before reading any further try to detect and explain the ambiguities in these examples, perhaps by discussing these sentences with your classmates. Try to give paraphrases that come as close to the desambiguated meaning as possible. Don't bother if in some cases you don't see (or feel) the ambiguity. Sometimes intonation may help to disambiguate. For instance, it makes for a subtle difference whether or not there is a rise on Butter in (39). (In the remaining cases, however, intonation is not very helpful.)

As you might have experienced, it's not at all easy to describe an ambiguity, and it's even more cumbersome to detect its reason or source. Nonetheless, giving an account of such intuitions about ambiguity is precisely what a semantic theory is about: such a theory should be able to explain the fact that we understand these sentences in different ways. Thereby, we hope to account for the more general ability of humans to understand sentences. Any theory (primarily and correctly) dealing with unambiguous cases should, to the extent that it is correct, also be capable of explaining the more complex ambiguous cases.

As discussed in the last section, an idea pursued by many (though not all) semanticist is that the additional complexity of ambiguous sentences can be due to the possibility of one and the same sentence having two different syntactic representations at the additional level of Logical Form (LF). According to this theory, ambiguous sentences are unambiguous at the level of LF. Others reject the idea of LF and try to account for additional complexity by much more complicated semantic rules. We cannot go into these matters here, they indeed provide for a rich field for research.

In the following subsections, we will discuss the ambiguity of (39), (35), (36), (37), and (38) in some detail. The impatient reader may skip these sections.
(40) Vor 20 Jahren waren die Professoren noch jung (als heutzutage)  
Ago 20 years were the profs PRTCL younger (than nowadays)

(40) can mean something very trivial, namely this:

(41) For each professor it holds that 20 years ago he was younger than he is today.

This is a self-evident truism, people simply get older, and because (41) is so obviously true, it is most probably not the intended meaning. Rather, one wants to say something like

(42) The average age of a professor twenty years ago was lower than the average age of a professor nowadays.

This assertion makes much more sense, it is not a self-evident truism (but probably wrong). The interesting question here is how these extra bits of meaning may creep into the meaning of the words. What is the mechanism behind this shift of meaning?

An answer to this question is indeed far from obvious. Observe that in the trivial reading the people we are talking about are the same in the comparison, we compare each individual’s age in time, whereas in the non-trivial reading we are comparing two entirely different groups of people, namely the professors today and the professors 20 years ago. Moreover, in this reading we are talking about the average age of professors. More technically speaking something like the “generic” professor, the proto-type of a typical professor, is involved. This concept of genericity (☞ Generizität; ☞ Generic mood) is an additional aspect and an additional complication.

If the paraphrases we have given in (41) and (42) come close to the different Logical Forms, the example shows that these paraphrases may contain semantic material that seems not to be included in the material of the original sentence, although it must be implicitly contained in it. Hence, the example suggests that the “distance” between a surface expression and its LF may be surprisingly big. It is a non-trivial task for the semanticist to show how to bridge the gap in a systematic, non-ad-hoc way.

3.4.2 Discussion of Example (36)

Let us now return to

(43) Gertrude sucht ein Buch für ihre Schwester
Gertrude seeks a book for her sister
One reading can be paraphrased as:

(44) Es gibt ein (bestimmtes) Buch das sich ihre Schwester zu Weihnachten gewünscht hat), das Gertrude sucht
There is a certain book (her sister requested as a Christmas present) that Gertrude is looking for

The other situation would be:

(45) Gertrude versucht, ein Geschenk für ihre Schwester zu finden, welches ein Buch sein sollte (aber sie weiß noch nicht welches)
Gertrude tries to find a present for her sister which should be a book (but she has no particular book in mind)

This ambiguity seems to be related to the different ways the indefinite article "a" can be used. The reading of "a book" we paraphrased as "a certain book" is called the specific reading of the indefinite NP, whereas the reading where the identity of the book does not matter is called the unspecific reading. Linguists in the tradition of Richard Montague (1973) have analysed this difference as a difference in scope. In the unspecific reading, the verb seek has wide scope over the indefinite NP, whereas the reverse holds for the specific reading. It is impossible for us to evaluate or appreciate this proposal, a more precise discussion would presuppose techniques to be developed in an advanced semantics course.\(^\text{14}\)

\(^{14}\)Nonetheless one remark (for the alert reader) might be in order: given the structure

(i) \[ VP \text{ seek } [NP \text{ a book }] \]

it is clear that seek c-commands a book, and a book c-commands seek. Therefore, due to the correlation between c-command and scope, both readings should be available interpretations of the one structure (i). Hence, the structure (i) permits representing a semantic ambiguity without being syntactically or lexically ambiguous. However, many linguists would prefer a representation (namely Logical Form) which unambiguously represents scope. In such a theory, ambiguous representations are not allowed. Rather, the structure in (i) would only serve to represent the unspecific reading, whereas the specific one would need a different representation at LF, one like

(ii) There is a book Gertrude seeks

Note finally, that above, when paraphrasing the specific reading, we assumed that it is either the speaker or the subject Gertrude having a particular book in mind. However, this additional aspect of meaning is in no way represented in the LF shown in (ii), and whether it should be, or whether this additional aspect of meaning belongs to pragmatics, is a matter of controversy.
3.4.3 *Discussion of Example (37)*

Now consider:

(46)  Paul weiß, was Gertrude bezweifelt
    Paul knows what Gertrude doubts

First put some stress on *Gertrude*. This helps to get the intended reading. The sentence then expresses a contrast between Paul and Gertrude, which can be paraphrased as follows:

(47)  If Gertrude doubts something, this something is known by Paul

For example: if Gertrude doubts that it is raining, Paul knows that it is raining. Since knowing something normally implies that the known proposition is true (I cannot know something that is false) this implies that all of Gertrudes doubts are unsubstantiated.

Now put stress on *weiß*. Then the following paraphrase seems possible:

(48)  If Gertrude doubts anything, then Paul knows that Gertrude doubts it.

This reading implies that Paul is fully aware of Gertrude’s mental states of doubtfulness. The difference is subtle, so it might help to illustrate it with another similar examples, namely the one already presented in (48)KB:

(49)  I know what I’m saying

The two readings can be paraphrased as in (50):

(50)  a. I only say what I know (to be true)
    b. I am totally aware of what I am saying

Likewise:

(51)  Bauknecht weiß, was Frauen wünschen
    Bauknecht knows what women desire
    a. If a woman wishes x, then Bauknecht knows that she wishes x
    b. If a woman wishes x, then Bauknecht knows x.

The sample sentence is a slogan from German TV-ads and Bauknecht is a brand for washing machines. The interesting question, however, is this: how can this ambiguity systematically be derived and whether it can be described as a structural ambiguity.

39
As it turns out, there are indeed two different syntactic constructions that can account for the ambiguity. We may distinguish between two types of complements of know: one is an indirect question, the other is called a free relative clause. Let us start with the latter by considering a sentence like

(52) I eat [ what(ever) you cook ]

The bracketed constituent is sometimes (misleadingly) called a concealed question, but in fact our semantic intuition tells us that (52) contains no question at all. Rather, intuitions would suggest (53) as a paraphrase:

(53) I eat [DP anything that you cook ]

This construction contains a relative clause “that you cook”, and for this reason, the construction in (52) is also called a free relative clause. It is called free because what you cook is syntactically analysed as a relative clause CP which is attached to an empty head:

(54) I eat [DP Ø [CP what you cook ]]

This is interpreted at the level of LF as something like:

(55) For any x: if you cook x then I eat x

For the purpose of our discussion, it is immaterial how this interpretation comes about. The point here is that, by analogy, we also get a free relative clause structure for (49), and a semantic interpretation that parallels (55):

(56) a. I [VP know [DP Ø [CP what I say ]]]
    b. For any x: if I say x then I know x

Now observe that (56-b) can also be paraphrased as “When I say something, I know that it’s true”, and this is almost identical to what we proposed as a paraphrase in (50-a) above.

Let us next turn to a second syntactic analysis. This is much simpler; according to this analysis, the CP is plainly a complement of know:

(57) I know [CP who came to the party ]

This complement is called an indirect question. Adopting a semantic analysis of indirect questions proposed by Hintikka (1962), we may assume that (58) is a good paraphrase for the meaning of (57):

(58) If some individual x came to the party, then I know that x came to the
Observe that in this analysis the complement of *know* is no more an indirect question, but an ordinary *that*-clause.

By analogy, it now follows that (49) can be paraphrased as (59):

(59) If $x$ is something I am saying, then I know that I am saying $x$

I thus claim that I am aware of what I am saying, and this is precisely the paraphrase we offered above in (50-b).

We have thus shown that the two syntactic analyses are mirrored in the semantics and that a syntactic ambiguity predicts a semantic one: Since each construction brings along its own semantics, we get different interpretations, despite a superficial identity of expressions.

### 3.4.4 Discussion of Example (38)

The two readings of (38) can be paraphrased as follows:

(60) a. For which number $n$ does it hold that $n$ dogs were fed by everyone?
    b. For which number $n$ does it hold that everyone fed $n$ dogs?

There is a subtle difference here, best explained by way of describing a situation in which two different answers to the question could be given.

Suppose there are three persons $a$, $b$, and $c$ and three dogs $x$, $y$, and $z$. Assume further that $a$ fed $x$ and $y$, $b$ fed $x$ and $z$, and $c$ fed $x$, $y$, and $z$. On one reading, then, the answer is “one” because $x$ is the only dog fed by everyone. According to the other reading, the answer is “two”, because everyone fed (at least) two dogs.

The solution proposed is two different LFs, corresponding to

(61) a. How many $n$: everyone fed $n$ dogs
    b. How many $n$: $n$ dogs everyone fed

In (b), the surface order of *dogs* c-commanding *everyone* is preserved, in (a) we have reconstructed *n dogs* into the object position, so that at LF the order is reversed. This can be exploited to derive the ambiguity.

### 3.4.5 Discussion of Example (39)

Two important features of (39) can be made responsible for the ambiguity to arise: First, the sentence must have the form of a so-called yes-no-question (*Entschei*
dungsfrage); second, it must contain a complex NP coordinated by the disjunction *or*. A similar sentence with the same features is:

(62) Hat jemand die Butter oder die Margarine gesehen?
    Has anyone the butter or the margarine seen

Suppose you don’t care about the difference between butter and margarine, then an appropriate answer could simply be “yes” or “no”.

Now imagine a situation where the difference matters; in this case the intonation requires a rise on *butter* and a fall on *margarine*. The question (62) will require a quite different answer: It is expected that the answer is either “(I’ve seen the) butter” or “(I’ve seen the) margarine”. In such a scenario, an answer like yes or no would be totally inadequate because the hearer feels obliged to make a decision between the two DPs, not one concerning the truth of the propositional content of the question. Note also that in this case there is an intonation peak on *butter*, which is not the case in the first context.

Now the question arises how this difference is a case of structural ambiguity. This is hard to see, because syntax doesn’t give us any clue for a syntactic ambiguity. Yet, linguists have argued that the reading requiring a choice between butter and margarine exhibits some kind of additional structure, called **focus structure**. As indicated by intonation, both *butter* and *margarine* are so-called *focussed constituents*, whose intonation highlights the alternatives one has to choose between. Writing F as a diacritic for focus, this kind of additional element may be represented as in (63):

(63) Did you buy [F butter] or [F margarine]?

The yes-no-reading, on the other hand, would lack this kind of additional focus structure.

Focus-structure must somehow influence meaning, and it is the task of one subbranch of semantics, namely focus-semantics, to show how focus influences the interpretation of utterances.

The one example we gave in (63) is only the tip of an iceberg. Consider

(64) Paul only insulted Gertrude

(64) is ambiguous, the different readings can be paraphrased in German as:

(65) a. Paul hat Gertrude nur beleidigt
    Paul has Gertrude only insulted

b. Paul hat nur Gertrude beleidigt
    Paul has only Gertrude insulted
The scope of *only* is the verb in in (a.) and the DP in (b.). Paraphrasing further, we understand (a.) as saying that Paul didn't do anything worse than insulting, whereas (b.) means that Paul didn’t insult anyone except Gertrude.

Though ambiguous, (64) does not show any sign of syntactic ambiguity. Rather, the readings differ in focus structure, which is also reflected in different intonation patterns:

(66)    a. Paul only [F insulted ] Mary
        b. Paul only insulted [F Mary ]

Putting stress on the F-constituents disambiguates the structure and must have an influence on meaning.

Note finally that an alternative way of disambiguating our earlier example (10), repeated in (67), is to assume different focus structures.

(67)    Paul doesn’t know Gertrude because she lives in Hamburg
        a. Paul doesn’t [VP [F know ] Gertrude because she lives in Hamburg ]
        b. Paul doesn’t [VP know Gertrude because [F she lives in Hamburg ]]

(67-b) asserts that the reason for knowing Gertrude is not that she lives in Hamburg, (67-a) asserts that living in Hamburg is the reason for not knowing Gertrude. In both readings the syntactic structure is the same (low attachment of the *because*-clause), but the focus structure differs, resulting in an obvious effect on meaning.

**HOMEWORK:** Discuss the following examples:

(68)    Mein Bruder  möchte eine Norwegerin heiraten
        my brother want-to a Norwegian marry

Another example, known as Russell’s ambiguity (discussed in Russell (1905)), is this:

(69)    Ich dachte  Ihre Yacht ist länger als sie ist
        I thought your yacht is longer than it is

In one reading, my belief is contradictory: its impossible that my yacht is longer than it in fact is, and therefore it is highly implausible that I entertain a belief in such a contradiction. In the normal natural reading no contradiction arises. Try to paraphrase this reading and analyse the ambiguity in terms of scope.
4 Introducing Extensions (Extensionen)

In the last sections, we have shown that many ambiguities can be traced back to an ambiguity of syntactic structure. The general principle that motivates such a move is that sentence meaning depends on the meaning of individual words and syntactic structure, i.e. the way these words are put together in syntax. For example, two unambiguous words can be arranged in different orders, as in

(1)  
  a. Fritz kommt  
  b. Kommt Fritz

Whereas the verb-second structure in (a) is normally interpreted as declarative sentence, the verb-first structure in (b) is interpreted as yes-no-questions. The two arrangements lead to different meanings, although the lexical material is the same (and there is no ambiguity of scope involved here).

The strategy we will pursue in what follows is to take the meaning of words and then combine them alongside and in tandem with the syntactic structure. Such a procedure is called **compositional** and the principle behind it is this: the meaning of a complex expression is fully determined by its structure and the meanings of its constituents. Once we know what the parts mean and how they are put together we have no more leeway regarding the meaning of the whole. This is the principle of compositionality, a fundamental assumption of most contemporary work in semantics.


(2) **Frege's Principle of Compositionality:**

The meaning of a composite expression is a function of the meaning of its immediate constituents and the way these constituents are put together.

[Die Bedeutung eines zusammengesetzten Ausdrucks ist eine Funktion (ergibt sich eindeutig) aus den Bedeutungen seiner unmittelbaren Teile und der Art ihrer Kombination.] (cf. ☞Frege-Prinzip)

It thus follows that not only do the meanings of the words determine the meaning of the whole; it also holds that the meaning of a complex expression can only depend on the meaning of its immediate constituents (its daughter nodes), together with the specific way of syntactic combination. Therefore syntactic structure is all the more important for any calculation of meaning. Each constituent must be assigned a meaning on the basis of the meaning of its immediate constituents. Immediate constituents may themselves be complex, having immediate constituents of their own. This way, the procedure matches the recursiveness of syntax: it must
also be recursive. The recursiveness of semantics then explains why it is possible that we understand sentences we might never have heard before.

In these notes, we will follow the tradition of the so-called logical semantics (logische Semantik) which was originally developed (at the end of the nineteenth century) as an attempt to make the language of mathematics more precise. As it turned out, the methods developed there proved to be flexible enough to be also applicable to the semantics of natural language.

Historical Remark

The most important pioneers of modern linguistic theories were the philosophers Gottlob Frege (1848-1925) and Bertrand Russell (1872-1970), who both worked on the foundations of mathematics at the end of the 19th century.

Interestingly, both authors considered natural language too irregular to be rigorously analysed by the logical methods they developed; their primary interest in this respect was the development of a language that is not like natural language in that it does not contain any ambiguities. Nonetheless, their influence on modern linguistics, notably that of Frege’s article Sense and Reference (Über Sinn und Bedeutung, 1892) (=Frege (1982) and Russell’s On Denoting (1905) (=Russell (1971)) cannot be underestimated.

The terms extension and intension are similar to Frege’s use of the terms Bedeutung and Sinn; they originate from the work of Rudolf Carnap (cf. Carnap (1947), = Carnap (1972)). The term intension should not be confused with the homophone intention, there is no relation whatsoever between the terms.

Who is who in philosophy?

4.1 Psychologism (Psychologismus)

Given Frege’s Principle, it is necessary to have a clear concept of the meaning of a word. In section (2.), we approached this question by considering relations between meanings: two meanings can be synonymous, incompatible or hyperonym, etc. We did not, however, develop a clear concept of the meaning of an individual word as such.
When we learn a new word, we learn how to combine a certain pronunciation, its phonetics and phonology (Lautgestalt) with its meaning. Thereby, a previously meaningless sequence of sounds like schmöll becomes vivid, we associate with it the idea of someone who isn’t thirsty any more. In that case, one might be tempted to say that the meaning of an impression is the idea or conception (Vorstellung) a speaker associates with its utterance.

Such a notion of meaning has been argued against by Frege, Russell and many other philosophers of language. They condemned it as “psychologism” and raised the following objections:

- **Subjectiveness:** Different speakers may associate different things with a single word at different occasions: such “meanings,” however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these so-called “private meanings” serve to communicate between different subjects.

- **Limited Coverage:** We can have mental images of nouns like horse or table, but what on earth could be associated with words like and, most, only, then, of, if, …?

- **Irrelevance:** Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.

- **Privacy:** The associations of an individual person are in principle inaccessible to other speakers, so how comes that they can be used for interpersonal communication?

In view of these considerations, many authors concluded that we need a more objective notion of meaning. Suppose you have just learned the meaning of schmöll. What you have acquired is not only associations, but also the facility to apply the expression in an appropriate way: you might refuse a glass of orange juice because you are schmöll. You say: “Danke, ich bin schmöll.” Given that your communicative partner has somehow acquired the same meaning, this common behavior is based on the following assumptions:

- each partner has learned the meaning of an expression in a similar way, most frequently by reference to the kinds of things, events, properties etc., that the expression is intended to denote: we refer to horses (or pictures of horses) when we make a child learn the word horse; we smile when we teach the word smile etc.
• each partner wants to convey information in a way that guarantees the content of the message to be identical for both the speaker and his audience; otherwise, misunderstandings were the rule rather than the exception.

• each partner is capable of extracting certain abstract meanings from the use of certain words like and which do not have a depictive meaning.

The first aspect of this notion of meaning captures the fact that by using words, we can refer to things in the “outside world”, i.e. in our environment; this is an objective feature of the word in relation to the world, which is called the reference (Sachbezug) or the referential meaning of an expression.

The second aspect of communication is that, while speaking, there is some flow of information that may change the mental state of the listener in a specific way, depending on what has been said (and of course how it has been said, but as we discussed before, this is not part of the literal meaning). In other words, an utterance is useful because it can change the state of information the listeners are in. How this can be achieved is part of a semantic theory that will be developed in the sections to follow.

Simplifying somewhat, we may say that any description of the semantics of an expression involves two aspects: a referential one that enables us to refer to things by using linguistic expressions — this will be called the extension (Extension, Sachbezug) of an expression — and another aspect that deals with the information structure, which will be called the intension (Intension) of an expression. In this section we deal only with extensions, we come back to intensions in section 7.

4.2 Simple Extensions (Einfache Extensionen)

For some expressions of natural language it is fairly obvious that they refer to things or persons, for others a little bit of reflection is necessary to find an appropriate extension, and for a few there seems to be no reference at all. Let us look at some examples:

(3) — Tübingen, Heidelberg, Prof. Arnim v. Stechow, Ede Zimmermann (proper names (Eigennamen))
— the president of the US (definite descriptions (Kennzeichnungen))
— table, horse, king (nouns (Nomina))
— bald, red, stupid (adjectives (Adjektive))
— nobody, nothing, no dog (negative quantifiers (negative Quantoren))

Proper names and descriptions are the simple cases, Tübingen and Heidelberg clearly refer to certain cities in Germany. What nobody and nothing refers to
is mysterious; adjectives and nouns are somewhere in between: a noun like table does not refer to a particular table, nonetheless a certain reference is recognizable. In what follows we will try to find certain kinds of objects (sometimes of a very abstract nature) that may serve as the reference of different types of expressions of natural language. These objects will be called the extension of the respective expression.

As a terminological side remark, note that above we said that a proper name like Heidelberg refers to the city of Heidelberg; some linguists, however, would object and say that it is not the name itself but the language user who refers to Heidelberg when uttering the word. Nonetheless, the city is the extension of the name, and in these notes we use the term reference ambiguously for both the act of referring and the thing that is denoted by an expression. It is always the thing that is important in what follows. As a more neutral term, we will also say that an expression denotes its extension. This is equivalent to saying that an expression (or its user) refers to its extension.

Let us now look at the above examples more closely. As already said, names and definite descriptions denote individuals. However, there is a difference between names and descriptions. The relation between the name and its bearer (its reference) is a completely conventional one: whoever has been baptized Arnim v. Stechow is Arnim v. Stechow. The act of baptizing is one that establishes a linguistic convention between a name and an individual. In contrast, the reference of a definite description cannot be determined purely by recourse to a convention. Barack Obama is not president of the US by linguistic convention, but because he has been elected president by the American voters. The reference of the definite description is the individual that, at a given time, happens to be president of the US. The determination of the reference thus depends on what the facts in a particular situation are. Obama is the president today, so the description denotes Obama when used today, whereas the description denoted George Bush in 2008. Thus, the reference (extension) of a description depends on time and circumstances. This is not true for names: once being Arnim v. Stechow means always being Arnim v. Stechow. But being the president does not mean always being the president.

Accordingly, the denotation of a description depends on situation and context. Some situations might be such that an expression has no extension at all. The most famous example is the expression the present king of France. The example was invented by Bertrand Russell (see above) at a time when there was no such king any more. The point is that the lack of a reference at that time is not the result of a linguistic convention, but the result of the French Revolution. This failure to refer to any thing or person rarely happens with proper names. We rarely baptize things, give them names and then discover that in fact they do not
exist. If this happened, we would conclude that something went wrong in the act of baptizing.\(^{15}\)

What both proper names and definite descriptions have in common is their reference to **individuals** (Individuen) or **singularities** (einzelen Gegenstände). These terms are technical terms coined by philosophers in search of a handy name for the entities referred to by linguistic expressions. Common nouns like *king* or *table* do **not** refer to singularities, they have, as extensions, what is sometimes called a **multiple reference** (they are “Gemeinnamen, Allgemeinnamen”). Instead of saying that such terms have more than one extension, we say that their extensions are **sets** of singularities. E.g. the extension of the term *table* is the set of all tables.

Sets play an important role in semantics. The notion derives from mathematics, namely from set theory (as you might have guessed). A set is an abstract collection of things or distinct objects; it is completely determined by its **elements**, the members of the set. Thus, if we are speaking of the set of cities, each city is an element of this set, and this is all we can find in there: it’s only cities in there.

In order to name a set, we can list its members; this is most often done by using curly brackets. E.g. the set of cities can be listed by specifying a list like

\[(4) \{\text{Madrid, Venice, Berlin, Tübingen, Rome} \ldots\}\]

The order of elements here is immaterial. Now, to express that Berlin is a city, we formalize this by saying that Berlin is an **element** of the extension of city. This is written as

\[(5) \text{Berlin} \in \{\text{Madrid, Venice, Berlin, Tübingen, Rome} \ldots\}\]

Of course this only works for small sets. It is impossible for anyone to give a complete list of German cities, but in principle, this could be done, and has been done, cf. \(\text{☞ Städteverzeichnis Deutschland}\).

Note that the denotation of *city* depends on some sort of convention, namely that a place can only be called a city if it has got certain forensic title, its town charter (Stadtrecht). It also depends on the facts of the world whether or not a settlement got that title. Moreover, things may change in time: what is a city now may not have been a city 100 years ago, or may lose its city status by becoming part of a larger city. Thus, the extension of *city* depends on the facts in the world.

This we observed already with definite descriptions. This common feature of descriptions and common nouns is explained by the fact that descriptions contain

\(^{15}\)Scientists might invent names for things they assume to exist: Eg. the Planet Vulcan was hypothesized by some 19th century astronomers, who wanted to explain irregularities in the planetary path of Mercury. They only later found out that no such planet exists. In that case, the initial act of baptizing went wrong: there was no real baptizing because there was nothing that could have been baptized.
common nouns as constituents. Eg., the description the largest city in Germany contains the common noun city, whose extension may vary. Consequently, the extension of the description can vary as well (eg. the denotation could have been different before and after reunification). Likewise, the extension of the king of France is nowadays empty (ie. there is no king of France) because the denotation of the (complex) noun king of France is the empty set. Note that the extension of the expressions king of France and king of Germany are identical (at the time of writing), both denote the empty set. Yet it is clear that the meaning is different; the extension only describes a limited aspect of the meaning.

It has been proposed that the extension of each common noun (at a given time) is a set. Granted that these extensions depend on the facts, and given that our factual knowledge might be limited, it follows that we sometimes simply don’t know the extension of a word. That is, we do not always know what elements belong to an extension, which means that we might not know the exact extension of a word like table in the actual world we are living in. But in practice and in theory (and as far as linguistic theorizing is concerned) this lack of knowledge is less important than one might think. First, one should bear in mind that the actual extension of a word should not be confused with its meaning. Hence, not knowing the actual extension does not imply not knowing its meaning. The fact that we do not know all the details of the world we inhabit has nothing to do with our linguistic conventions and abilities. Second, not knowing the actual extension does not imply that we are unable to decide (on demand) whether a given entity is a table or not: of course we can apply the notion to things we have never seen before and whose existence we didn’t know anything about. This implies that we are endowed with ways to determine the extension of a word without knowing the extension in advance. This is an important aspect of a word’s meaning, though not the entire meaning itself.

Thirdly, in scientific inquiry we are often entitled to abstract away from certain insufficiencies. Eg., the meaning of almost any noun is vague: there can always be borderline cases. Consequently, also the extensions can be vague. Although it would be possible to capture this in a vague set theory (called fuzzy set theory) we may well ignore this additional complication. This does not imply that vagueness is always unimportant: for example, we might be uncertain where to draw a line between sphere (Kugel) and ball (Ball, Kugel), yet the word ball pen translates into German not as Ballschreiber, but as Kugelschreiber. And a ball in american football would hardly be called a Ball by a German soccer player (it’s probably called an egg). So for some intents and purposes, it is important where to draw the line between Ball/ball and Kugel/sphere, but for the calculations we will make in the present introduction, such differences will play no role, and hence will be
Apart from nouns, adjectives can be said to denote sets, too. Again, though, numerous questions arise: e.g., color terms like red and green are notoriously vague: Where exactly is the borderline between red and green? For the time being, these difficulties can be ignored.

Sets can also be used as the extension of intransitive verbs. For example, the verb sleep has as its extensions the set of all sleepers, which is of course the set of sleeping individuals. The sentence

(6) John is sleeping

can be said to be true if and only if the individual John is an element of that set.

For transitive verbs, however, we get into difficulties. Take the verb kiss as an example. Intuitively, in a sentence like

(7) John kisses Mary

two individuals are involved. The two are connected by the relation of kissing. The relation of kissing thus applies to the pair consisting of John and Mary. This is an ordered pair, which is why (7) is different from

(8) Mary kisses John

Let us introduce a notation for ordered pairs: a pair is enclosed in angled brackets \( \langle a, b \rangle \) with \( a \) as the first element of the pair and \( b \) as the second element. Note that although the set \( \{a, b\} \) is the same as \( \{b, a\} \), this does not hold for ordered pairs: the pair \( \langle a, b \rangle \) is different from \( \langle b, a \rangle \) (unless \( a = b \)).

We might say, then, that (7) holds if and only if the pair \( \langle \text{John}, \text{Mary} \rangle \) is an element of the relation of kissing, whereas (8) is true if and only if the pair \( \langle \text{Mary}, \text{John} \rangle \) is such an element. For this to make sense we must assume that kiss denotes a relation, and that relations are sets of ordered pairs. This is precisely how the notion of a relation is formalized in mathematics.

(9) a. sleep (= schlafen): the set of sleepers
b. kiss (= küssen): a relation between kissers and kissees, i.e. the set of pairs \( \langle x, y \rangle \) such that \( x \) kisses \( y \).
c. give (= geben): a three-place relation, a set of triples.

16 Other complications arise with nouns like milk which are called substance words or a mass nouns (Massennomen) because they refer not to individuals but to bits and parts of a substance. We might then say that any bunch of molecules that makes up a quantity of milk is an element of the milk-set. This method raises a number of questions we cannot discuss here; let us therefore ignore these kinds of expressions and turn to expressions we can easily handle in terms of sets.
The notion of a triple should be obvious: whereas a pair is a sequence of two elements, a triple is a sequence of three elements. We may thus summarize our descriptions of certain extensions in the following table.

(10)

<table>
<thead>
<tr>
<th>type of expression</th>
<th>logical type of extension</th>
<th>example</th>
<th>extension of the example</th>
</tr>
</thead>
<tbody>
<tr>
<td>proper name</td>
<td>individual</td>
<td>Fritz</td>
<td>Fritz Hamm</td>
</tr>
<tr>
<td>definite description</td>
<td>individual</td>
<td>die größte dt. Stadt</td>
<td>Berlin</td>
</tr>
<tr>
<td>noun</td>
<td>set of individuals</td>
<td>Tisch</td>
<td>the set of tables</td>
</tr>
<tr>
<td>intransitive verb</td>
<td>set of individuals</td>
<td>schlafen</td>
<td>the set of sleeping individuals</td>
</tr>
<tr>
<td>transitive verb</td>
<td>set of pairs of individuals</td>
<td>essen</td>
<td>set of pairs (eater, eaten)</td>
</tr>
<tr>
<td>ditransitive verb</td>
<td>set of triples of individuals</td>
<td>schenken</td>
<td>set of triples (donator,recipient,donation)</td>
</tr>
</tbody>
</table>

Recall that our list in (3) also contains the negative expressions *nobody, nothing*, and *no dog*. We have not attempted yet to describe their extension; this will be done in section 5.4.

4.3 Truth Values as Extensions of Sentences (Wahrheitswerte)

Looking at the verbs in (9), one may detect an ordering, the so-called **arity** of verbs: 1-place verbs need only a subject; 2-place verbs (called transitive) need a subject and an object; 3-place verbs are called ditransitive: they require a subject, a direct object, and an indirect object. Corresponding to these types of predicates there are three-place tuples (triples), two-place tuples (pairs) and one-place tuples (individuals). The generalization here is that predicates can be represented by sets of \(n\)-place tuples. So there is a simple connection between the valency of a verb and the \(n\)-tuples in its extension: the higher the former, the longer the latter. We thus arrive at the following observation:

(11) **Parallelism between valency and type of extension:**
The extension of an \(n\)-place verb is always a set of \(n\)-tuples.

What is remarkable about this parallelism is the fact that it does not only hold for lexical expressions, it holds for complex expressions as well. E.g. *walk* is a one-place predicate, and so is *walk slowly*, therefore *walk slowly* will also denote a set 1-tuples. *slice* is two-place, and so is *slice slowly/carefully* etc. Moreover, by adding an object to a two-place relation, eg. adding *the salami* to *slice*, we get *slice the salami*, which itself only requires a subject. This implies that adding an object turns a two-place relation into a one-place predicate. Likewise:

(12) give (3-place)
give a book (2-place)
give a book to the student (1-place)

(13) I give a book to the student (0-place)

The last step in (12) suggests that one-place predicates are one-place “relations”; this terminology might seem somewhat counterintuitive, since relations are normally conceived of as two-place. But there is nothing wrong with extending the terminology and this is indeed standard practice in mathematics.

What might be even more puzzling is the step from (12) to (13); the qualification “0-place” is in fact intended to suggest that the arity of a sentence is zero and that sentences are nothing but zero-place verbs or zero-place relations. This is quite remarkable, but still somewhat mysterious, unless we know how to deal with zero-place relations.\footnote{Speaking of sentences as of 0-place verbs might be felt as undue terminological hardship. Perhaps a more intuitive conception is to replace the notion of an \( n \)-place verb by that of a sentence with \( n \) gaps. Thus, a transitive verb is a sentence with 2 gaps, an intransitive verb is a sentence with 1 gap, and a sentence a sentence with no gaps. The connection would then be that a sentence with \( n \) gaps denotes a set of \( n \)-tuples.}

Let us illustrate this in a table:

(14)

<table>
<thead>
<tr>
<th>verb or verb phrase</th>
<th>valency</th>
<th>extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>zeigt</td>
<td>3</td>
<td>set of all triples ( \langle a,b,c \rangle ) such that ( a ) shows ( b ) to ( c )</td>
</tr>
<tr>
<td>zeigt dem Präsidenten</td>
<td>2</td>
<td>set of all pairs ( \langle a,b \rangle ) s.th. ( a ) shows ( b ) to the president</td>
</tr>
<tr>
<td>zeigt den Vatikan</td>
<td>1</td>
<td>set of all 1-tuples ( \langle a \rangle ) s.th. ( a ) shows the Vatican to the president</td>
</tr>
</tbody>
</table>

We might then continue in the following way:

(15)

<table>
<thead>
<tr>
<th>sentence</th>
<th>valency</th>
<th>extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Der Papst zeigt dem Präsidenten den Vatikan</td>
<td>0</td>
<td>set of all 0-tuples ( \langle \rangle ) s.th. the Pope shows the Vatican to the president</td>
</tr>
</tbody>
</table>

But what is a zero-tuple? It’s a list of length zero! Mathematicians assume that such lists exists. Of course, there is only one such list, the empty list. For the sake of simplicity, we identify this list here with the empty set. We thus have the following:

(16) A zero-tuple is an empty list; there is only one such list, and we identify this list with the empty set \( \emptyset \). The set of all zero-tuples contains just one element, namely the empty set \( \emptyset \).
Frege’s Generalization:
The extension of a sentence is a set of 0-tuples.

According to this generalization, the extension of a sentence is the set of all 0-tuples, such that the Pope shows the Vatican to the president. What set is this? Apparently, there is only one 0-tuple, which we identified with the empty set. Now, if it is indeed the case that the Pope shows the president the Vatican, then this set contains all 0-tuples (and nothing else). That is, this is the set which contains the empty set as its only element. This is \( \{ \emptyset \} \). If the Pope does not show the president the Vatican, then there is no such 0-tuple and the extension is empty: this is \( \{ \} \). But this is exactly the same as the empty set, which we have written as \( \emptyset \). We thus get the two possible extensions \( \{ \emptyset \} \) and \( \emptyset \).

Our next step is to note that this does not only work for the particular sentence under discussion. It works for all sentences the same way! That is, if a sentence is true, its extension is \( \{ \emptyset \} \), and this holds for all true sentences. This means that all true sentences have the same extension, namely \( \{ \emptyset \} \). Likewise, all false sentences have the same extension, namely the empty set \( \emptyset \). These two sets are also called truth values (Wahrheitswerte). In the context of linguistic semantics, they are also represented by the letters \( T \) and \( F \) or by the numbers 1 and 0:

\[
\{ \emptyset \} \text{ is also written as } T \text{ (for true; } W \text{ für wahr), 1, or sometimes } T \\
\emptyset \text{ is also written as } F \text{ (for false; } F \text{ für falsch), 0, or sometimes } \bot
\]

Recall that the extension of an expression was called its reference (Sachbezug). It should have become clear by now, that the extension of a sentence (its reference), being its truth value, cannot be identified with its meaning, or otherwise all true sentences would be synonymous. But we already remarked above that there is more, namely the information conveyed by a sentence, its intension, that contributes to the meaning of a sentence. Before going into intensions, let us see what we can do with simple extensions. The basic question we want to answer is this: how can we determine the extensions of phrases and sentences, given the extensions of words?

5 Composing Extensions

The Principle of Compositionality stated in Chapter 4, goes a long way toward explaining how speakers and hearers are able to use and understand expressions they have not come across before: starting with the smallest ‘atoms’ of syntactic structure, the words or morphemes provided by the lexicon, the meanings of ever more complex expressions can be determined by combining the meanings of their parts. Hence the language user only needs to learn and know the meanings of the
lexical expressions and the ways in which they are combined.

The meanings thus determined in turn may serve to relate to the extra-linguistic world around us in ways not accessible by lexical meanings alone. Insects are cases in point. They rarely have names\textsuperscript{18} and usually cannot be referred to by lexical expressions other than pronouns. However, even a nameless bug long gone and far away can be singled out by a definite description like the creature that bit me in my left earlobe half a year ago. And compositionality explains how this is possible: the lexical meanings of the parts combine into the meaning of the entire description, which in turn determines a certain animal (provided there is one that fits the description). Now, whatever this meaning is, it somehow encodes information that suffices to determine a particular nameless insect—reference to which thus becomes possible by a suitable composition of lexical meanings.

Although in general the extensions of complex expressions are determined by compositionally determining their meanings first, it turns out that more often than not, there is a more direct way. It is a remarkable fact about language that in many (though not all) cases, the referent of an expression can be determined by combining the extensions of its parts in a compositional way. In this chapter we will look at a variety of such cases, some of which will also help finding out what the extensions of particular expressions are in the first place. Only thereafter, in Chapter 7, will we look at the limitations of the composition of extensions and the nature of meaning in somewhat more general terms.

5.1 Connectives and Truth Tables

Identifying the extensions of (declarative) sentences as truth values has important—and somewhat surprising—consequences for the semantic analysis of complex sentences. For it turns out that, in certain cases, the extension of a complex sentence is entirely determined by the extensions of its immediate parts. (1) is a case in point:

(1) Harry is reading and Mary is writing

Under the (disputable) assumption that the conjunction (and) and the two underlined sentences form the immediate parts of (1), we may observe that the truth value of the entire sentence is fully determined by the truth values of the latter: if either of them is false, then so is (1); otherwise, i.e. if both are true, (1) is as well. In a similar vein, we observe that the truth value of (2) also depends on the extensions of the underlined sentential parts:

\textsuperscript{18}A. A. Milne’s beetle Alexander (http://blog.ewanscorner.com/2010/07/alexander-beetle/) may be an exception, but then again it may also be a piece of fiction…
(2) **Harry is reading or Mary is writing**

In the case of (2), the whole sentence is true as long as one of the underlined sentences is; otherwise, i.e. if both are false, then so is (2). Hence the extensions of coordinated sentences like (1) and (2) depend on extensions of the sentences coordinated in a way that is characteristic of the respective conjunction. These dependencies can be charted by means of so-called *truth tables*:

(3) **Harry is reading and Mary is writing**

<table>
<thead>
<tr>
<th><strong>Harry is reading</strong></th>
<th><strong>Mary is writing</strong></th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(4) **Harry is reading or Mary is writing**

<table>
<thead>
<tr>
<th><strong>Harry is reading</strong></th>
<th><strong>Mary is writing</strong></th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(3) and (4) show the possible distribution of truth values of the sentences coordinated, and the effect they have on the truth value of (1) and (2), respectively. In both cases this effect may be thought of as the output of a certain operation acting on the inputs given by the constituent sentences. As the reader may immediately verify, the operation described in (3) always outputs the maximum of the input values, whereas the one in (4) uniformly yields their minimum.

Using standard formal logic notation, these operations may be indicated by the symbols ‘∧’ and ‘∨’. Hence the truth value of (1) can be written as ‘\( p \land q \)’, where \( p \) and \( q \) are the truth values of its constituent sentences; similarly ‘\( p \lor q \)’ denotes the truth value of (2).

Since truth values are the extensions of sentences, (3) and (4) show that the extensions of coordinations like (1) and (2) only depend on the extensions of the coordinated sentences and the coordinating conjunction. Let us write the extension of an expression \( A \) by putting double brackets ‘\([\ [ ] \ ]\)’ around \( A \), as is standard in semantics. The extension of an expression depends on the situation \( s \) referred
to; so we add the index $s$ to the closing bracket. Thus, $\llbracket [\text{Harry is reading}]s \rrbracket$ denotes a truth value, viz. the truth value of (1), i.e. $\llbracket (1) \rrbracket_s$.

Generalizing from the above example to any declarative sentences $A$ and $B$, we thus conclude:

(5) $\llbracket A \text{ and } B \rrbracket_s = \llbracket A \rrbracket_s \land \llbracket B \rrbracket_s$

(6) $\llbracket A \text{ or } B \rrbracket_s = \llbracket A \rrbracket_s \lor \llbracket B \rrbracket_s$

(5) and (6) show in what way the extension of a sentence coordination depends on the extensions of the sentences coordinated and the choice of the coordinating conjunction. The latter, it would seem, contributes a combination of truth values. It is therefore natural and, indeed, customary to regard this contribution itself as the extension of the respective conjunction. In other words, the extension of and (in its use as coordinating declarative sentences) is the combination $\land$ of truth values, as charted in (3). Similarly, the extension of or may be identified with the operation depicted in (4). We thus arrive at:

(7) $\llbracket \text{and} \rrbracket_s = \land$

(8) $\llbracket \text{or} \rrbracket_s = \lor$

(7) and (8) allow us to compositionally derive the truth values of (1) and (2), respectively, directly from the extensions of their immediate parts, without first determining their meanings:

(9) $\llbracket (1) \rrbracket_s$

$= \llbracket [\text{Harry is reading}]s \rrbracket \land \llbracket [\text{Mary is writing}]s \rrbracket$

$= \llbracket [\text{Harry is reading}]s \rrbracket \llbracket \text{and} \rrbracket_s \llbracket [\text{Mary is writing}]s \rrbracket$ by (7)

(10) $\llbracket (2) \rrbracket_s$

$= \llbracket [\text{Harry is reading}]s \rrbracket \lor \llbracket [\text{Mary is writing}]s \rrbracket$

$= \llbracket [\text{Harry is reading}]s \rrbracket \llbracket \text{or} \rrbracket_s \llbracket [\text{Mary is writing}]s \rrbracket$ by (8)

It is easily seen that the third lines in (9) and (10) each derive from their predecessors by applying the equations (7) and (8) respectively. If they still look unfamiliar, this may be due to the fact that—just like ‘$\land$’ and ‘$\lor$’—the notations ‘$\llbracket \text{and} \rrbracket_s$’ and ‘$\llbracket \text{or} \rrbracket_s$’ both stand for arithmetical operations that can be applied to truth values. And it is precisely this application to the truth values of the sentences conjoined that delivers the truth value of the entire coordination. Hence according to (9), the extension of (1) is obtained by applying the extension of one of its parts, viz. and, to the extensions of the other two parts; and similarly for (2). Hence in both (1) and (2) the extension of the entire expression is determined by the extensions of its (immediate) parts in the same way, i.e. by applying the extension of one of
them to the extensions of the others. As we will see in the remainder of this chapter, this combination of extensions is frequent enough to deserve its own name. Since it consists in \textit{applying a function} (denoted by one of the parts) to its arguments (given by the other parts), we will say that the extensions of coordinated sentences like (1) and (2) are obtained by \textit{functional application}.

Let us now see how the truth values of simple sentences like the ones conjoined in (1) and (2) can be obtained from the extensions of their parts.

\textbf{5.2 Subject and Verb}

Now that we've learned that the extension of a sentence is a truth value, it's easy to see how extensions can be combined in accordance with Frege's Principle:

\begin{equation}
\text{Extensional Principle of Compositionality:}
\end{equation}

The extension of a compound expression is a function of the extensions of its immediate parts and the way they are composed.

Note that we slightly modified the principle: the difference from the general principle of compositionality stated earlier is that we now take into account only the \textit{extensions} of the expressions involved (rather than their meanings or intensions). This is a simplification to which we return in section 7.

Let us now apply the principle to a simple sentence like

\begin{equation}
\text{Paul schnarcht}
\end{equation}

\begin{equation}
\text{Paul snores}
\end{equation}

\begin{equation}
\text{‘Paul is snoring’}
\end{equation}

The immediate parts are Paul and schnarcht. The extension of the proper name is the individual Paul, the extension of schnarcht is the set of snoring individuals (at a certain time in a certain situation). Can we determine a truth value by looking at these extensions? Surely we can: Having adopted the framework of set theory, we only need to say that the sentence is true if the extension of Paul is an element of the extension of schnarcht (= the set of individuals snoring). If Paul is not an element of that set, the sentence is false. The same is true for Tim schnarcht. And for Paul schläft (Paul is sleeping), Tim schäft etc. Therefore, a general pattern is at work here:

\begin{equation}
\text{The extension of a sentence of the form “proper name + verb” is the truth value T if and only if the extension of the proper name is an element of the extension of the verb; otherwise its extension is F (the sentence is false).}
\end{equation}
In a similar way we can ‘calculate’ the extensions of simple clauses whose subject is a not proper name but a definite description or any expression whose reference is an individual. For descriptions, an English example is the following:

(14) The president of the USA walks

The only thing our truth conditions require is this: Whoever should turn out to be the president (e.g., Barack Obama, at the time of writing), check if this individual, namely the extension of ‘the president of the US,’ is among the individuals walking (at the time of writing). If so, the sentence is true, if not, the sentence is false. We thus have:

(15) \[[t\text{he president of the USA walks}]_s = T \text{ if and only if } [\text{the president of the USA}]_s \in [\text{walk}]_s, \text{ otherwise } [t\text{he president of the USA walks}]_s = F.\]

### 5.3 Verb and Object

Let us next look at sentences with an object and a transitive verb. As before, we would like to be able to calculate the extension of sentences of the form “subject + verb + object”:

(16) Paul liebt Mary

Paul loves Mary

We already know the extensions of the names, namely the individuals called Paul and Mary, and we also know the extension of liebt, which is a set of $n$-tuples, namely the set of pairs $\langle x, y \rangle$, such that it holds that $x$ loves $y$. For (16) to be true it must therefore be the case that the pair $\langle \text{Paul}, \text{Mary} \rangle$ is an element of the extension of liebt.

(17) \[[\text{Paul liebt Mary}]_s = T \text{ if and only if } \langle [\text{Paul}]_s, [\text{Mary}]_s \rangle \in [\text{liebt}]_s\]

At this point, however, we encounter a difficulty: (17) requires us to consider $\langle [\text{Paul}]_s, [\text{Mary}]_s \rangle$, i.e. the pair consisting of the subject and the object. However, when looking at the syntactic tree for (17), it turns out that the subject and the object do not form a constituent. Rather, we learned that in order to derive (16) two transformations were needed, namely topicalization and verb-second movement. In order to simplify things a bit, let us assume that these movements are undone, so that we now have to interpret the still simplified structure in (18):

(18) $[s \text{ Paul } [v\text{P Mary liebt }]]$

Equivalently, the structure for English would be (19):
It is easy to see that the pair consisting of the subject and the object mentioned in (17) still does not form a constituent in either structure. Is that a problem?

Well, yes and no. “Yes,” because the rule (17) does not conform to the Principle of Compositionality, and “no” because it’s easy to find a way around the problem so as to still conform to the principle. Taking (11) seriously means that the extension of the sentence, its truth value, must be calculated from its immediate constituents. These are Paul and the VP, which implies that we first have to calculate the extension of the VP \textit{loves Mary}/\textit{Mary liebt} before we can determine whether the sentence \textit{S} is true or false. The main question therefore reduces to the problem of assigning an extension to the VPs in (18) (or (19)). The problem seems to be that our original encoding of transitive verbs as two-place relations does not fit with the syntax.

The good news is that it is quite easy to overcome this problem. The key to an understanding of the method of compositional interpretation is to look at the set theoretical objects that correspond to the syntactic categories. In the above example, we thus ask what type of denotation the VP has, and we’ve already seen in the last section that this must be a set. But which set? Of course this must be the set of Mary’s lovers. So Paul loves Mary if and only if Paul is an element of the set of Mary’s lovers, which is the extension of the VP.

Now the only remaining problem is to determine that set on the basis of the relation \textit{love}. But again this is easy. We only have to look at those pairs in \textit{\texttt{\textit{liebt}}}_{s} (or \textit{\texttt{\textit{love}}}_{s}) whose second member is Mary. These are the tuples whose common property is that someone loves Mary. Now out of this set of pairs, we collect all the first members and let them form a new set. This of course is the set of Mary’s lovers.

What remains to be done is to state a general rule that interprets complex verb phrases along these lines. Before turning to this rule, observe that our old rule (13) should be revised. Previously, we only looked at combinations between a subject and a verb. Now, we are looking at a combination between a subject and a VP. But since an intransitive verb is also a VP, our old rule can simply be restated as follows.

\begin{equation}
\text{The extension of a sentence of the form “proper name + VP” is the truth value T if and only if the extension of the proper name is an element of the extension of the VP; otherwise the sentence is false.}
\end{equation}

For intransitive verbs, the extension of the VP is simply the extension of the verb. For complex VPs containing a transitive verb and its object, we need a new rule:
The extension of a VP of the form “verb + proper name” (or “proper name +
verb” for German) is the set of all individuals such that any pair consisting
of that individual and the extension of the proper name is an element of
the extension of the verb.

This sounds complicated enough, so let us demonstrate the method by way of
a small example. Assume the relation of loving is represented by the following
n-tuples:

Let the relation love be the following set of 2-tuples:
{⟨a, b⟩, ⟨b, c⟩, ⟨b, d⟩, ⟨b, e⟩, ⟨d, d⟩, ⟨e, d⟩}

Note that this extension is purely stipulated, any other two-place relation would
do the same job. Technically speaking, what we did by stipulating a certain (arbi-
trary) extension like (22) is to assign a particular interpretation to the word
liebt (or rather the infinitive form lieben); any other set of pairs would also be an inter-
pretation and would do the same job. The idea is that instead of having to specify
the real extensions in the real world, which is often impossible because we lack
knowledge about the real extensions, we can simply choose a representation that
goes proxy for the real extension. Any such collection of extensions is also called
a model for a language. Thus, a model contains all extensions of all expressions
of a language. In our case, the very small model contains just the extension of the
predicate love and the extension of proper names.

Let us assume that in our model (22) d is the extension of Dorothy, and a is
the extension of Albert. Is it true in our model, that Albert loves Dorothy? Of
course we could inspect (22) directly by looking for a pair ⟨a, d⟩. But this is not
the point of the exercise. What we want to calculate is the extension of the VP love
Dorothy, so we first look at all pairs containing d as an object. This is

⟨b, d⟩, ⟨d, d⟩, and ⟨e, d⟩

Now each of b, d, and e in (23) loves Dorothy; putting them into a set we get the
set of Dorothy’s lovers:

{b, d, e}

This is the extension of the VP. Is Albert an element of that set? No. Therefore
the sentence Albert loves Dorothy is false in this model.

Now, after having seen an example, you should return to (21); the condition
sounds rather complicated, but we hope that by having gone through an example,
its content has become much clearer. Nonetheless, the wording of the rule still
looks cumbersome. We can, however, simplify (21) enormously if we are allowed
to use a little bit of set theory. Some bits of terminology and notation will be introduced in the following “Digressions in Set Theory”.

**Digression in Set Theory (1)**

As already explained above, a set is any collection of objects. There are a number of ways to characterize sets. According to one such method we simply list the members of a set, as we already did in an earlier example:

(25) \{Madrid, Venice, Berlin, Tübingen, Rome \ldots\}

The dots here say that this characterization is not quite complete.

Another way to characterize sets is by stating a property that is common to all and only the members of a set. This is a property that qualifies an individual as a member of that set. For this one normally uses a special notation:

(26) \{x : x is a natural number that can be divided by 5\}

The letter \(x\) is called a variable. As usual, variables function as a kind of place holder standing in for objects without specifying which. The colon following the variable \(x\) is read ‘such that’. The whole expression now reads: “the set of those \(x\) such that \(x\) is ...” The use of these auxiliary symbols is highly conventionalized; for variables one normally uses letters from the end of the alphabet.\(^{20}\) Thus, instead of saying (27-a), we will also write (27-b):

(27) a. Let \(A\) be the set of all cats

b. \(A := \{x : x\text{ is a cat}\}\)

Read (27-b) as “\(A\) is defined as the set of all \(x\) such that \(x\) is a cat”. Note that \(A = \llbracket\text{cat\rrbracket}\).

\(\text{*** End of digression ***}\)

\(^{20}\)Sometimes, we can characterize infinite sets by giving an instruction or a rule that tells us how to generate such a set (such sets are called “recursive”). For example, the set \(A\) of all integers that can be divided by 5 can be characterized as:

(i) a. \(0 \in A\)

b. If \(x \in A\), then \(x + 5 \in A\),

c. Nothing else is in \(A\).

The a.-clause states the beginning case of the induction. The (b.) part states that by adding 5 over and over again, we generate new members of that set. The third part ensures that no number is in the set other than those being generated by the operation described in (i-b).
Given these conventions let us return to rule (21). By applying the set notation and also the bracket notation for extensions, we can simplify (21) considerably:

\[(28) \quad \llbracket \text{verb } + \text{ proper name} \rrbracket_s := \{ x : (x, \llbracket \text{proper name} \rrbracket_s) \in \llbracket \text{verb} \rrbracket_s \}\]

Now, this looks quite easy again, as it is, and therefore we will continue to use this kind of notation when stating further rules for the combination of extensions.

HOMEWORK: In our discussion of (28) we assumed that the verb is two-place. What changes are necessary for three-place verbs? Try to analyse:

\[(29) \quad (\text{dass}) \text{ Paul Bernadette Geraldine zeigt}
\]
\[(\text{that}) \text{ Paul Bernadette Geraldine shows}
\]
\[\text{'(that) Paul shows Bernadette to Geraldine'}\]

Note that we do not have to change rule (28); we only need a new rule that transforms three-place verbs into two-place verbs.

Another rule is needed for the interpretation of

\[(30) \quad \text{Paul is old}
\]

Assume that old denotes the set of old things. What (30) says, then, is that Paul is an element of that set. It seems, then, that the verb be in this context has no meaning at all. This is exactly what (31) says:

\[(31) \quad \llbracket \text{is } + \text{ adjective} \rrbracket_s := \{ x : x \in \llbracket \text{adjective} \rrbracket_s \} (= \llbracket \text{adjective} \rrbracket_s).
\]

Later on we will also describe rules for adjectives in expressions like the old man. Before doing so, however, we have to discuss the semantics of quantifiers. This will also include finding an extension for mysterious negative expressions like no man or nothing.

5.4 Quantifiers (Quantoren)

The grammatical subject of the following sentences is neither a proper name nor a description:

\[(32) \quad \text{a. Jeder Student schnarchte}
\]
\[\text{Every student snored}
\]
\[\text{b. Eine Frau schnarchte}
\]
\[\text{A woman snored}
\]
\[\text{c. Keine Fliege schnarchte}
\]
\[\text{No fly snored}
\]
There is no relevant difference here between German and English, so it might be easier to continue our discussion with reference to English. In order to exclude unwanted (generic) readings, we switched to the past tense of the verb, but this detail is otherwise immaterial and will also be ignored in what follows.

The problem already mentioned earlier is that the subject expressions do not seem to refer to anything in particular and that, therefore, these expressions cannot have an extension. Fortunately, however, Frege found a way to treat these quantified DPs as abstract objects that do have an extension. It is clear that the extensions of nouns and verbs are sets: For example, in a normal situation during the night the predicate *snore* might contain a lot of individuals, during daytime and in particular during this lecture in the present situation the extension should rather be the empty set. Let us ignore details of this sort. Even if we don’t know the extension precisely, we can say under what conditions these sentences are true and false. We know that the extension of the entire clause is a truth value; so the task is this: given the two sets corresponding to the predicate *snore* and the predicate *student*, what is the role of *every* in determining the truth value of the sentence *every student snores*?

The trick here is to think about what must be the case for the sentences to become true in terms of a comparison between the extensions of the two predicates. Conceived of this way, we may say that

\[(33) \text{“Every student snores” is true if and only if the set of snoring entities contains the set of students.}\]

This sort of “containment” is a set theoretical notion which is called the *subset* relation. It’s useful at this place to do a little bit of set theory again.

*Digression in Set Theory (2)*

When every member of a set \(A\) is also a member of a set \(B\), we call \(A\) a *subset* of \(B\). This is formally written as \(A \subseteq B\). The notation already suggests that for \(A \subseteq B\) to hold it is not necessary that the sets are different. If every member of \(A\) is also a member of \(B\) this does not exclude the possibility of \(A\) and \(B\) having exactly the same members.\(^\text{21}\)

We also say that if \(A\) is a subset of \(B\), then \(B\) is a *superset* of \(A\). These relations are often visualized by using so called *Euler-diagrams*, as shown in (34):

\[(34)\]

\(^\text{21}\)The notation \(\subseteq\) is actually composed out of \(\subset\) and =, which suggests that \(\subset\) is the proper subset relation, whereas \(A \subseteq B\) means \(A\) is either a proper subset of \(B\) or equal to \(B\). According to the *extensionality* axiom of set theory, \(A\) and \(B\) are identical if and only if \(A \subseteq B\) and \(B \subseteq A\).
In order to describe the composition of meanings in terms of set theory, we will introduce two operations on sets. Both take pairs of sets as input and yield another set as output.

The first is **union** of sets. The union of $A$ and $B$ is defined as the set whose members are precisely the members of $A$ together with the members of $B$. These are just the objects which are elements of $A$ or $B$ (or both). The notation for this is $\cup$:

\[
A \cup B = \{ x : x \in A \text{ or } x \in B \}
\]

**Intersection** of two sets is the second operation; it's written as $\cap$ and defined as

\[
A \cap B = \{ x : x \in A \text{ and } x \in B \}
\]

Intersection produces a set whose members are just the members of both $A$ and $B$. As an example, take the sets of socks and shirts. Since nothing is both a shirt and a sock the intersection is the empty set. As another example, the set of red entities and the set of socks. Then the intersection of these sets are the set of red socks.

Note that if $A$ is a subset of $B$, then $A \cup B = B$ and $A \cap B = A$.

Both the union and the intersection of sets are often visualized by using Euler digrams:

\[
(37) \quad \text{(a.) is meant to represent intersection, (b.) union. In each digram, the shaded region represents the set that result from performing either operation to $A$ and $B$. The big disadvantage of this method is that the digrams illustrate the workings of union and intersection by representing only a special case (non-empty intersection, an overlap of $A$ and $B$) suggesting that $A$ and $B$ have specific properties they would not have in general, e.g., if $A$ is a subset of $B$, in which case the intersection...}
\]
tion of A and B would be A, the union would be B. This outcome is not directly represented in the above graphics, but of course it follows from the definition of union and intersection. Therefore Euler digrams are not really suited to define these operations, although they may nicely illustrate how the definitions work in the situations depicted by the digrams.

*** End of digression ***

Let us now return to (33) and reformulate the truth conditions by using set-theoretic notation:

(38) a. (32-a) is true if and only if the extension of student is contained in the extension of snore; that is: the set of students is a subset of the set of snoring entities. — Formally: \([\text{student}] \subseteq [\text{snore}]\).

b. (32-b) is true if and only if the extension of woman and the extension of snore have a common element; that is: the set of women and the set of snoring entities are not disjoint. — Formally: \([\text{woman}] \cap [\text{snore}] \neq \emptyset\).

c. (32-c) is true if and only if the extension of fly and the extension of snore have no common element; that is: the set of flies and the set of snoring entities are disjoint. — Formally: \([\text{fly}] \cap [\text{snore}] = \emptyset\).

Make sure that you understand these conditions and convince yourself that they are intuitively correct. Having stated (38), it seems that we are done.

However, this is not quite right. First observe that in (38) we are talking about particular sentences. What we want to formulate, however, is a general rule. So what we want so say is something like:

(39) a. \([\text{every} + \text{noun} + \text{VP}] = 1\) if and only if \([\text{noun}] \subseteq [\text{VP}]\).

b. \([\text{a} + \text{noun} + \text{VP}] = 1\) if and only if \([\text{noun}] \cap [\text{VP}] \neq \emptyset\).

c. \([\text{no} + \text{noun} + \text{VP}] = 1\) if and only if \([\text{noun}] \cap [\text{VP}] = \emptyset\).

So far, so good. But although we’ve made some progress, the rules in (39) still do not conform to the Principle of Compositionality.

There are in fact two problems to be solved. The first is that we have not yet defined an extension for the quantifier expressions every, a, and no themselves. That is, we’ve not yet said what \([\text{every}]\) is. The second problem is to harmonize (39) with syntactic structure.

Turning to the first problem, observe that the quantifiers in (39) compare the extensions of two sets and therefore describe a relation between them: one might say that every is the subset relation, some is the common element relation and no is the relation of disjointness. Thus, we can indeed assign some sort of extension
to each of the quantifiers in (38), namely a particular relation between sets.

(40) \[ \begin{align*}
\mathrm{\langle \text{every} \rangle,} & \text{ is the set of pairs } \langle X,Y \rangle \text{ such that } X \subseteq Y; \\
\mathrm{\langle \text{a} \rangle,} & \text{ is the set of pairs } \langle X,Y \rangle \text{ such that } X \cap Y \neq \emptyset; \\
\mathrm{\langle \text{no} \rangle,} & \text{ is the set of pairs } \langle X,Y \rangle \text{ such that } X \cap Y = \emptyset.
\end{align*} \]

So conceived, we can state the semantic analysis of the sentences in (32) as in (41):

(41) \[ \begin{align*}
\langle \mathrm{\langle \text{student} \rangle,} & \text{\langle \text{snore} \rangle} \rangle \in \langle \text{every} \rangle, \\
\langle \mathrm{\langle \text{woman} \rangle,} & \text{\langle \text{snore} \rangle} \rangle \in \langle \text{some} \rangle, \\
\langle \mathrm{\langle \text{fly} \rangle,} & \text{\langle \text{snore} \rangle} \rangle \in \langle \text{no} \rangle.
\end{align*} \]

However, it is obvious that the ordered pair is not a constituent in the syntactic tree. In order to get into the position to tackle this problem it’s worthwhile again to do a little bit of set theory.

**Digression into Set Theory (3)**

The only additional ingredient needed is that sets may themselves have sets as their members. Here are some examples:

(42) \[ \begin{align*}
\{\{a,b\},\{b,c\}\} \\
\{\{a,b,c\}\} \\
\{a, b, c, \{d,e\}\} \\
\{\{a\}, \{a,b\}, \{a,b,c\}\} \\
\{\emptyset\} \\
\{\emptyset\{\emptyset}\}^{22} \\
\{\{a,b,c\},\{a,b,c,d\},\{a,b,c,e\},\{a,b,c,d,e\}\}
\end{align*} \]

\[^{22}\text{This set has two members, namely the empty set and the set containing only the empty set. Such sets may look weird, but in fact they play an important role in mathematics. For instance, John von Neumann has shown that natural numbers can be represented as sets (ie. have a model within set theory) by proposing the following definition of natural numbers:}\]

\[ \begin{align*}
i & := 0 \\
1 & := \{0\} = \{\emptyset\} \\
2 & := \{0,1\} = \{\emptyset,\{\emptyset\}\} \\
\ldots \\
n+1 & := \{0,1, \ldots, n\} = n \cup \{n\}
\end{align*} \]

Note that each natural number contains its predecessor as an element and as a subset. In fact, if it were not for sets having themselves sets as their members, the notion of a set would be totally boring and irrelevant for mathematics (and linguistics).
Note that the number of elements in these sets are 2, 1, 4, 3, 1, 2, and 4 respectively. Note also that in the last example, all sets contained in this set are supersets of \{a,b,c\}. If our model contains exactly five individuals, namely \textit{a, b, c, d, e} the last set described in (42) consists of all possible supersets of its smallest element. Using the notation introduced earlier, this set can also be described as shown in (43):

(43) \( \{X : \{a, b, c\} \subseteq X\} \)

The variable \(X\) does NOT range over individuals, rather, its values must be sets. The general convention is to use capital letters from the end of the alphabet as variables standing in for sets, whereas small letters are used as variables for individuals. Thus, \( \{X : \{a, b, c\} \subseteq X\} \) is the set of all supersets of \( \{a, b, c\}\)

**** End of digression ****

With this in mind, let us return to the analysis of \textit{every student snores}. The syntactic structure is roughly:

(44) \( [S \ [DP \ every \ student \ ] [VP \ snores \ ]] \)

What is needed is an extension for the DP. Assume that the extension of \textit{student} is \{a,b,c\}. Then (44) is true if this set is a subset of the snore-extension. Or equivalently, \( [[\text{snore}]_{s}] \) must be a superset of \( [[\text{student}]_{s}] \). But if \(X\) is a superset of \(Y\), it is an element of all supersets of \(Y\), that is

(45) \( X \subseteq Y \) if and only if \( X \in \{Z : Y \subseteq Z\}\)

Utilizing this equivalence for solving our problem, it is follows that

(46) \( [[(44)]_{s}] = T \) if and only if \( [[\text{snore}]_{s}] \in \{Z : [[\text{student}]_{s}] \subseteq Z\}\)

(46) allows us to derive an extension for the DP: Comparing (46) with (45) suggests that \( \{Z : [[\text{student}]_{s}] \subseteq Z\} \) is the extension of \textit{every student}.

Assuming so, we have reached our first goal: we found an extension for the immediate constituents of the sentence. It only remains to combine this DP-extension with the verb. For this to work properly we state the following rule:

(47) \( \) The extension of a sentence of the form “quantified DP + verb” or “quantified DP + verb phrase” is \(T\) if and only if the extension of the verb (phrase) is an element of the extension of the quantified DP; otherwise, its extension is \(F\).

Given that all intransitive verbs are VPs, a more symbolic version of (47) is this:
(48) \([\text{quantified-DP + VP}]_s = 1 \) if and only if \( [\text{VP}]_s \subseteq [\text{quantified-DP}]_s \)

Now we still have to analyse the internal structure of the DP *every student*. It is clear that the following must hold:

(49) \([\text{every + NP }]_s = \{X : [\text{NP}]_s \subseteq X\}\)

But (49) does not show how the extension of the whole is composed from the extension of *every* and the extension of the NP. Recall that the extensions are defined as in (40). We may now apply the same trick we already exploited with transitive verbs:

(50) \([\text{every + NP }]_s = \{X : (\langle [\text{NP}]_s, X \rangle \in [\text{every}]_s)\}\)

Thus, *every + NP* denotes the set of \( X \) that are supersets of NP, because *every* denotes the superset relation.

Of course, this works the same way not only for *every*, but also for any other quantifier:

(51) a. \([\text{a + NP }]_s = \{X : (\langle [\text{NP}]_s, X \rangle \in [\text{a}]_s)\}\)
    b. \([\text{no + NP }]_s = \{X : (\langle [\text{NP}]_s, X \rangle \in [\text{no}]_s)\}\)

Generalizing still further by assuming that all determiners denote relations between sets, we finally arrive at the following rule:

(52) \([\text{D + NP }]_s = \{X : (\langle [\text{NP}]_s, X \rangle \in [\text{D}]_s)\}\)

Before closing this section, let us briefly discuss the denotation of German *nichts* (nothing, no one). This is a DP that can be paraphrased as *no entity*. It thus follows that the extension of *nichts* is:

(53) \([\text{nichts}]_s = \{X : [\text{entity}]_s \cap X = \emptyset\}\)

But now, if the set of entities comprises all the things there are, then

(54) \([\text{entity}]_s \cap X = X\)

Therefore, the only \( X \) that gives the empty set as a result is the empty set itself, it follows that

(55) \([\text{nichts}]_s = \{\emptyset\}\)

It thus follows that the extension of *nichts* is not nothing (*nichts*), but a set with an element that turns out to be the empty set.
HOMEWORK: What is the extension of the DP *etwas* (= something or someone)?

### 5.5 Calculating the Ambiguity of *red socks and shirts*

Now that we’ve got familiar with the idea that sets may consist of sets, we can calculate the ambiguity of *red socks and shirts*.

The one big problem we must come to grips with is the denotation of plural objects. In principle, all plural nouns and adjectives denote sets of sets. For instance, the denotation of *men* is the set of all non-empty sets of men. This is the set of all non-empty subsets of the extension of *man*. The same way, the plural of *child* denotes all (non-empty) sets of children.

One might wonder why the denotation of a plural noun is allowed to contain elements that are not at all plural, since it also contains sets with only one element. The reason for this can be derived from inspecting the following dialogus:

\[(56)\]
- a. Do you have children? Yes, I have a daughter
  - b. No, I don’t have children

In (56-a), the answer is affirmative, although I might have only one child. But if *children* always referred to two or more of them, then I would be entitled to reply (56-b) in case I only have one child. As this is absurd, it makes more sense to include the one-child case into the definition of *children*. On the other hand, in plain sentences like (57) (uttered out of the blue),

\[(57)\]
- I have children

we do understand *children* as referring to more than one. The choice between singular and plural phrases must therefore be regulated by considerations that belong to pragmatics.

Plural denoting sets can be quite large and not easy to handle. Therefore we will do a little bit of cheating here: we will ignore the plural morphology entirely and try to keep with the denotations we already have at hand. We will see later that everything we are doing can also be done in a compositional way that also takes care of the plural morphology.

Let us first look at the combination of adjectives and nouns (or noun phrases); its semantics is given in the following rule:

\[(58)\]
- \[[\text{adjective + noun}]_s = [\text{adjective}]_s \cap [\text{noun}]_s\]

For example, assume that \[[\text{sock}(s)]_s = \{\alpha, \beta, \gamma\}\] and \[[\text{red}]_s = \{\beta, \gamma, \delta, b, g\}\]. Then the denotation \[[\text{red sock}(s)]_s\] is the intersection of these sets, that is, the set of all things that are both red and socks. This is the set \{\beta, \gamma\}, the extension of the
noun phrase *red socks*.

Does this sound reasonable? I hope you will agree it does. We then have to
state a rule for the conjunction of two noun phrases. This is where sets of sets
come into play. Intuitively, the extension of *socks and shirts* contains all sets with
at least a sock and at least a shirt as a member (and containing only socks and
shirts). Let’s try to make this more precise in the notation of set theory.

First we want to say that an element $X$ of the complex denotation contains
nothing but socks and/or shirts:

$$X \subseteq ([\text{\textit{sock}}]_s \cup [\text{\textit{shirt}}]_s)$$  \hspace{1cm} (59)

Next, we say that $X$ contains at least a sock…

$$X \cap [\text{\textit{sock}}]_s \neq \emptyset$$  \hspace{1cm} (60)

…and at least a shirt:

$$X \cap [\text{\textit{shirt}}]_s \neq \emptyset$$  \hspace{1cm} (61)

Putting these conditions together, we obtain:

$$[[\text{\textit{socks + and + shirts}}]]_s = \{X : X \subseteq ([\text{\textit{sock}}]_s \cup [\text{\textit{shirt}}]_s) \text{ and } X \cap [\text{\textit{sock}}]_s \neq \emptyset \text{ and } X \cap [\text{\textit{shirt}}]_s \neq \emptyset\}.$$  \hspace{1cm} (62)

With \{a,b\} as the set of shirts, the extension of *socks and shirts* is:

$$\{\{a, \alpha\}, \{a, \beta\}, \{a, y\},$$
$$\{a, \alpha, \beta\}, \{a, \beta, y\}, \{a, \alpha, y\},$$
$$\{a, \alpha, \beta, y\},$$
$$\{\{b, \alpha\}, \{b, \beta\}, \{b, y\},$$
$$\{b, \alpha, \beta\}, \{b, \beta, y\}, \{b, \alpha, y\},$$
$$\{b, \alpha, \beta, y\},$$
$$\{\{a, b, \alpha\}, \{a, b, \beta\}, \{a, b, y\},$$
$$\{a, b, \alpha, \beta\}, \{a, b, \beta, y\}, \{a, b, \alpha, y\},$$
$$\{a, b, \alpha, \beta, y\}\}$$  \hspace{1cm} (63)

(63) illustrates that sets denoting plural objects can be quite complicated, and
that’s why we try to do without interpreting the plural morphology.

As a generalization of (62), the following rule suggests itself:

$$[[\text{\textit{NP}}_1 + \text{\textit{and}} + \text{\textit{NP}}_2]]_s = \{X : X \subseteq ([\text{\textit{NP}}_1]_s \cup [\text{\textit{NP}}_2]_s) \text{ with } X \cap [\text{\textit{NP}}_1]_s \neq \emptyset$$
$$\text{ and } X \cap [\text{\textit{NP}}_2]_s \neq \emptyset\}.$$  \hspace{1cm} (64)

Inevitably, (64) generates a plural object, so that we now have the task to
combine the denotation of an adjective with that object. One way is to generate the plural object \[\text{[red+pl]}\], and then apply rule (58). As illustrated by (65), this is not at implausible, since adjectives have plural morphology in German, which motivates the assumption of some invisible semantics also in English.

\[(65)\] \[\text{red} \text{ Socken und Hemden}\]

red socks and shirts

However, as already demonstrated above, this requires the calculation of very large sets and is left to the reader as an exercise (cf. below).

So let’s try a shortcut by combining the simple extension \[\text{[red]}\], with the plural object in (64). The price we have to pay for this to work properly is a new rule that combines a singular denotation of the adjective with the plural denotation of the conjunct:

\[(66)\] \[\text{[adjective + plural-NP]} = \{X : X \subseteq \text{[adjective]}, \text{and } X \in \text{[plural-NP]}\}\]

This rule creates a new plural denotation from the plural NP by adding the requirement that all sets contained in it must be subsets of the adjective, in our case they must be red things. This, we hope, is obvious.

Let us apply this rule to the extension of red repeated in

\[(67)\] \[\text{[red]} = \{\beta, \gamma, \delta, b, g\}\]

So there is only one red shirt, namely b, and there are two red socks, namely β and γ. Neither δ nor b are socks or shirts, so we can forget about these elements. We then have to see whether which subsets of the remaining set \{β, γ, b\} are elements in (63). These are the following:

\[(68)\] \[\{\beta, \gamma, b\}, \{\beta, b\}, \{\gamma, b\}\]

This is the extension of red [ socks and shirts ]

Let us now calculate the extension of [ red socks ] and shirts. This is again a combination of two nouns as in (63), but now we have to take only red socks into account. That is, instead of combining all socks we can ignore α, which is not red. This yields the following extension (we simply omitted from (63) all sets containing α).

\[(69)\] \[\{a, \beta\}, \{a, y\},
{a, \beta, y},
{b, \beta}, \{b, y\},
{b, \beta, y},
{a, b, \beta}, \{a, b, y\},\]
\[ \{a, b, \beta, \gamma\} \]

This set is much larger than the one in (68)—a difference that proves the referential ambiguity.

HOMEWORK: Calculate through the ambiguity in the same model under the assumption that all morphology is interpreted semantically. Note that we need a new rule that that conjoins plural denotions. Here it is:

\[
(70) \quad \llbracket\text{NP-1}_{pl} \text{ and NP-2}_{pl}\rrbracket_s = \{X : X = (Y_1 \cup Y_2) \text{ with } Y_1 \subseteq \llbracket\text{NP-1}\rrbracket_s \text{ and } Y_2 \subseteq \llbracket\text{NP-2}\rrbracket_s \}
\]

5.6 The Verb *be*

One of the most vexing problems in the analysis of natural language is the verb *be* (also called the copula). Above we considered sentences like

\[(71) \quad \text{Paul is smart}\]

and we decided that *is* has no meaning of its own. However, there are other uses of *is*, attested in the following examples:

\[(72) \quad \text{a. Obama is the president of the USA} \]
\[(72) \quad \text{b. Paul is a nerd}\]

In (72-a), *Obama* and *the president of the USA* each denote an individual, so it is obvious that the semantic content of *is* is the identity relation =. In this case, *is* is not meaningless but denotes the set of all pairs \(\langle x, x \rangle\). It thus follows that *is* expresses two different verbs, depending on whether it combines with an adjective or a definite description (or a proper name).

(72-b) represents still another case. Here we are combining *is* with a DP whose grammatical function is that of a so-called predicative noun (Prädikatsnomen). Recall that we have already calculated the meaning of *a nerd* as the set of all (non-empty) \(X\) that overlap with the set of nerds. But this type of denotation does not easily fit with the denotation of the subject. It seems, then, that we need still another type of *is* that combines a quantified DP with a subject. Such a denotation for *is* has indeed be proposed in the literature, but it is rather complex.\(^{23}\)

A more simple solution would be to postulate that in such constructions neither the copula *be* nor the indefinite article *a* have any meaning of their own.

\(^{23}\)The interested (ambitious) reader might try to check that the following relation between individuals \(x\) and DP-denotations \(Q\) does the job:

\[
(i) \quad \llbracket\text{is}\rrbracket_s = \{\langle x, Q \rangle : \{x\} \in Q\}
\]
Instead of postulating another ambiguity of *is* we now postulate one of *a*. Thus, in ordinary constructions, *a* still denotes a quantifier, but in predicative constructions it does not. These constructions are syntactically characterized as combining an indefinite DP with verbs like *be* or *become*. Some evidence for this alternative way of treating the indefinite article as vacuous can be drawn from the fact that the German analogue *ein* in these contexts is optional. Thus, the following pairs of sentences are identical in meaning:

(73) a. Mindestens ein Deutscher ist ein Weltmeister
    at least some German is a world champion
   b. Mindestens ein Deutscher ist Weltmeister

(74) a. Jeder Weltmeister ist ein Medalliengewinner
    every world champion is a medal winner
   b. Jeder Weltmeister ist Medalliengewinner

The optionality of *ein* thus suggests that *ein* in these constructions is as meaningless as *ist*. If so, [[nerd]] = [[a nerd]] = [[is a nerd]], and the rule for combining a subject with a VP may apply as usual.

### 5.7 Unifying the Subject-Predicate-Rule

In this section we show how to solve a problem that comes up frequently in semantics. To illustrate, recall the rule for combining subjects with predicates. You may have noticed that we actually had to stipulate two different rules: One rule that applies to names and descriptions, saying that the [[subject]] must be an element of the [[predicate]] (cf. (13)), and another one which applies to quantified DPs, saying that the [[predicate]] must be an element of the [[quantified subject]] (cf. (47)). This duality seems a little bit strange, and one might wonder whether it is really necessary to maintain such an asymmetry. Ideally, there should be only one rule here, despite the two distinct modes of semantic composition.

The standard solution in such cases is this: Instead of saying that the extension of a proper name or a description is simply an individual, eg. Paul, we now say that the extension is a set of sets, namely, the set of sets *X* such Paul is an element of *X*. Intuitively, this is the set of all the properties Paul has. Saying now that Paul snores amounts to saying that the extension of *snore* is an element of that set of properties of Paul’s. This works, since every individual has its own characteristic set of properties. In particular, the individual Paul has the property of being identical with Paul—a property that cannot be shared with any other individual.

The subject-predicate rule can now be unified by eliminating the rule for names. Both the quantified subject and a “simple” subject denote sets of sets. To illustrate, starting off with
the following holds (as is common in mathematics, we abbreviate “if and only if” as “iff”):

(76)  \[
\text{[[Paul schnarcht]]}_s \text{ is true iff }
\text{[[schnarcht]]}_s \in \text{[[Paul]]}_s \text{ iff }
\text{[[schnarcht]]}_s \in \{X : \text{Paul} \in X\} \text{ iff }
\text{Paul} \in \text{[[schnarcht]]}_s \text{ if and only if }
\text{Paul} \in \{y : y \text{ snore}\} \text{ iff }
\text{Paul snores}
\]

Given this modification, the unified rule that applies to all kinds of subject-predicate combinations reads as follows:

(77)  The extension of a sentence of the form “DP + VP” is T iff the extension of the VP is an element of the extension of the DP; otherwise, the sentence denotes F. More formally: \[
\text{[[DP +VP]]}_s = T \text{ iff } \text{[[VP]]}_s \in \text{[[VP]]}_s.
\]

In particular, (77) makes no difference between quantified subjects and proper names; it assimilates the simple case (individual) to the more complicated one (quantifier). This method is known as type shifting: we shifted the simple set-theoretical type ‘individual’ to the more complicated type ‘set of sets of individuals’ and thereby enabled ourselves to unify our semantic rules of combination.

Although type shifting is an elegant and popular method to unify rules for composition, it is flawed by a conceptual objection: one wouldn’t hesitate to say that one refers to an individual by using a name, but do we thereby also refer to a set of properties? This seems a bit doubtful. Be this as it may, an alternative way to overcome the problem is to dispense with particular rules in favor of a general scheme that interprets binary branchings in the following way:

(78)  Interpret a structure [A B] as either
a. \(A \in B\), or \(B \in A\), or
b. \(A \cap B\),
whatever works.

(78) is meant as a kind of default rule which basically says that the two set theoretical relations “element of” and conjunction come “for free”, i.e. can be applied whenever A and B are set-theoretical objects of the correct “type”. For example, (78) would not be applicable if both A and B are individuals. If both are sets, (78-a) applies only if either is a set of sets; (78-b) is restricted to sets of the same type (both sets of individuals of sets of sets).
The mechanism is meant to replace the rules that previously called for type shifting. It's easy to see that it works fine for the case in point: If $A$ is the subject, then $A \in B$ applies if $A$ is a name, and $B \in A$ applies if $A$ is a quantified DP. The whatever-works-method is known as type-driven interpretation.\(^{24}\)

5.8 Quantifier DPs in Object Position*

Let us finally discuss transitive verbs and quantified expressions in object position. Unfortunately, looking back to the rule that combines a verb and an object, we are in trouble again. The reason is that the rule only works for names and descriptions, but not for quantified objects as in (79).

(79) Paul loves every girl

This is because our semantics so far expects an individual in the position of the object, not a set of sets. Again, a unified rule will generalize to the worst case by interpreting all objects as sets of properties. We can then formulate a more complicated rule for the composition that applies to all DPs alike. Let's see what such a rule would look like:

(80) If $V$ is a transitive verb, then $[[V + DP]] := \{x : \{y : \langle x, y \rangle \in [[V]]\} \in [[DP]]\}$.

Now, since this is difficult to paraphrase in plain words, perhaps the best way of making (80) plausible is by applying the rule to the VP in (79). So let's look first at the set $\{y : \langle x, y \rangle \in [[V]]\}$ contained in (80). For $V = love$, this is the set of entities being loved by $x$. (80) then says that this denotation is an element of the DP-extension, so the property of being loved by $x$ is one that every girl has. In other words, every girl is loved by $x$. Finally, (80) says that we have to form the set $\{x : \text{every girl is loved by } x\}$, or equivalently, $\{x : x \text{ loves every girl}\}$. If Paul is an element of that set, (79) is true. This way, it should become clear that (80) does exactly what it should do, namely describing the property of loving every girl.

The same method also works for much simpler sentences like *Paul loves Mary*, if we assume that *Mary* is type shifted, i.e. has the same logical type as quantificational DPs. Then, according to (80), being loved by Paul must be one of the properties Mary has, which is exactly how it should be.

HOMEWORK: Give a precise account of *Paul loves Mary* by specifying an interpretation and by explicitly stating each step of the calculation of truth.

\(^{24}\)The question arises whether we can dispense with all rules in favor of type-driven interpretation. We are skeptical; technically this might be feasible but for us the question remains whether it is reasonable on linguistic grounds.
HOMEWORK: Design a new rule for quantified objects of three-place verbs, exemplified by *every boy* in:

(81) John \[ V_P [ V \text{ bought every boy } V] \text{ a toy } V_P \]

HOMEWORK: State an interpretation function for which (82-a) is true and (82-b) is false:

(82) a. Mary kisses a doll
    b. Mary kisses every doll

Now calculate the truth conditions explicitly, thereby stating a formal proof that the interpretation function does what it’s supposed to do.

6 Logic and Semantics

6.1 Reasoning with Sentences

In the above sections we ventured the proposal that the meaning of sentences is amenable to formal analysis; in particular, we discussed one aspect of meaning, the so-called truth conditions of sentences which analyse the conditions for truth of falsity in terms of the internal structure of sentences. But formal semantics is not only concerned with the meaning of sentences in isolation. Rather, the meaning of sentences can often only be determined by looking at semantic relations *between* sentences. Traditionally, that is, before semantics turned into a branch of linguistics, the study of meaning of natural language sentences was confined to intersentential aspects of meaning; in particular, philosophers investigated so-called *laws of thought*/*Denkgesetze* that reveal themselves when we consider logical relations between sentences. An example of such an investigation are the *syllogisms* (*syllogisms*/*Syllogismus*) of Aristotle (4th century B.C). Each syllogism consists of two premises and a conclusion. For example, the two sentences in (1) are the *premises* (*Prämissen*), and the sentence in (2) is called the *conclusion* (*Folgerung*).

(1) All men are mortal
    Socrates is a man

(2) Socrates is mortal

The conclusion is also called a logically valid *inference* from (1). The fact that (2) can be validly inferred from (1) shows that there is an important semantic relation between sentences and reflects the ability of us humans to draw conclusions from sentences; this ability is at the core of all deductive reasoning, whose nature
constitutes one of the major topics of philosophy.

As linguists we might hope to learn something about the meaning of sentences by investigating why certain conclusions or inferences are valid, and why others are not. In fact, by doing truth conditional semantics, linguists should be able to account for these inferences solely by accounting for the truth conditions of the sentences involved. This means that we should be able to prove, on the basis of an analysis of the extensions of the sentences, that the syllogisms are correct. For the above example, this is easy enough. The only obstacle is Aristotle’s wording of the premises by using plural terms like all men, some men, or no men. Above, we refrained from precisely analysing plural expressions; this does not handicap us too much now, since nothing gets lost by replacing all men with every man, and some women with a woman or at least one woman, and no men with no man. The above inference then reads:

(3) Every man is mortal
    Socrates is a man
    Therefore:
    Socrates is mortal

In order to show why the conclusion holds, we only have to semantically analyse the sentences that make up the syllogism. Starting with “Every man is mortal” we know from section 5.4 that the VP denotes a set (the set of mortal things) and that the subject DP denotes the set of supersets of man. The entire sentence is true if $[[mortal]]_s$ is an element of that set, i.e. is a superset of $[[man]]_s$. This holds if and only if $[[man]]_s \subseteq [[mortal]]_s$. The second sentence states that Socrates is an element of $[[man]]_s$. If this is the case then by definition of the subset-relation, Sokrates is also an element of the set $[[mortal]]_s$. But this is precisely what the conclusion says. We thus have shown that the conclusion is a necessary consequence of the premises, thus a valid inference.

As another example, consider the following two sentences:

(4)  a. Fred is a cat.
     b. Fred is black.

From this we can draw the following inference (or conclusion):

(5)  Fred is a black cat.

This follows from the definition of $[[black \text{ cat}]]_s$ as the intersection of two sets, namely of $[[black]]_s$ and $[[cat]]_s$. If Fred is an element of both sets (which is what the premises say) than Fred is also an element of the intersection (which is what the conclusion says). Again, this follows from the definition of intersection.
We may assert that (5) is a valid inference drawn from (4). That is, as rational human beings, we are compelled to believe that if the premises are true (the sentences in (4)) then the conclusion is also true (the sentence in (5)).

Note that the premises need not be true in the actual world or situation. For example, none of the following sentences must be true:

(6) 

Einige Deutsche sind Holländer  
Some Germans are Dutch

Alle Holländer sind höflich  
All Dutch are polite

Therefore:

Einige Deutsche sind höflich  
Some Germans are polite

Nonetheless the inference as such is valid. This is because we do not claim that the conclusion is true, it is only true given the truth of the premises. But even if they are actually false, the argument only says that if they were true (counterfactually), the conclusion would also have to be true (counterfactually). Thus, valid inferences tell us nothing about the truth of the sentences themselves, but only about the relations between the sentences.

Moreover, Aristotle’s basic insight was that the inferences hold regardless of the predicates involved. He therefore developed general schemes or patterns of inferences, like the following corresponding to (6):

(7) Some A are B  
All B are C  
Therefore:

Some A are C

Aristotle identified quite a number of such valid inferences that can be drawn from only four types of clauses, containing the three letters A, B, C and the logical expressions all, some, no, are and its negative counterpart are not. Restricting ourselves to paraphrases that avoid plural morphology, we can easily show that the inference (7) is a mathematical truth, hence valid, given our semantic analysis of its components. Consider the following instantiation of (7). In (8) we translated some as mindestens ein; we assume that mindestens ein has the same semantics as ein, namely non-empty intersection of A and B. Moreover, the term B is translated as (ein) B, making it clear that this is a predicative NP; its semantics and the optionality of (ein) was already discussed above in section 5.6.
(8) Mindestens ein Deutscher ist (ein) Weltmeister
(at least one German is (a) world-champion)

Jeder Weltmeister ist (ein) Medalliengewinner
every world-champion is (a) medal-winner

Therefore:

Mindestens ein Deutscher ist (ein) Medalliengewinner
(at least one German is (a) medal-winner)

The first premise says that \([\text{Weltmeister}] \cap [\text{Deutscher}] \neq \emptyset\). The second clause says that \([\text{Weltmeister}] \subseteq [\text{Medalliengewinner}]\). This can be depicted as:

(9)

And from this it immediately follows that \([\text{Deutscher}] \cap [\text{Medalliengewinner}] \neq \emptyset\). And this statement is of course the conclusion of the premises.

The reasoning is true irrespective of what sets the extensions of the nouns actually are. Precisely this indifference justifies the use of schematic letters A, B, and C that stand in for any extension whatsoever; it's this invariance in the patterns that make some syllogisms “laws of thought”.

Note that syllogisms are defined as any combination of sentences that conform to a certain pattern. This implies that some syllogism are valid, but others are not. As an example for the latter, consider the following:

(10) (Mindestens) ein Deutscher ist (ein) Weltmeister
(Mindestens) ein Weltmeister ist (ein) Medalliengewinner

*Therefore:
(Mindestens) ein Deutscher ist (ein) Medalliengewinner

This is a syllogism, but the inference is wrong! Even if the premises were true, it’s not the case that the conclusion follows from the premises. Thus, (10) is among the schemes that Aristotle classified as incoherent reasoning. It is easy to see, why this is so. We must only find a model that satisfies the two premises (a model in which the two sentences are true) but which at the same time falsifies the conclusion. Such a model is easy to construct. Look at the following diagram:
Now identify the circles as either A, B, and C and show that the conclusion is wrong!

(11) ![Diagram](image)

The remarkable fact is that we are now in a position to prove in terms of set theory why some syllogisms are valid while others are invalid. No such proof existed until philosophers like Frege, Russell, and Alfred Tarski in Tarski (1936) developed a way of interpreting sentences that bridged the gap between logic, grammar, and semantics as outlined above and in the previous section. In fact, one of the goals of linguistic theory is to give an account of our ability to distinguish between valid inferences and invalid ones. Part of the problem was already solved in the nineteenth century, but only for the examples that Aristoteles considered. However, the task is much more general. It’s for any arbitrary set of sentences that we want to know which conclusion can be drawn from it. So the much more ambitious hope is to develop a semantic theory for natural language that ultimately gives an answer to the following question:

(12) How are able to tell that certain inferences are valid but others are not?

Unfortunately, Aristotles’ methods were much too simplistic to be useful for an analysis of natural language: He simply gave a list of valid inferences of certain types (see also the history of logic) treating these as axioms without proof. For over 2000 years the development of logic was hampered by the great authority Aristotle enjoyed during the Middle Ages. It was only at the end of the 19th century that logic went semantic; only then did philosophers try to construct models that allowed for a semantic justification of Aristotle’s laws of thought.25

Even in the first half of the 20th century, natural language was not a scientific subject matter at all: it was felt that meaning in natural language is much too evasive and too complex to be studied seriously. It was only by the work of philosophers and logicians in the last century that the marriage between natural language and logic came into reach/sight. Most important figures were Richard Montague (1930-1971) and David Lewis (1941-2001), who managed to apply Frege's ideas to natural language by giving a precise syntax and semantics for a small but

25Interestingly, one of them was Charles Lutwidge Dodgson, better known under by the pen-name Lewis Carroll, author of “Alice in Wonderland”. Cf. Carrol (1887) or Carroll (1897) (various reprints should be available).
interesting fragment of English (cf. Montague (1970a,b) and Lewis (1972)).

Who is who in philosophy?

6.2 Truth Tables and Truth-Functional Connectives

To summarize, the most important aspect of the above discussion is that our semantics allows us to give an account of rational reasoning that manifests itself in our ability to draw logically valid inferences, i.e., logical conclusions from sentences or sets of sentences. For example, we showed above that the set consisting of (13-a) and (13-b) implies (13-c).

(13)  a. Fred is a cat.
     b. Fred is black.
     c. Fred is a black cat.

And conversely, (13-c) implies (13-a) and (13-b). In what follows, read the traditional (but admittedly old-fashioned) symbol \(\therefore\) as “therefore”. We can thus show that both (14) and (15) hold:

(14) Fred is a black cat.
    \(\therefore\) Fred is black.

(15) Fred is a black cat.
    \(\therefore\) Fred is a cat.

Somewhat surprisingly, however, we are not yet in a position to prove the following:

(16) Fred is black and Fred is a cat
    \(\therefore\) Fred is a black cat.

Nor can we show (17) or (18):
(17) Fred is black and Fred is a cat
∴ Fred is black.

(18) Fred is black and Fred is a cat
∴ Fred is a cat.

The reason for this deficiency is that we have not yet provided a semantics for the word *and*. This little word is also called “*conjunction*” (Konjunktion). In what follows we will provide a method for describing the semantics of conjunction and connectives like *or*, called “*disjunction*” (Disjunktion).

6.2.1 *Conjunction*

Consider the following inferences:

(19) Bart likes skateboarding.  
    Homer likes donuts.  
∴ Bart likes skateboarding and Homer likes donuts.

(20) Wendy takes care of her siblings.  
    Peter flies home every night.  
∴ Wendy takes care of her siblings and Peter flies home every night.

These all seem like pretty compelling inferences. If the premises are true, then the conclusion must be true as well, right? Check them out and try to abstract away from the actual sentences in these arguments. There’s a type of schema that is being followed here.

(21) \( A \)
    \( B \)
∴ \( A \) and \( B \)

Can you see (21) in (19) and (20)? It doesn’t matter what sentences you use for \( A \) and \( B \), the conclusion should be valid, regardless of the ‘content’ of \( A \) or \( B \)!

Likewise, sentences like

(22) Wallace likes cheese and Gromit wants to fly to the moon.  
∴ Gromit wants to fly to the moon.

instantiate another scheme. For it is also quite obvious that regardless of the content of the constituent sentences, both (23-a) and (23-b) are valid inferences:

(23) a. \( A \) and \( B \)
    \( \therefore A \)

   b. \( A \) and \( B \)
    \( \therefore B \)
Side note: when we say “sentence” here, we really mean a “proposition” (Proposition). That’s the word philosophers use. A proposition is simply a sentence together with its semantic content. As for the semantic content we subscribe to the following slogan (basically Frege’s):

(24) The semantic content of a sentence is its potential of being true or false, ie., of having a truth value.

That is, a proposition can be either true or false depending on the situation described when using the sentence. For instance, “Wolfgang is 4 feet tall” is patently false. It is a proposition, but in this world it is false. Above we explained that the extension of sentences is truth values, they express propositions. Some sentences, however, cannot denote a truth value, for example sentences like *Do the dishes!* and *Do you want some coffee?* Let’s stick to only declarative sentences for our propositions. We’ll use the terms proposition and sentence interchangeably in class, but this is what we mean by sentence.

Philosophers developed a way of representing the meaning of “A and B”. For this, they draw a so-called truth value chart or a truth table (Wahrheitswerttabelle) like the one in (25):

(25)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A and B</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>

(25) is actually not as difficult to read as it looks. The first two columns contain the possible extensions for sentence A and sentence B. Sentence A can be either true (T) or false (F) and sentence B can be either T or F. Therefore, there are four logical combinations: both are true (in the first row that contains truth values), A is true and B is false (the second row), A is false and B is true (the third row), and both are false (the last row). Now, let’s fill in the missing truth values for the complex expression “A and B”. We can do this based on our own intuitive judgements. Substitute your own sentences for A and B to check your intuitions. The basic question is this: if sentence A and sentence B have these truth values (either true or false) what is the truth value of the sentence “A and B”?

(26)
If A is true and B is true, then we can say something like “A and B is true” . . . it has to be true, right? It doesn’t matter what the sentences A or B are, the new sentence “A and B” should be true as long as A is true and B is true. More precisely, A and B is true if and only if both A and B are true, if either of them is false, so is “A and B”.

This is actually the reason why we can make the inference in (22). If the premise with the and in it is true then both A and B must also be true. . . it couldn’t be otherwise: this can be read off the chart by looking at the first row: this is the only row in which (A and B) is true; reading the chart from right to left it tells us that A must also be true, as well as B. Therefore, A by itself is true and B by itself is true. The table in (26) therefore reveals the precise truth conditions for the logical word and.

For evaluating inferences like (19) and (20), we must read the chart from left to right. We look at all rows that make the premises true. There are two lines for the premise A and two for B, but only one, where both premises are true, namely the first row. Now we have to check whether in this case the conclusion, namely (A and B) is also true, and in fact it is. Hence the conclusion is valid.

We can also visualize the truth conditions for and (and for other connectives, as we will see) by using a so-called Venn diagram. Such a diagram for two propositions A and B is shown in (27):

We consider the region within the circle A as the situations in which A would be judged true, and the same for B. Thus, every situation outside A is one where A is false. This way, the diagram allows for all of the four possibilities we already listed in the truth value chart: situations with A true, A false, B true, B false, both true, and both false. No situation is excluded; in particular, the diagram does not tell us anything about what the world being talked about looks like.
Now, by shading regions of the diagram, we may exclude certain situations that are incompatible with the content of a sentence under discussion. For example, by asserting A, we exclude situations in which A is false. Shading thus means excluding possible situations that lie in the shaded region; asserting A is thus represented as:

(28)

\[ A \cap B \]

Accordingly, if one claimed that “A but not B” this would in addition exclude all B-situations from consideration, which would graphically corresponds to:

(29)

\[ A \cap \neg B \]

Likewise, “B but not A” is represented as:

(30)

\[ \neg A \cap B \]

With both A and B being true, the diagram that corresponds to “A and B” is this:

(31)

\[ A \cap B \]
So this diagram corresponds to “and” in an obvious way: a representation for the conjunction *and* should pick out the situations where A and B hold, and this means that we must shade all the situations that are incompatible with “A and B” being true. That is, the remaining white space represents the situations where “A and B” is true. We may therefore say that this graphics represents the truth conditions for “A and B”.

**WARNING:** There is a small but important difference between Venn-diagrams and Euler diagrams. The important thing to note here is that the shading in Euler diagrams excludes possibilities, whereas in Euler diagrams for sets the shading represents the elements of a set. Therefore the shading for $A \cap B$ in an Euler diagram is just the opposite of that in (31)!

Here is another Venn diagram:

(32)

You may ask yourself how the situation described in this picture could be expressed in natural language. Your answer should be that (32) corresponds to “neither A nor B”. This could of course also be expressed by using a truth value chart; here it is:

(33)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>neither A nor B</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

As it turns out, the method of truth value charts and the representation by Venn diagrams are equivalent; as the reader can figure out for herself there are 16 different possibilities of shading a Venn diagram, and there are also 16 corresponding truth value charts, some of which will be discussed further below.

Before discussing other connectives than *and*, another warning might be useful. First of all, a common mistake is for students to think that semanticists claim true sentences to be grammatical and false sentences to be ungrammatical. NO! We are claiming that you know how to find out if the sentence is true or false. The truth value can be either true or false, and you know what the situation described must look like to find out.
Another common misunderstanding is this: Knowing the truth conditions of a sentence does not mean that we know whether or not the sentence under discussion is true or false. On the contrary. We need no factual knowledge for setting up the table in (33). The actual truth of A or B is in fact totally irrelevant for the conclusion to be true, that's why we look at all four possibilities for A and B being true or false.

So, in order to find out whether “Bart likes skateboarding and Homer likes donuts” is true or false, according to (26) we look to see if “Bart likes skateboarding” is true or false and then if “Homer likes donuts” is true or false. And then we know that the whole clause can be true only if both of the simple clauses are. But this does not mean that we have to have any factual knowledge as to whether either of the sentences is in fact true! This might even be impossible for someone to find out. Nonetheless, one knows the conditions under which a complex sentence would be judged true, assuming that we knew the truth or falsity of its components.

This is the point we are getting at here: there are ways to combine things syntactically, and we have patterns (rules) that govern the way in which the meaning of the parts make up the meaning of the whole.

Now keeping in mind that the denotation of sentences is truth values, the above considerations have made it clear that the semantic rule for coordination with and is the following:

\[(34) \quad \llbracket S_1 \text{ and } S_2 \rrbracket_s = T \text{ (or } 1) \text{ if and only if } \llbracket S_1 \rrbracket_s = T \text{ and } \llbracket S_2 \rrbracket_s = T \text{ (otherwise } \llbracket S_1 \text{ and } S_2 \rrbracket_s = F \text{ (or } 0)\]

This sounds rather trivial; in fact, it is.

The alert reader might have noticed that we still have not defined \([\text{and}]_s\). This seems to be a problem because the functioning of and does not depend on the actual situation. Recall from section 5.4 that the extension of quantifiers did not depend on extensions, either. We were thus able to directly define the meaning of quantifiers. Likewise, we can directly define the meaning of the logical connectives. There are several ways to do that. One method is to represent the meaning of and as a two-place relation between truth values. It then follows that

\[(35) \quad \llbracket S_1 \text{ and } S_2 \rrbracket_s = T \text{ if and only if } \langle \llbracket S_1 \rrbracket_s, \llbracket S_2 \rrbracket_s \rangle \in [\text{and}]_s.\]

For this to be equivalent to (34), it must be the case that \([\text{and}]_s\) contains only one element, namely the pair \(\langle T, T \rangle\) (or \(\langle 1, 1 \rangle\)). This implies that for any two sentences A, B, the conjunction of A and B is true only if the pair of their extension \(\langle T, T \rangle\) is an element of \([\text{and}]_s\), which means that both A and B must have the value T (or 1). As the extension of and does not vary from situation to situation, it holds that
\[ [\text{and}] = [\text{and}]_s \text{ in all situations } s. \]

### 6.2.2 Disjunction

We hope this is starting to make some sense. Let's try some other inferences and hopefully this will clear up some things.

(36) Bart likes skateboarding. 
Homer likes donuts. 
\[ \therefore \text{Bart likes skateboarding or Homer likes donuts.} \]

(37) Wendy takes care of her siblings. 
Peter flies home every night. 
\[ \therefore \text{Wendy takes care of her siblings or Peter flies home every night} \]

(38) Wallace likes cheese or Gromit wants to fly to the moon. 
\* \[ \therefore \text{Gromit wants to fly to the moon.} \]

(36) and (37) are a little tricky, so let's look at (38) first. When using the conjunction \textit{and} in (22), a similar kind of inference went through, but now that we are using \textit{or} instead of \textit{and} it doesn't (I've used the * to indicate this.) Take a look at it: the premise \textit{Wallace likes cheese or Gromit wants to fly to the moon} doesn't necessarily mean that Gromit wants to fly to the moon. Gromit may want to fly to the moon, but we don't know that because the premise says that either one of A or B could be true but there is no guarantee that both of them are true. This is the difference between \textit{or} and \textit{and}. For \textit{or}, either A or B could be true, and both of them could be true, but it is impossible that both are false. For \textit{and}, both have to be true. Here, then, is the truth value chart for the connective \textit{or}:

\[
\begin{array}{ccc}
\text{A} & \text{B} & \text{A or B} \\
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]

This chart says that
1. whenever A and B are true, then (A or B) is true;
2. whenever A is true and B is false, then (A or B) is true;
3. whenever A is false and B is true, then (A or B) is true; and
4. whenever A is false and B is false, then (A or B) is false.

This can again be represented in a Venn-diagram:

(40)
The diagram only excludes situations where neither A nor B is true. In other words, *or* requires that at least one of the sentences it is connecting be true if the whole sentence is to be true.

Let us now calculate why the inference in (38) is invalid. The entire sentence of the form (A or B) is true, but this doesn't say which of A or B is true. Look at all the rows in (39) that make (A or B) true. For the conclusion to be true it must be true in all the rows that satisfy the premise. Now if A is the conclusion, we find that A is not true in all these cases; in fact A is false in the second row. Therefore, we can't go from the (A or B)-sentence to A. Nor can we go to B, by the same reasoning considering the third row in the chart.

On the other hand, we can show that the conclusions drawn in (36) and (37) are valid. Since the premises must both be true, we look at the first line of the truth table and find that (A or B) is also true. This suffices to justify the conclusion. Intuitively, it is clear that if (A and B) is true, then (A or B) must also be true, although there is something strange going on here: one would never claim (A or B) in a situation in which we know that A and B are both true. This is so, because (A or B) would convey much less information than would be accessible to the speaker, and as long as the speaker does not want to hide something, saying (A or B) in such a situation would simply be inappropriate for pragmatic reasons. This, however, does not impede the validity of the inference, rather it shows that more is going on here that belongs to pragmatics (and will be discussed in the pragmatics course next term).

### 6.2.3 Exclusive ‘or’

The alert reader might object that our treatment of *or* does not always correspond to our linguistic intuitions. In many cases, like the following,

\[(41) \quad \text{The cat gets some food or she visits the neighbor}\]

we reason that it is not the case that the cat went to the neighbor when she got some food. And conversely, that going to the neighbor precludes getting food. In other words, we conclude from (41) that (42) is false:
The cat gets some food *and* she visits the neighbor

This reasoning, however, does not conform to the truth table given for *or* in (39): rather, one would expect a truth table with A or B being false when A and B are true:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

A connective with the meaning shown in (43) has also been called 'Exclusive Or' for the obvious reason that it excludes the possibility of both A and B being true. By contrast, the connective shown in (39) is called 'Inclusive Or'. Inclusive Or allows both A and B to be true, while Exclusive Or only allows one or the other to be true, but never both. Here is the Venn diagram for Exclusive Or:

It has therefore been claimed that *or* is ambiguous between inclusive and exclusive *or*. Such an ambiguity would be surprising, since we previously found out that ambiguities are rare. Should it really be the case that a simple word like *or* is ambiguous? Let us discuss this question in more detail.

According to the ambiguity test applied in section 3 we should be able to find two paraphrases that are incompatible with each other. Let’s write $or_e$ for exclusive *or* and $or_i$ for inclusive *or*. Finding a paraphrase for the exclusive reading is simple:

$$A or_e B = T \text{ if and only if } A \text{ or } B = T \text{ and possibly both } A \text{ and } B = 1.$$ 

Somewhat surprisingly, there is no natural language expression that unambiguously encodes Exclusive Or. Moreover, the easiest way to paraphrase $or_e$ is with the help of $or_i$. And an even bigger problem is to paraphrase $or_i$ itself.\(^{26}\) Can you

\(^{26}\)Of course, a paraphrase like that on the right hand side of “if and only if” in (i)

(i) $A or_i B = T \text{ if and only if } A \text{ or } B = T \text{ and possibly both } A \text{ and } B = 1.$

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think of a way of doing so without using or\textsubscript{i} itself?

Before presenting a solution to this problem, note that so far, in discussing structural ambiguities, we tried to show that for two paraphrases P\textsubscript{1} and P\textsubscript{2} we can always find a situation with P\textsubscript{1} true and P\textsubscript{2} false, and vice versa. For lexical ambiguities, we might also require that the two meanings be distinct in that one should not be a hyponym of the other.\textsuperscript{27} But now, inspecting the truth conditions for or\textsubscript{e} and or\textsubscript{i} we see that we can find a situation with or\textsubscript{i} true and or\textsubscript{e} false, but the reverse, or\textsubscript{e} true and or\textsubscript{i} false, is impossible. This should make you think and suspect that something special is going on.

For reasons we cannot discuss here in detail, linguists have argued that there is no real ambiguity involved. One motivation for this claim comes from the observation that we are not free to choose either meaning. For example, consider:

(46) If John shows up (A) or Mary starts singing (B), I will leave the party (C)

Clearly, the possibility that both A and B are true at a time cannot be excluded, and it is equally clear that in such a case this would definitely not lead to my staying at the party. This implies that or in (46) is understood as having an inclusive meaning. But if or could also have the exclusive reading, we would expect that I stay if both A and B are true. The problem is that (46) simply has no such reading with A and B being true and me staying at the party (i.e., C being false). In consequence, only the inclusive reading is possible here. The question is, of course, why this should be so.

Another thing to be explained is why natural language has not invented two different expression in order to disambiguate the connectives, if there were indeed such an ambiguity.

This is an interesting problem which belongs to pragmatics and cannot be answered here in full. But here is a hint: Roughly put, the answer pragmatics provides is the following: In fact, there is no ambiguity of or (and no pressure to resolve an ambiguity) because in semantics, or unambiguously means inclusive or. The fact that or is nonetheless understood in an exclusive way in some contexts (but, as we saw, not in all) is due to additional pragmatic considerations of the following sort:

Suppose I say “A or B”. The reason for expressing myself this way is that I don’t know whether A or B, otherwise I should straightforwardly say A, if I know that A, and B if I know that B. And I should also say “A and B” if I know that both A and

---

\textsuperscript{27}This does not imply that meaning generalizations of meaning specializations (Begriffserweiterungen, Begriffsverengungen) are impossible, but normally the new meaning develops further in such a way that it does not comprise the old one.
B are true. If this were the case I would have uttered “A and B”. But I haven’t. So, from the fact that I cannot utter the stronger statement, one may conclude that the stronger statement is false. But then, if “A or B” is true but “A and B” is false, we have exactly the truth conditions for exclusive or.

This also explains why we do not need a separate expression for Exclusive Or: there is no need for an additional expression because the job of expressing exclusiveness is already done in the pragmatic component of natural language.

On the other hand, there are a number of steps in the above argumentation that deserve closer examination (and elaboration). This goes into the field of formal pragmatics. Anyway, the result of the discussion is that in natural language there is only one connective or, namely Inclusive Or.

This much said, we can now define the extension and the meaning of or as a two-place relation the elements of which are given in (47):

\[
(47) \quad \llbracket \text{or} \rrbracket := \{\langle T,T \rangle, \langle T,F \rangle, \langle F,T \rangle\}
\]

Let us now turn to the question of how to paraphrase Inclusive or without using or. Given the result of our discussion, this is perhaps unnecessary because, according to the argument above, Inclusive Or is the only or we have in natural language, so why bother to paraphrase it?

On the other hand, we already paraphrased Exclusive Or with the help of a negative statement: we stated that at least in some cases our intuitive understanding implies that not both A and B should be the case. This leads us naturally to the semantics of negation. We will see below that or can easily be paraphrased as a combination of negation and conjunction.

6.2.4 Negation

Negation is indeed a central concept of logic. Reasoning from negative premises was crucial in syllogistics, and is at the heart of any kind of logic. Natural language represents the negative value by several kinds of expressions:

\[
(48) \quad \begin{align*}
a. & \quad \text{It's not true that Homer likes donuts.} \\
b. & \quad \text{It's incorrect that Homer likes donuts.} \\
c. & \quad \text{It's false that Homer likes donuts.} \\
d. & \quad \text{That Homer likes donuts is false.} \\
e. & \quad \text{Homer doesn't like donuts.}
\end{align*}
\]

28“Stronger” here means logically stronger in the sense that A and B logically implies A or B.

29Observe also that we didn’t yet explain why there is no exclusive reading for (46). Roughly put, the answer is that the exclusive reading is okay if it makes the entire proposition “stronger”. But this would not happen in (46), because “A or B” is more deeply embedded into a conditional. We cannot discuss this here as we did not yet discuss the semantics of the conditional.
There are more, but we hope you get the picture. How do we represent this in terms of a Venn-diagram? This is easy:

(49) ![Venn Diagram]

In a truth value chart, we do the same thing that we did for and and or: we abstract away from the proposition *Homer likes donuts*. We can represent the proposition just like we did for the other cases with an A or a B.

(50) Truth value chart for “not A”:

<table>
<thead>
<tr>
<th>A</th>
<th>not A</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

All that (50) says is that when A is true, not A is false, and when A is false, not A is true. Let’s use an example to clear it up a bit.

*Homer likes donuts* is either true or false. Let’s say it’s true. Then *Homer does not like donuts* is false. Let’s say *Homer likes donuts* is false. Then *Homer does not like donuts* is true.

Let’s now turn to a situation with A or B being true, but not both A and B being true. The second clause states that the conjunction of A and B is false. A natural candidate to express this would be

(51) Not A and B

But now observe that (51) is structurally ambiguous. The scope of negation could be either A alone, or the entire conjunct. Syntactically, we can draw two different trees:

(52) a.  

```
   not A
```

```
   and B
```

b.  

```
   not
```

```
   A
```

```
   and B
```

In linear notation (the one used by logicians), we can disambiguate the two readings by different bracketings. As logicians most often use the sign ¬ for not, an unambiguous representation would be:
(53)  
a.  \((\neg A) \text{ and } B\)

b.  \(\neg (A \text{ and } B)\)

As a general convention, the brackets in (53-a) are omitted. The brackets in (53-b), however, are essential. Let us now prove the ambiguity by calculating the truth tables for (53-a) and (53-b). The truth table given for negation said that the truth values for \(A\) reverse. In the case at hand, its the truth values of \((A \text{ and } B)\) that must reverse. This shows that the actual statement of truth value charts is not meant to apply to specific sentences only but to arbitrary complex expressions. And the same holds for the other truth value charts. To avoid confusion, we restate them here in using variables \(p\) and \(q\) that stand in for any sentences the charts apply to.

(54)

<table>
<thead>
<tr>
<th></th>
<th>not (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

(55)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>(A and B)</th>
<th>((\neg A) \text{ and } B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

To apply the chart for negation to the case at hand, we only have to replace the \(p\) in (54) by the expression \((A \text{ and } B)\) in (55). Doing this yields the opposite truth value for \((A \text{ and } B)\), because this is what (54) tells us to do.

Next assume that negation has only scope over \(A\). The resulting truth table is the following:

(56)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>(\neg A)</th>
<th>((\neg A) \text{ and } B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>
We first calculated $\neg A$. In the second step we applied the truth value chart for conjunction, but now we replace the $A$ in (26) by $\neg A$. This works since the letters $A$ and $B$ can stand in for any complex expression.

HOMEWORK: Draw two different Venn diagrams that illustrate the ambiguity.

Let us now return to the problem of paraphrasing $\lor_i$. Although we noticed above that there might be no need to invent a paraphrase, it is easy to see now how $\lor_i$ can be expressed with the help of negation and conjunction. Inspecting the truth table of $\lor_i$ reveals that this kind of disjunction is false only if both $A$ and $B$ are false. That is, the disjunction is false if not $A$ and not $B$ are true. But then the disjunction is true if and only if not (not $A$ and not $B$) is true. Thus, $A$ or $B$ is true if and only if not (not $A$ and not $B$) is. Hence the paraphrase for $\lor$ is this: it is not the case that not $A$ and not $B$.

HOMEWORK: Try to give a paraphrase of conjunction by using negation and $\lor$.

Finally, consider the role of negation in $\neg\lor$, whose truth table was given in (33). It seems that this expression is composed out of negation and $\lor$. Can we make this intuition more precise? We can! First look again at the truth value chart of $\lor$; the result for ($A$ or $B$) is shown in the third column below. Let’s now apply negation to this column, which means that we simply reverse the truth values, as shown in the fourth column:

$$
\begin{array}{c|c|c|c}
\hline
A & B & (A \lor B) & \neg(A \lor B) \\
\hline
T & T & T & F \\
T & F & T & F \\
F & T & T & F \\
F & F & F & T \\
\hline
\end{array}
$$

Now compare this result with (33). As you will see by comparing the last columns, the truth conditions are exactly the same. This means that $\neg\lor$ is in fact equivalent to $\neg (A \lor B)$. This result is what we expected: it shows that both negation and disjunction play a role in defining the semantics for $\neg\lor$. What might come as a surprise, however, is the fact that we do not have a simple word (let’s say ‘nand’) to express this connective. And similarly: in order to express the idea ‘not both $A$ and $B$’ we simply have to use the combinatorial means supplied by negation and conjunction; there is no shortcut for expressing this as one simple (one-word) connective. This seems to be a universal property of natural languages and it is an interesting question why this should be so. See eg. Lang (1991) for further discussion.
6.2.5 Computing the Ambiguity of A and B or C

Next, consider a sentence like

(58) He bought wine or beer and chips

For the sake of simplicity let’s assume that this is represented by

(59) He bought wine or he bought beer and he bought chips

Let’s abbreviate this by the following scheme:

(60) A or B and C

As it turns out, (60) is structurally ambiguous, and this ambiguity can be represented by different trees:

(61) a. 

```
  •
 /\  
•  and  C
/   \
•    •
A   or  B
```

b. 

```
  •
 /\  
•  A or  •
/   \
•    •
B   and  C
```

It’s quite obvious that in (61-b) the conjunction and is in the domain of or, whereas in it’s the other way round in (61-a). It follows that scope relations also differ. In (62-a) the disjunction or has narrow scope with respect to the conjunction and, and in (A or B) and C, it’s just the other way around.

(62) a. (A or B) and C
b. A or (B and C)

Is this difference in bracketing also accompanied by a difference in meaning? If not, then (62-a) and (62-b) should be synonymous. Thinking about it for a while, you should agree that from the truth of (62-a) we can infer that C must also be true, but this conclusion cannot be drawn on the basis of (62-b), because we do not know whether (B and C) is true. (62-b) could be true just on the basis of A’s being true, while both C and possibly B are false.

This sort of reasoning exploits the following idea: the sets of conclusions that can be drawn from each of two sentences X and Y are identical if and only if X and Y are synonymous. We have just demonstrated that (62-a) and (62-b) do not allow for exactly the same conclusions. This implies that the meaning of the complex expressions must be different, which suggests that (61) is ambiguous.
If our intuitive reasoning is correct, then the truth value charts for (62-a) and (62-b) must differ for some assignment of truth values to the elementary sentences A, B, and C, so there are three different propositions that may influence the result. Above, we looked at only two propositional variables A and B and considered all combinations of truth values for A and B. Now we have to consider all possible combinations of three variables A, B, and C. That is, we have to look at a table with eight different rows and three columns, one for each variable:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>(A or B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
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<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
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<td>F</td>
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</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

The next step is writing down the formulas to be evaluated. Let us start with (A or B) and C. According to this bracketing, we first have to calculate the values for (A or B). Let's do that by applying the rules for disjunction from subsection 6.2.2:

Now we no longer need the values for A and B, since these variables do not occur anywhere else in the formula. From the values of C and those we calculated above in (64), we finally calculate the chart for the entire formula:
Having calculated this, we now apply the same method to A or (B and C). Here we have to start with (B and C), as shown in (66):

We may now forget about B and C and calculate the disjunction:

Comparing the two results, we see the charts differ at two positions in the second and fourth row. This may also be represented by Venn diagrams.
Summarizing the above demonstration, we have shown that a simple ambiguity at the level of sentence coordination can be calculated by using the technique of truth tables. These tables represent the meaning of the logical connectives.

HOMEWORK: What about “A and B and C”? Can we get semantic ambiguities?
HOMEWORK: Design a truth table for “(A and B) or (C and D)”.
HOMEWORK: Check Wikipedia for a Venn diagram with the four variables A, B, C, and D.

6.2.6 Other Connectives

As mentioned above there might be more connectives besides and and or. A case in point is neither . . . nor already discussed. In fact, truth value charts (and Venn diagrams) would allow us to define 16 different connectives. Most of them, however, have no analogue in natural language. The reason for this is quite obvious. Consider the following truth tables:

\[
\begin{array}{ccc}
\text{A} & \text{B} & \text{C} \\
\text{T} & \text{F} & \text{F} \\
\text{T} & \text{T} & \text{F} \\
\text{F} & \text{T} & \text{F} \\
\end{array}
\]

Neither of them is expressed in natural language, and the reason for this seems to be clear: the resulting column does not in any way depend on the truth values of A and B together. In the first table, A and B play no role whatsoever for the result, in the second table, B does not influence the outcome: it would thus simply be possible to say A instead of something like “A regardless of B”. So there is not
much practical use for those potential connectives.\textsuperscript{30}

This consideration drastically reduces the number of possible candidates for natural language connectives. Two of the remaining possibilities would be the following truth tables:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
A & B & T \\
\hline
T & T & T  \\
T & F & F  \\
F & T & T  \\
F & F & F  \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
A & B & T \\
\hline
T & T & T  \\
T & F & F  \\
F & T & F  \\
F & F & T  \\
\hline
\end{tabular}
\end{table}

These may indeed be potentially relevant for natural language (and also for logical theory). The connective defined in (70-a) has been called \textbf{material implication} by logicians and philosophers, and the closest corresponding natural language expression is “If A then B”. Inspecting the truth table in (70-b) will reveal that it could be expressed simply by “If B then A”, so we need not have a particular natural language connective for the second table. So let’s concentrate on the first table.

Traditionally, A is called the antecedent, B the consequence. This reminds us of our concern with valid inferences, and in fact there is a close connection. Let us first illustrate the use of “If A then B” in inferences of the following sort:

\begin{proof}
\begin{align*}
\text{If Gromit likes cheese, then he wants to fly to the moon} \\
\text{Gromit likes cheese}  \\
\therefore \text{Gromit wants to fly to the moon.}
\end{align*}
\end{proof}

Here again it seems that the inference is valid regardless of the content of the non-logical material.

\begin{proof}
\begin{align*}
\text{If A then B}  \\
A  \\
\therefore \text{B}
\end{align*}
\end{proof}

This is a classical valid inference, called \textit{modus ponens}.

The question then is whether we can find a definition of “if … then” that makes the inference valid. And in fact, this is precisely the one in (70-a). When a sentence like “If A then B” is true, it cannot be the case that A is true while B is false. This is exactly what is stated in the following table (repeated from above):

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
A & B & T \\
\hline
T & T & T  \\
T & F & F  \\
F & T & F  \\
F & F & T  \\
\hline
\end{tabular}
\end{table}

\textsuperscript{30}Or perhaps “regardless of” itself is the wanted connective? But see below for the influence of pragmatics in cases where truth values alone do not capture the meaning of expressions.
The symbols \( A \rightarrow B \) read: If A, then B. We can paraphrase the meaning of \( \rightarrow \) as:

\[
\text{(74)} \quad \text{It’s not the case that A and not B.}
\]

In fact, the truth value chart for (74) is equivalent to (73), as can easily be verified:

\[
\begin{array}{ccc|ccc|c}
A & B & \neg B & (A \land \neg B) & \neg (A \land \neg B) \\
T & T & F & F & T \\
T & F & T & T & F \\
F & T & F & F & T \\
F & F & T & F & T \\
\end{array}
\]

Let us now check whether our inference (modus ponens) is valid. We proceed as usual: We first look at all the rows X that make the first premise true. Then we ask which of these rows also make the second premise true. This will be a set Y so that \( Y \subseteq X \). Then we check whether in all rows of \( Y \) the conclusion also holds. If so, the inference is valid, if not, it is invalid.

In the case at hand, \( X \) contains the first, the third and the fourth row. Of these, the second premise, namely A is false in the third and in the fourth row, it therefore remains to look at the first row (\( Y \) is the first row only). And indeed, B is true in the first row, therefore the inference is valid.

### 6.2.7 Contradictions and Tautologies

The method of truth value charts allows us to define certain important classes of sentences. For example, a complex sentence is a contradiction (Kontradiktion, Widerspruch) if and only if its truth value chart contains only F’s. Such a sentence can never be true. An example is the following:

\[
\text{(76)} \quad (A \land \neg A), (A \implies \neg A)
\]

Tautology [to be written]

\[
\text{(77)} \quad C \text{ is a valid inference from } A \text{ and } B \text{ if and only if } ((A \land B) \rightarrow C) \text{ is a tautology.}
\]

\[
\text{(78)} \quad A \text{ and } B \text{ are synonymous iff } ((A \rightarrow B) \land (B \rightarrow A)) \text{ is a tautology}
\]
HOMEWORK: Show that “A and A” is synonymous with A, and that “A or A” is synonymous with A. Can we conclude that “A and A” is synonymous with “A or A”? Can we conclude from this that and or are synonymous? Justify your answer.

6.3 Beyond Truth Functionality

What we have done so far was define the meaning of conjunction, disjunction, negation, and implication. The important thing to note is that the meaning is presented by the entire truth value chart, not by any single row in it. For example, when looking at the two sentences in (79),

(79) a. I have two daughters and I am a professor
    b. I have two daughters or I am a professor

one might assume that both I have two daughters and I am a professor are true, and therefore both (79-a) and (79-b) are true. But this does not imply that (79-a) and (79-b) have the same meaning (are synonymous)! In order to get the meaning, we must consider all possible combinations of truth values, and this is exactly what truth conditions do: they provide a value for all possible cases.

Inspecting the truth tables for conjunction and disjunction will reveal that they make no difference whatsoever between “A and B” on the one hand, and “B and A” on the other. Hence, the sentences in (79) and those in (80) should not differ in meaning:

(80) a. I am a professor and I have two daughters
    b. I am a professor or I have two daughters

Yet there are sentences in natural language that appear to be counterexamples to this symmetry:

(81) a. She married and (she) got pregnant
    b. She got pregnant and (she) married

The relevant difference seems to be due to the influence of time, a topic that has hitherto been ignored completely. It is clear that there is something in the meaning of and that goes beyond considerations of truth values alone; there seems to be a pragmatic component involved that matches the order of sentences with the order or events reported by the sentences. This way, the meaning of and seems to turn into the meaning of and then. The use of then here points to the need of accounting for temporal relations implicitly expressed in (81). The analysis of tense and time is an important branch of semantics that will not be dealt with in
Another case in point where truth tables alone are insufficient is the natural language use of if ... (then) exemplified in (82):

(82) If it rains, (then) the street is wet

(82) is usually understood as making a causal or law-like statement. However, this additional meaning component cannot be expressed by using so called truth-functional connectives (wahrheitswertfunktional) like material implication. Intuitively, we want to express a causal relation between raining and being wet.

Closely connected to this failure is the fact that a conditional is true as soon as the antecedent is false. This seems counterintuitive because falsity of the antecedent is felt as being irrelevant for the conclusion. Thus, the sentence

(83) If it rains (then) the moon will melt

is true (counterintuitively) in case it does not rain. Intuitively, however, (83) expresses a causal relation between raining and melting, and therefore the sentence is judged false. Such a relation is not expressed by material implication. Since causality is a relation totally absent from the realm of mathematics, it is clear that logical reasoning is perfectly consistent with the truth conditions for material implication; it’s only the additional “non-logical” aspect of meaning that is not yet captured by material implication. Nor is there any other of the 16 connectives that could do the job. The reason for this is that all connectives work truth-functionally, which means that for each connective the resulting truth values of the complex expression can be calculated by looking at the truth values of A and B in each row, so that we can proceed rowwise, by looking at just two truth values for A and B. This complies with Frege’s principle of extensional compositionality: the extension (= truth value) of the whole is calculated from the truth values of the parts. Unfortunately, however, most connectives of natural language are more complicated, so that this method does not always work; they do not work in a truth-functional manner.

As an illustration, consider the connective because. The meaning of A because B cannot be captured by only looking at the respective truth values of A and B alone. For example: assume that A = The street is wet, B = I feel sick, C = It is raining, and that all of A, B, and C are true in a given situation. In that situation, the sentence the street is wet because I feel sick = A because B is false, whereas the street is wet because it is raining = A because C is true. Now, if the meaning of because were truth-functional this would be impossible: since B and C are both true in the situation described, the result of applying because to A and B should be exactly the same as applying because to A and C. The result cannot be both
T and F. Thus, the truth value of a complex sentence is not always a function of the truth values of its parts. In order to calculate the truth conditions, we need more, namely a causal connection between the propositions. For this reason, such conjunctions are not truth-functional. What makes “logical” connectives unique is that they indeed are truth-functional, whereas most connectives in natural language are not. Their truth value depends on the intensions of their component parts, a matter to which we return in the next section.

HOMEWORK: Discuss which aspect of the meaning of or in (84) makes the connection between the sentences A and B not truth-functional:

(84) I bekomme jetzt mein Bier (=A) oder ich gehe (=B)
    I get now my beer or I leave

Another limitation of truth functional connectives is that they can only be used to combine or modify clauses. Yet, in natural language the same expressions are also used as a means to combine or modify phrases, as exemplified in (85):

(85) a. At the zoo, we saw [DP penguins and bears]
    b. One of the girls does not [VP sleep]

Nonetheless, it seems possible to give the following paraphrases:

(86) a. At the zoo, [we saw penguins] and [we saw bears]
    b. There is a girl and it is false [that she sleeps]

The obvious advantage of these paraphrases consist in being able to use truth functional connectives again: In going from (85-a) to (86-a) we replaced the and between two DPs by an and that connects sentences; in (b) we replaced the VP-modifying negation by an expression that takes a sentence as an argument and that can be interpreted as sentential negation.

The relation between (86) and (85) seems quite systematic. It is tempting, therefore, to design a semantic theory that takes advantage of this relation in assuming that the semantic computation of the meaning of (85) does in some way or other involve a calculation that corresponds to that in (86). The task of the linguist is to accomplish this in a general rule-governed way.

One might pursue the idea that the shift from (85) to the paraphrases in (86) should be accomplished by syntactic transformations so that (86) can be regarded as the Logical Form of (85). This would be a solution in the spirit of Generative Semantics, a theory that came up in the sixties of the 20th century. Unfortunately, there are a number of counterexamples that militate against such an idea. Consider (87):
(87) a. In this zoo, penguins and bears live together in one enclosure
   b. Eines der Mädchen schläft nicht hier
      One of the girls sleeps not here

Taking (87-a) as an example, the transformation that lead us from (85-a) to (86-a) would yield the wrong result: (88) says that penguins live together on their own, as do bears, whereas (87) just says that bears and penguins share an enclosure.

(88) In this zoo, penguins live together in one enclosure and bears live together in one enclosure.

This shows that simple syntactic transformations won’t do. What we need is a semantic theory that can handle both cases. It is by no means trivial how this could be done, there is more behind the meaning of these words than meets the eye. We cannot pursue these matters here.

HOMEWORK: Are the formulas in (a.) equivalent to those in (b.)?

(89) a. If A, then B
   b. If not B, then not A

(90) a. (A and B) or C
   b. A and (B or C)

Which of the following is a tautology?

(91) a. A and not A
   b. A or not A
   c. If (A and B) or C, then A
   d. If A and (B or C), then A

6.4 Appendix: A Formal Language and Its Semantics

In this section, we will briefly explain the mathematician’s way of dealing with truth-functional connectives; moreover, we introduce some terminology and some notational conventions used by semanticists, philosophers, and logicians alike. Whether or not you should read this section depends on how far into semantics you want to get, and whether you intend to pursue the matter any further in your studies. If so, the following is a must; if not, you may skip the appendix without harm.

   Above, we symbolized negation as \( \neg \) and if...then as \( \rightarrow \); in what follows and in large parts of the literature, and is symbolized as \( \wedge \), and or as \( \vee \). Other conventions can be found; here is an (incomplete) overview:
A and B | A \land B; A \& B; less common: A.B
A or B | A \lor B, less common: A|B
if A, then B; A implies B | A \rightarrow B; A \Rightarrow B; less common: A \subseteq B
it’s false that A | \neg A, \sim A, less common: A, !A

In computational applications we still find more idiosyncratic notation, due to the fact that many programming languages have only a very restricted stock of symbols (that of an ordinary typewriter).

We will now set up a formal language that consists of sentences whose internal structure cannot be analysed any further, this simplifies matters considerably, as the only semantic objects we can talk about are truth values. Keeping in mind that our own notation is \neg, \land, and \lor the first thing to do in order to precisely define the language of propositional logic is to define the set of well-formed expressions, that is: its syntax. This is usually done by a recursive definition like the following:

Syntax:

L is a language of **propositional logic** (Aussagenlogik) if and only if
1. L contains a set of propositional constants \{A, B, C, \ldots\};
2. if p is in L, then \neg p is in L;
3. if p and q are in L, then (p \land q), (p \lor q), and (p \rightarrow q) are in L;
4. nothing else is in L.

The reader should be familiar with this kind of inductive definition. (In case there is a problem in understanding, we have to advise you to attend the course “mathematics for linguists” next term.)

HOMEWORK: Decide which formulas are well-formed according to the above definitions: A \neg B; A \lor A; (A \lor A); (A \land B) \lor C; \neg(A \land B); \neg(A \land B \land C); \neg(A \land B (\neg C)); ((A \lor B) \lor (C \lor D)).

The next step is to define the truth tables for the connectives. This is done by a recursive definition of truth for all formulas. We proceed in two steps. First we define an **interpretation** (or a **model**) for all the constants of L. Such an interpretation is a function that assigns to each constant one of the values T or F. Each such function corresponds to a row in a truth table. Let us use the letter I for such a function. Then for each constant p in L, I(p) ∈ \{T, F\}.

The next step is to determine the truth value of all (complex) formulas. This is done by a recursive definition of truth:

(93) a. If p is a constant of L, then p = T iff I(p) = T.
b. If p = \neg q, then p = T iff q = F.
c. If \( p = (q \land r) \), then \( p = T \) iff \( q = T \) and \( r = T \)
d. If \( p = (q \lor r) \), then \( p = F \) iff \( q = F \) and \( r = F \)
e. If \( p = (q \rightarrow r) \), then \( p = F \) iff \( q = T \) and \( r = F \)

Make sure that the above definition tells you nothing new, we only repeated here what we already established by the use of truth value charts.

This gives us all we need in order to define the notions we already introduced above. We summarize in the following

Definitions:

(94) \( p \) is **logically valid** (a tautology) iff \( p = T \) in all models, ie. for all interpretation functions \( I \).

(95) \( p \) is a **contradiction** iff \( p = F \) for all interpretation functions \( I \).

(96) \( p \) and \( q \) are **synonymous** iff the truth values of \( p \) and \( q \) are the same for all interpretation functions \( I \).

(97) \( p \) **entails** \( q \) (\( q \) is a logical consequence of \( p \)) iff all interpretations that assign \( T \) to \( p \) also assign \( T \) to \( q \).

Facts and Theorems:

(98) \( p \) **entails** \( q \) iff \( (p \rightarrow q) \) is a tautology.

(99) \( (A \lor B) \) is synonymous with \( \neg(\neg A \land \neg B) \); therefore, \( \lor \) can be defined in terms of negation and conjunction.

(100) \( (A \rightarrow B) \) is synonymous with \( \neg(A \land \neg B) \); therefore, \( \rightarrow \) can be defined in terms of negation and conjunction.

(101) \( (A \land B) \) is synonymous with \( \neg(\neg A \lor \neg B) \); therefore, \( \land \) can be defined in terms of negation and disjunction.

(102) \( p \) and \( q \) are synonymous iff \( p \) entails \( q \) and \( q \) entails \( p \).

(103) Any truth functional two-place connective \( \alpha \) can be defined by using negation and conjunction. In other words, for any of the 16 possible combinations of truth values, we can find a formula built up of \( A, B, \) negation, and conjunction alone, that is equivalent to that combination. It is said that such a system (containing negation and conjunction) is **functionally** (or expressionally) **complete** (☞ funktional vollständig).

(104) Since conjunction can be defined in terms of negation and disjunction (cf. (101)), a system containing disjunction and negation only is also functionally complete.
HOMEWORK: Look up "Sheffer stroke". Explain how negation and conjunction can be defined by the Sheffer stroke. Note that this result implies that a language of propositional logic with only one single connective can be functionally complete.

The above definition is what you find in elementary textbooks on logic. Linguists prefer to use a notational variant of the above definition that makes it more conspicuous that every sentence of propositional logic has an extension, and that the extensions are built up from other extensions. They usually start with a function $I$ as above, and then they extend this function by defining values for complex expressions. Such a function from complex expressions into truth values is a special case of the function $[[\cdot]]$, we used above in order to define extensions for all parts of speech. The specialization here consists in the restriction to sentences and truth values.

Using this kind of notation we may say that to each situation, represented as an index $s$ on $[\cdot]$ there corresponds an interpretation $I$. Different interpretation functions for the constants describe different situations. Therefore we could, by convention, equally well replace $s$ by $I$. Another convention is to replace $s$ by $w$. The letter $w$ means “possible world”. Possible worlds are big situations. The intuition is that the function $I$ gives truth values to all the sentences in our language; these sentences describe every fact that can be expressed in L, therefore they leave no expressible aspect of a situation undetermined. But if for any sentence we determine whether or not it holds, we not only describe a partial situation, but $I$ determines all facts of the world. Thus, any function $I$ determines a totality of facts, and such a totality is called a possible world. The world-terminology goes back to Wittgenstein (1921), Wittgenstein (1922).

Following this tradition, we say that each interpretation function corresponds to a possible world. That is, if $I(p) = T$ then $p$ is true in the possible world determined by $I$. We can then reformulate and simplify (93) as:

\[
(105) \quad \begin{align*}
\text{a. } & \text{If } p \text{ is a constant of } L, \text{ then } [p]_w = I(p). \\
\text{b. } & \neg q \equiv T \iff [q]_w = F. \\
\text{c. } & (q \land r) \equiv T \iff [q]_w = [r]_w = T. \\
\text{d. } & (q \lor r) \equiv T \iff [q]_w = [r]_w = F. \\
\text{e. } & (q \implies r) \equiv F \iff [q]_w = T \text{ and } [r]_w = F \text{.}
\end{align*}
\]

Adjusting terminology, we now say that a tautology is true in all possible worlds, a contradiction is false in all possible worlds.

The alert reader might have noticed that the procedure described above does not define an extension for the logical operators $\neg, \land, \lor,$ and $\implies$. The reason for this is that their meaning is implicit in the definitions above. We could, of course,
make their meaning explicit by extending the interpretation function $I$ to these operators. Recall from above that we already defined the meaning of the two-place operators as two-place relations. The meaning of negation is a one place relation, namely the set $\{F\}$. Note that the logical symbols are constants of the language $L$ whose interpretation is constant, i.e. is the same for each $I$. We can then reformulate (105) as:

$$(106) \begin{array}{ll}
\text{a.} & \text{If } \alpha \text{ is a constant of } L, \text{ then } \llbracket \alpha \rrbracket_w = I(\alpha) . \\
\text{b.} & \llbracket \neg q \rrbracket_w = T \text{ iff } \llbracket q \rrbracket_w \in \llbracket \neg \rrbracket_w . \\
\text{c.} & \llbracket (q \land r) \rrbracket_w = T \text{ iff } \langle \llbracket q \rrbracket_w, \llbracket r \rrbracket_w \rangle \in \llbracket \land \rrbracket_w . \\
\text{d.} & \llbracket (q \lor r) \rrbracket_w = T \text{ iff } \langle \llbracket q \rrbracket_w, \llbracket r \rrbracket_w \rangle \in \llbracket \lor \rrbracket_w . \\
\text{e.} & \llbracket (q \rightarrow r) \rrbracket_w = T \text{ iff } \langle \llbracket q \rrbracket_w, \llbracket r \rrbracket_w \rangle \in \llbracket \rightarrow \rrbracket_w . \\
\end{array}$$

Finally, note that the structure of $A$ and $B$ in natural language might actually be binary branching, as shown in

$$\begin{array}{c}
A \\
\bullet \\
\text{and} \\
B
\end{array}$$

This requires a new syntax and a new compositional semantics with $\bullet$ as a constituent. Let’s say that this is an ad-formula. We then have to replace the syntactic rule for $(A \text{ and } B)$ by the following “natural language” equivalent.

$$\begin{array}{ll}
\text{(108) } & \text{a. If } p \text{ is a sentence, “and } p \text{” is an ad-formula.} \\
& \text{b. If } q \text{ is a sentence and } \alpha \text{ is an ad-formula, then “} A \alpha \text{” is a sentence.}
\end{array}$$

It now remains to provide a semantics for ad-formulas and for the two syntactic rules in (108). This problem is analogous to the one we already solved when discussing transitive verbs as two-place relations. Actually, we can employ the same semantic rules that combine a verb with its object and then with its subject.

**HOMEWORK:** Show this in detail.

7 Intensions (Intensionen)

In previous sections we argued that the meaning of an expression cannot be equated with its extension: if this were the case, all true sentences would have the same meaning (and so would all false sentences). We also learned that the extension of a sentence is a function of the extensions of its parts (by extensional
compositionality). Extensions are that part of a theory of meaning that ensures that linguistic expressions can be used to refer to entities in the world. We will see in this section that extensional compositionality is not always adequate: we need a broader theory that goes beyond extensions. This part of the theory of meaning is concerned with the information conveyed by a sentence.

7.1 Intensional Contexts (Intensionale Kontexthe)

Consider the following two sentences:

(1) a. Hamburg is larger than Cologne  
   b. Pfäffingen is larger than Breitenholz

As it turns out, both sentences are true\(^{31}\), which means that they have the same extension, and both extensions can be calculated from the extensions of the names and the relation \textit{larger than} (the set of pairs \(\langle x, y \rangle\) with \(x\) larger than \(y\)). But now consider so-called \textit{propositional attitude} reports, i.e. sentences that tell us something about the information state of a person:

(2) a. John knows that [ Hamburg is larger than Cologne ]  
   b. John knows that [ Pfäffingen is larger than Breitenholz ]

Let us assume that the extensions of the words \textit{John}, \textit{knows}, and \textit{that}, whatever they are, are exactly the same in (a.) and (b.), that is: none of these terms is lexically ambiguous. Moreover, no structural ambiguity is detectable. We also know that the embedded sentences (i.e. the sentences in (1)) have the same extensions. Now, regardless of what exactly the extensions of \textit{know} and \textit{that} are, given the Principle of Extensional Compositionality, we can infer that the extensions of the more complex sentences in (2) must also be identical, simply because the extensions of (1-a) and (1-b) and that of all other relevant lexical items and constructions are the same. But now we face an obvious dilemma. It is surely not the case that anyone who knows that Hamburg is larger than Cologne also knows that Pfäffingen is larger than Breitenholz. In particular, assume that John knows (1-a) but not (1-b). In consequence, (2-a) is true, whereas (2-b) is false in the same situation, despite the fact that the extensions are the same. In fact, our theory of extensional compositionality predicts identical truth values, contrary to fact. What went wrong?

It seems intuitively clear that the complement (the object) of a verb like \textit{know} cannot be a truth value. If this were the case then any piece of knowledge would

\(^{31}\)Pfäffingen and Breitenholz are districts in the municipality of Ammerbuch; \(\Rightarrow\)http://de.wikipedia.org/wiki/Ammerbuch#Gemeindegliederung.
imply omniscience. This means that extensional compositionality fails in a context like ... know that ... In plain words: the principle cannot hold in full generality; there seem to be exceptions. Such contexts where extensional compositionality fails are called intensional contexts. If we embed an expression (e.g. a subordinate clause) into an intensional context, then the contribution of the embedded expression cannot be its extension. But what else could it be?

7.2 Propositions (Propositionen)

The difference of truth conditions of the sentences in (2) seems to be due to the state of information John is in. More precisely, what the sentences claim is that John's state of information comprises the information expressed by the embedded sentences. But the information conveyed by the embedded sentences differ, they express different facts. Hence the truth value of the entire sentence depends on the information expressed by the embedded sentence. In semantics, the technical term for this information is the proposition (Proposition) expressed by the sentence. The truth values may differ because the propositions do.

What is a proposition? What is the information contained in a sentence? To answer this question consider the following sample sentences:

(3) 4 fair coins are tossed
(4) At least one of the 4 tossed coins lands heads up
(5) At least one of the 4 tossed coins lands heads down
(6) Exactly 2 of the 4 tossed coins land heads up
(7) Exactly 2 of the 4 tossed coins land heads down

(3) is the least informative of the five sentences, because it does not inform us about the result of the tossing. The other sentences are more informative in this respect. (4) is less informative than (6), (7) is more informative than (5). Presupposing that each coin either lands heads up or down (and excluding thereby a third possible outcome of the tossing), (6) and (7) are equally informative. Whether (4) and (5) are also equally informative depends on our understanding of “informative”: in a certain sense, both contain the same amount of information; they amount to the same quantity of information. But qualitatively, they are of course totally different. To see this, assume that all four coins land heads up. Then (4) is true, but (5) is false. According to the so-called most certain principle of semantics, repeated in (8),

(8) If a sentence $p$ is true but a sentence $q$ is false in the same situation, then $p$ and $q$ cannot have the same meaning.
we must conclude that although the amount of information might be the same, the meaning, and therefore the propositions are still different.

Tossing coins is reminiscent of probabilities, and this is not accidental. As one can easily verify, the quantitative informativity of these sentences corresponds to the probability of a certain event taking place. This probability can be measured by counting the positive cases in relation to all possible cases. Moreover, a sentence A is more informative than a sentence B if the number of cases that make A true is smaller(!) than the number of cases that make B true.

On the other hand, the qualitative differences between the sentences depend on the kind of situations that make sentences come out true or false. Thus, the qualitative difference between (4) and (5) consists in the fact that the positive cases, i.e. the cases that make the sentence true, are not the same, although their number may be the same.

It should be obvious that it is the quality of the information that is relevant for a theory of meaning. Two sentences that are qualitatively equally informative apply to exactly the same positive cases; their informational content can be identified with the set of cases that make the sentence true. This content is usually identified with the notion of proposition:

(9) The proposition expressed by a sentence is the set of possible cases in which that sentence true.

At this point it will be useful to recall that possible cases can be characterized by interpretation functions. Such functions define for all elementary sentences whether they are true or false; each such function defines a possible outcome, that is, a possible case or a way how the facts might turn out to be. Let us apply this to the scenario just described, taking (3) to (7) as our example sentences. To simplify things a bit, let us abstract away from the internal structure of sentences, assuming that an interpretation function assigns truth values directly to the four elementary sentences “coin-1 lands heads up”, “coin-2 lands heads up”, “coin-3 lands heads up”, “coin-4 lands heads up”. Each assignment function that assigns false to “coin-X lands heads up” is one that interprets “coin-X lands heads down” as true. The possible assignments are arranged in the following table (with cX = “coin X lands heads up”):
The table just lists all possible outcomes of a tossing; each of the sixteen different rows represents one type of situation (you should fill in the missing cases (12)-(14)). Now, the informational content of (and thereby the proposition expressed by) the sentences (3) to (7) can be identified with the following sets of possibilities:

(11) a. At least one coin lands heads up = {1-15}
b. At least one coin lands heads down = {2-16}
c. Exactly 2 coins lands heads up = {6-11}
d. Exactly 2 coins lands heads down = {6-11}

(12) a. Exactly one coin lands heads down = {2-5}
b. Exactly one coin lands heads up = ? (you are asked to answer this by yourself)
c. c3 lands heads down = ?

It seems that the propositions expressed by (11-c) and (11-d) are identical. But this is not really intended, if we identify the informational content with the meaning of the sentences, it seems that the sentences still have different meanings. At this point, a further consideration is to be taken into account: the interpretation functions are in general much larger. In the above cases we only considered the values for four elementary sentences c1-c4. But of course we would also like to interpret a sentence like

(13) a. 4 coins were tossed when John coughed
b. 4 coins were tossed and noone coughed
The assignment function $I$ must therefore also supply a value for $x$ coughed, for any individual $x$. In general, this function must be large enough so as to decide for every elementary sentence whether it is true or false. What it does is this: it gives us a complete description of a possible state of affairs. Such a complete description must take into account anything we can conceive of: the number of hairs on my head, the names of all my ancestors, the position of all atoms in our world etc. Such a completely specified possible state of affairs is also called a possible world. Moreover, in order to capture the meaning of a sentence, we must consider all possibilities. To illustrate, consider again the two sentences (11-c) and (11-d). We were a little bit concerned that they differ in meaning, but they seem to realize the same possible cases. But this is not really the case. In order to capture the meaning of the sentences, we have to consider all possibilities. Above, we simply didn’t take into account enough possible assignments (= enough possible worlds); we only considered cases with exactly 4 coins being tossed. In a situation with five (or more) coins being tossed, the set of assignments to be considered is much larger, although the quantitative amount of information expressed by the two sentences still remains the same. In such a situation, the proposition expressed by the sentences differ, which can easily be checked by imagining a possible situation in which (11-c) is true and (11-d) is false. Such additional possible assignments account for the difference in meaning. In effect then, we can say that only sufficiently many assignment functions can capture all potentially relevant aspects of (literal) meaning. We therefore identify the meaning of a sentence with a sufficiently large set of possible worlds, namely the set of all possible worlds in which the sentence is true.

Therefore, propositions are sets of possible worlds. And possible worlds are highly specific, completely specified (and very big) situations. This change of perspective is also a change of terminology. In linguistics, we do not speak of possible cases or possible states of affairs, but of possible worlds. And likewise, rather than saying that a sentence is true in a particular situation, we now say that it is true in a possible world. Possible worlds are particular situations, namely very very big complete situation where every possible case is determined (“die Welt ist alles, was der Fall ist”).

This change in terminology also induces a change in notation. From now on, we use $\omega$ (for world) rather than $s$ (for situation) as an index on the extension function $[\cdot]$. By definition, then, $[p]_\omega$ is the denotation or extension of $p$ in a world $\omega$, namely its truth value in $\omega$. We may now reformulate (9) as follows:

(14) The proposition expressed by a sentence is the set of possible worlds in which that sentence true.

(15) A sentence $S$ is true in a possible world $\omega$ if and only if $[S]_\omega = 1$. 

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(16) By $[[S]]$ we mean the proposition expressed by $S$. $[[S]] := \{ w : [[S]]_w = T \}$

Hence it follows that

(17) A sentence $S$ is true in a possible world $w$ if and only if $w \in [[S]]$.

Or symbolically:

(18) $[[S]]_w = T$ if and only if $w \in [[S]]$.

This is basically the definition of Wittgenstein (1921) (= Wittgenstein (1922)) and Carnap (1947) (= Carnap (1972)). Adopting Wittgenstein’s terminology, we may say that all possible combinations of circumstances (Sachverhalte) make up a logical space (logischer Raum) and each sentence cuts this space into two parts: in one part the sentence is true, in the other the sentence is false.

(19) cuts the logical space represented by the rectangle into two, the A-worlds and the non-A-words. We can thus identify a possible world as a point within the rectangle.

While Wittgenstein conceived of propositions as cuts in the logical space of possible facts, so that possible worlds can be identified with points in the logical space, Carnap reduced possible world to maximally contradiction-free set of sentences (so-called state descriptions (Zustandsbeschreibung)). To each world there corresponds a contradiction free set of sentences that describes this world (worlds are assumed to be contradiction-free) so that a maximally consistent set describes all fact in a possible world. This view reduces possible worlds to linguistic entities, namely to certain sets of sentences. In later work, possible worlds were no longer identified with sentences, they were considered as genuine metaphysical entities on their own, independent of the language under consideration and alongside with the real world that also exists independently of language. Nevertheless there is considerable disagreement among philosophers about the correct ontological interpretation of possible worlds. Should we imagine possible words as representing abstract units of information or should we conceive of
them as being real, i.e. made up of atoms exactly as our real world is? Perhaps it would be wiser to talk about only small situations, but formally, size alone does not matter. So we leave things open for now, suggesting that anyone not comfortable with the possible worlds ontology might consider only very small “worlds".\footnote{A most provocative extreme position is taken by the American philosopher David Lewis in his 1986 book \textit{On the Plurality of Worlds}. Cf. also \texttt{http://www.zeit.de/1999/44/199944.lewis1.xml}.}

\textit{Propositions as Sets}

The topology inside the box that displays a logical space plays no role in the representation of a proposition; we could represent the division equally well as

\begin{equation}
\text{(20)}
\begin{array}{c}
\text{A}
\end{array}
\end{equation}

with a region inside A and a region outside A. Inside A are the facts that make A true, this corresponds to \([A]\). Outside A we have the facts that hold when \([\neg A]\) is true. It's also immaterial whether or not all A-worlds are in one circle, they need not occupy a connected region and could be distributed in any regions of the logical space.

As a side effect of defining propositions as sets, we can now define the proposition that A and B as an operation on sets, namely the intersection of A with B:

\begin{equation}
\text{(21)}
\begin{array}{c}
[A \land B] := [A] \cap [A]
\end{array}
\end{equation}

That is, the set of worlds where “A and B" holds is precisely the intersection of all worlds in which A is true and all worlds in which B is true; cf. also example (31) on page 86. And likewise, disjunction corresponds to set-union, as can easily be verified by looking back to (40), page 89.

\subsection*{7.3 From Propositions to Intensions (Von Propositionen zu Intensionen)}

Let us now define the notion of intension. Previously, we identified the intension with the informational content of a sentence. We will show now that the intension of a sentence S is practically the same as the proposition expressed by S, but for
reasons that will become clear in a minute our definition of intension is a little bit more involved.

Our starting point is again the box in (20)—a very large, possibly infinite set of possible words $W$, among them our actual world. The proposition in (20) can also be represented as a table in which every possible world is assigned a truth value for $A$:

<table>
<thead>
<tr>
<th>world $w_i$</th>
<th>truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$T$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$T$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>$F$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$w_n$</td>
<td>$F$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

This table is only another way of representing a proposition, namely as a function from possible worlds to truth values. We will call this function the intension of a sentence. For example the intension of a sentence like

(23) Barschel wurde ermordet
Barschel$^{33}$ was murdered

is a function whose arguments are possible worlds, assigning the value $T$ to every world in which Barschel was murdered, and assigning $F$ to any other world.

The intension of a sentence shows how its extension varies in logical space. The most interesting aspect of this characterization is that it works not only for sentences, but for arbitrary expressions like nouns, verbs, adjectives etc. The only thing we have to do is combine the previous functions that determined the extensions of these expressions for a possible world $w$ into a large function that depends on possible worlds as arguments. Such a function assigns extensions to each possible world and to each expression. It can be exemplified by the table whose upper part was already discussed above:

(24) $w_1$: [$c_1$]$_{w_1}$ = [$c_2$]$_{w_1}$ = [$c_3$]$_{w_1}$ = [$c_4$]$_{w_1}$ = $T$
$w_2$: [$c_1$]$_{w_2}$ = [$c_2$]$_{w_2}$ = [$c_3$]$_{w_2}$ = $T$; [$c_4$]$_{w_1}$ = $F$
$w_3$: [$c_1$]$_{w_3}$ = [$c_2$]$_{w_3}$ = [$c_4$]$_{w_3}$ = $T$; [$c_3$]$_{w_3}$ = $F$
$w_4$: etc.

$^{33}$Uwe Barschel (1944-1987) was a German politician (prime minister of Schleswig Holstein) who had to resign under scandalous circumstances (comparable to the Watergate affair) and who was found dead in the bathtub of his hotel room a few days after his resignation. The circumstances of his death could never been clarified. Cf. http://www.jurblogs.com/de/was-weiss-helmut-kohl-ueber-barschels-tod.
Apart from assigning extensions to sentences, the big function shown in (24) also assigns extensions to definite descriptions, names, predicates, verbs etc. It assigns George Bush as the extension of a definite description the president of the US for $w_1$ and $w_2$, but it assigns Hilary Clinton to $w_3$ (and would assign Barack Obama to $w_1$ if this is our actual world). And if this function assigns values to a future world or to any other possible world, its value might well be myself, or the Pope, or any other individual, depending on the possible world (possible circumstances) these individuals live in, however far-fetched these circumstances might seem. Similarly, given a predicate like snore, the function exemplified in (24) assigns to each world the set of individuals that snore in that world.

We may now define the intension of an expression $\alpha$ as a function that assigns an extension to each possible world. More formally, we can build up intensions from extensions in the following way:

\[(25) \text{ The intension of } \alpha, \text{ written as } [ [ \alpha ] ]_i, \text{ is that function } f \text{ such that for every possible world } w, f(w) = [ [ \alpha ] ]_w. \]

According to (25), the intension of a sentence S is a function that assigns to S one of T and F depending on a possible world $w$. If the value of that function is T for a world $w$, then S describes a fact in $w$. If not, S does not describe a fact, and we say that S is false in $w$.

### 7.4 Combining Intensions

The above discussion has revealed that the concept of an intension applies to arbitrary expressions—as long as we can describe their extensions. Intensions can now be combined in order to form new intensions of complex expressions, in accordance with the

\[(26) \text{ Principle of Intensional Compositionality:} \]

The intension of a complex expression is a function of the intensions of its immediate parts and the way they are composed.

Let us illustrate this with the following example:
The intension of (27) is a function that assigns to any world the truth-value T if Paul is sleeping in that world, and false otherwise. How can this be calculated on the basis of the intensions of Paul and schläf? The intension of schläf works analogously to the intension of president above: it is a function assigning the set of snoring entities to each possible world. What is the intension of Paul? In reality, the name refers to a particular person, namely Paul. But what about other possible worlds? Could someone else have been Paul? Hardly. Of course, another person could have been called “Paul”, but calling her or him Paul wouldn’t make this person Paul. Paul could have had another name, but he would still be Paul. When considering the possibility that Paul’s parents almost called him Jacob, we would have to say that Paul (and not Jacob) could have gotten another name different from the one he actually has. Thus, with the name ‘Paul’ we always refer to the same person—regardless of what this person would be called in other circumstances. We conclude from this that the extension of Paul looks like this:

<table>
<thead>
<tr>
<th>world</th>
<th>entity</th>
</tr>
</thead>
<tbody>
<tr>
<td>w₁</td>
<td>Paul</td>
</tr>
<tr>
<td>w₂</td>
<td>Paul</td>
</tr>
<tr>
<td>w₃</td>
<td>Paul</td>
</tr>
<tr>
<td>...</td>
<td>Paul</td>
</tr>
<tr>
<td>wₙ</td>
<td>Paul</td>
</tr>
<tr>
<td>...</td>
<td>Paul</td>
</tr>
</tbody>
</table>

This table of course reveals a certain redundancy, because the extension of the name does not depend on any of the worlds. But for systematic reasons we do assume that all names, like all other expressions, have an intension which determines the extension in every possible world. The only difference from other expressions is that this is a constant function that yields the same individual for each possible world.

---

34 Of course, in a possible world there could be millions of people called Paul. The convention is that if in a given situation only one person comes to mind, then we can use the name to refer to that person. In a text-book context, however, no such specific context is given. Here the convention is that Paul should really be understood as Paulᵢ, which the index i disambiguating between the millions of Pauls by saying to which of them the expression is intended to refer.

35 This interesting fact about names was overlooked in many philosophical discussions prior to the pioneering work of Saul Kripke (1972) (= Kripke (1993)).
How can we combine these intensions in order to get a complex intension? This is quite simple. We only have to compute the extensions for every possible world. Having done so we get another table, which for each world \( w \) contains a row with the value T if and only if \( \llbracket \text{Paul} \rrbracket_w \in \llbracket \text{schläft} \rrbracket_w \). This new table is again an intension, ie. a function that assigns truth values to each possible world. This way, the combination of intensions is reduced to the combination of extensions which has already been described in section 5. It’s only that the results of this computation for each possible world now make up a new intension, a function that assigns to each world \( w \) a truth value, namely the result of calculating the extension in \( w \).

Let us next, after this long detour, return to our analysis of:

\[
(29) \quad \begin{align*}
\text{a. John knows that } & \llbracket \text{ Hamburg is larger than Cologne } \rrbracket \\
\text{b. John knows that } & \llbracket \text{ Pfäffingen is larger than Breitenholz } \rrbracket
\end{align*}
\]

It is clear by now that the embedded sentences express different propositions. The Principle of Intensional Compositionality (26) says that the intensions of the embedded sentences are responsible for the difference in meaning of the entire sentences. Hence, the key to the solution of our problem must be that the object of \textit{know} (the \textit{that}-clause) is an intension (rather than an extension, ie. a truth value).

For the present purpose, however, we may simplify things a little bit by making use of the fact that the intension of a sentence can be identified with the proposition expressed by that sentence. This is so because any set of possible worlds \( A \) can be identified with a function \( f \) having the set of all possible worlds \( W \) as its domain, and having as its value \( f(w) \) the truth value T if and only if \( w \in A \). And conversely, for each function \( f \) into truth values T and F, there is exactly one set \( S \), such that \( x \in S \) iff \( f(x) = T \). This correspondence allows us to identify the intension of a sentence with a set of possible worlds. This is explicitly stated as a theorem in (30):

\[
(30) \quad \text{For any sentence } S \text{ the proposition } \llbracket S \rrbracket \text{ is that set of possible worlds } A \text{ such that } w \text{ is an element of } A \text{ if and only if the result of evaluating the function } \llbracket S \rrbracket_i \text{ at the world } w \text{ is } T.
\]

Put more formally, the following equivalences hold for all sentences \( S \):

\[
(31) \quad \llbracket S \rrbracket = \{ w : \llbracket S \rrbracket_i (w) = T \} = \{ w : \llbracket S \rrbracket_w = T \}
\]

And conversely:

\[
(32) \quad \llbracket S \rrbracket_i = \text{ the function } f \text{ such that } f(w) = T \text{ iff } w \in \llbracket S \rrbracket.
\]
We have thus shown that propositions and intensions of sentences are interdefinable, so we can take one for the other at will. In what follows we will work with sets (propositions) rather than with the more complicated functions (intensions).\footnote{Note that for every set $A$ there is a function $f$ with $f(x) = \text{T}$ iff $x \in A$. This function is called the \textit{characteristic function} of $A$. But this does not mean that we could replace all functions with sets, because this is legitimate only in case the function has just the two values true and false. In general we cannot replace intensions with sets, this only works for intensions of sentences.}

Given this simplification let us now ask for the extension of $\text{know}$. Earlier we assumed that a two-place predicate is a relation between two individuals. For a verb like $\text{know}$, however, it is clear by now that we need a relation between an individual (the subject) and a proposition (representing the meaning of a sentence). In other words, the verb $\text{know}$ in (29) expresses a relation between John and a proposition, i.e., the intension of a sentence. For that reason, verbs like $\text{know}$ are called \textit{intensional verbs}.

Given that the propositions in (29) are not synonymous we might easily imagine a situation (a possible world) in which (29-a) is true but (29-a) is false, so that John might know one proposition but not the other. In other words, what we proposed above as the truth conditions for Johann weiss dass $p$ is the following:

\begin{equation}
\boxed{\text{[Johann weiss dass } p\text{]}}_w = \text{T} \iff \langle \boxed{\text{[Johann]}}_w, \boxed{[p]} \rangle \in \boxed{\text{[weiss]}}_w.
\end{equation}

There is a technical point here that should be mentioned: once again analysis ignores syntax and compositionality. Strictly speaking, we would need a rule that first takes an object (a proposition) to form a property, and only in a second step does this property combine with a subject (an individual) to yield a proposition. This works just as above with other transitive verbs. We only replace an object $x$ with a set $p$. Moreover, the complementizer dass ($\text{that}$) apparently has no meaning of its own. We leave it to the reader to reformulate (33) in analogy to the rules in section 5.

Summarizing so far, all we did was replace the extension of the complement of $\text{know}$ with its intension. Therefore the extension of the complex sentence is not calculated on the basis of the extensions of its parts, but in the case at hand on the basis of the intension of the sentential complement. As already mentioned in connection with clausal connectives in section 6.3, most connectives are not truth-functional, which means that their interpretation is based on the intensions of their complements, rather than their truth values. This also applies to all verbs and adjectives that take sentences (or infinitives) as their complement.\footnote{An exception is adjectives (and corresponding verbs) like those occurring in \textit{it is true that} \ldots, \textit{it holds that} \ldots, etc.}

As another case in point, consider (34):

\begin{equation}
\boxed{\text{[John believes that } p\text{]}}_w = \text{T} \iff \langle \boxed{\text{[John]}}_w, \boxed{[p]} \rangle \in \boxed{\text{[believes]}}_w.
\end{equation}
Paul seeks an exciting German detective story

The problem is the following: assume that the actual extension of *exciting German detective story* is actually the empty set, so there is no exciting German crime novel. Assume further that there is no cheap French Bordeaux wine either. Then the two predicates have the same extension, namely the empty set. Nonetheless we cannot conclude from (34) that (35) holds.

Paul seeks a cheap French Bordeaux wine

This, however, should be the case if the Principle of Extensional Compositionality were applicable. From this we must conclude that the verb *seek* creates an intensional context, turning the object of *seek* into something intensional (the different intensions of the properties mentioned above). This may then also explain the ambiguity of examples like the one discussed in section 3.4.2.38

7.5 From Intensions to Extension and Back Again

Let us now take a closer look at the methodology we pursued above. One reason for denotations (or extensions) not to represent meaning was that there are vastly more propositions (indeed infinitely many) than there are denotations of sentences (only two). Another reason is that anyone who learns the meaning of an expression does not automatically know its denotation. Even if your German (or English) is perfect, you don’t know the truth value of all sentences. Think about such sentences as (23): of course we know the meaning of this sentence, but its extension is known only by very few (who have every reason to keep a secret).

Now what about knowing an intension? Doesn’t anyone who knows the intension of (23) also know its extension? Given a table with truth values and possible worlds, we only have to pick out the actual world in order to get the truth value. That’s correct, but how do we know which of the worlds is our actual one? Worlds are maximally detailed, maximally specific states of affairs. And even if I should know that any case in which Barschel committed suicide is one where he was not murdered, I do not know which of these cases is actually a fact. And even if I knew he committed suicide I would still not be able to pick out the actual world because there remain infinitely many details I do not know about the world I inhabit. If I did, I would be omniscient.

Knowing the intension of a sentence therefore does not imply knowing its truth value. One only has to know which hypothetical state of affairs would make it true. And this seems to be an adequate criterion for understanding a sentence.

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38 An explanation of the ambiguity along these lines can be found in Quine (1960) (= Quine (1980)).
“Einen Satz verstehen, heißt, wissen was der Fall ist, wenn er wahr ist” (Wittgenstein 1921, Tractatus 4.024) [look up translation]. Adopting this slogan we agree that the intension of a sentence exhausts its meaning. Informativity is therefore the central aspect of meaning. This does not mean that extensions are unimportant: the extension results from the intension as its value for the actual world. Anyone who wants to know the extension of an expression must know enough about the reality in order to be able to uniquely (eindeutig) identify the extension. But given the vastness of worlds, this is practically impossible, we must be content with the disposition to know an extension that is implicitly described by the intension.

So there is a road from intensions to extensions: anyone who knows enough about the real world, also knows, at least in principle, where this road leads us to. But is there also a way back from extensions to intensions? In a sense, the answer is yes, because the totality of possible extensions makes up the intension; this is the definition of intension. On the other hand the step from extension to intension involves the concept of a possible world. Possible worlds are stipulated, they cannot be discovered. So they are not given to our experience independently from the concept of intension. In a sense then, there is no way back, as is already clear from the fact that two extensions may be identical (the chancellor (die Kanzlerin), the party leader (die Parteivorsitzende)) without identity of the intensions.

A final remark on terminology might be in order. Note that for sentences, we use the following three notions as synonymous: Meaning of a sentence, intension of a sentence, proposition expressed by a sentence. All mean the same. For other parts of speech (those that are not sentences) there is no equivalent to the notion of a proposition. Frege's Principle of Compositionality can therefore be stated in two variants, both being equivalent:

(36) **Principle of Compositionality of Meaning:**
The meaning of a compound expression is a function of the meaning of its immediate parts and the way they are composed.

(37) **Principle of Intensional Compositionality:**
The intension of a compound expression is a function of the intension of its immediate parts and the way they are composed.

Frege himself did not formulate it precisely this way, he used notions like “parts of thought” in Frege (1923) (= Frege (1956)) and the notion of Sinn (“sense”) in Frege (1982) (= Frege (1969)). So a Fregean sense is roughly what we called an intension. Unfortunately, Frege also used the term Bedeutung (in ordinary contexts translated as “meaning”) as a technical term for reference, denotation, or extension. In consequence, a sentence “means” (= bedeutet) its truth value. This terminology is a bit confusing, and we thought this kind of a warning is appropriate in case you
want to read Frege (1982).

The relation between sense and meaning in Frege’s work is concisely discussed in Stepanians (2001), chapter 7.

7.6 Intensions and Sense Relations

When talking about sense relations in section 2.3 we had not yet introduced sets. From a set theoretical point of view, however, it is easy to see that many sense relations can be represented as set theoretical relations between extensions. For example, the hyponymy relation between man and human being can be represented as a subset relation between the extensions of $\llbracket \text{man} \rrbracket_w$ and $\llbracket \text{human being} \rrbracket_w$. One is tempted, therefore, to say that a noun phrase $A$ is a hyponym of a noun phrase $B$ if and only if $\llbracket A \rrbracket_w \subseteq \llbracket B \rrbracket_w$.

However, we did not present it that way because there is a potential danger of misunderstanding: the correspondence between sense relations and relations between sets needs further qualification. For it could well be that $\llbracket A \rrbracket_w \subseteq \llbracket B \rrbracket_w$ without $A$ being a hyponym of $B$. For example, assume that each professor is an adult, though adult is not a hyponym of professor. The fact that there is no hyponymy here reveals itself in that it is not inconceivable that there are younger than adult professors. Hence sense relations are not a matter of extension. Rather they are a matter of logical space. Since it is conceivable that some professors are non-adults, this is reflected in logical space in that some possible worlds are inhabited by underage professors. Although it is clear that such worlds are purely hypothetical (Denkmöglichkeiten) the mere existence of such worlds also blocks inferences. For example, from

(38) Saul Kripke is a professor.

we cannot logically infer

(39) Saul Kripke is an adult.

although in our world this is more than probable (it’s a practical inference, not a logical one). The reason is that it is at least conceivable that Saul became professor before reaching the age of adults (18 years in Germany). This means that the set of
possible worlds that validate (38) is not a subset of possible worlds that represent (39).

On the other hand, from

(40) This is a bus.

we can validly infer

(41) This is a vehicle.

The proposition (40) is a subset of (41), and this is so because this time there is no possible world in (40) that is not contained in (41). And this is so because there is no possible extension of bus that is not contained in the extension of vehicle. In consequence, when depicting sense relations as relations between sets, we must always additionally keep in mind that these relations between extensions must hold in all possible worlds:

(42) A is a hyponym of B if and only if \([A]_w \subseteq [B]_w\) in all possible worlds \(w\).

In fact, sense relations are intensional relations, and it is for this reason that we felt reluctant to represent it as a mere relation between extensions.

We also demonstrated above that there is a close connection between inferences between sentences and sense relations. Let us look at another example. Usually it is assumed that cat and dog are incompatible. Two propositions are incompatible if and only if they do not share a common world, and likewise the sense relation of incompatibility can be represented by two extensions that do not share an element. For the sense relation to hold, this non-overlap must hold in every possible world. Among others, the following inferences should be valid:

(43) Fido is a dog

\(\vdash\) Fido is not a cat

(44) Fido is a cat

\(\vdash\) Fido is not a dog

cat and dog are natural kind terms and it is in agreed among philosophers that if something is a cat in a possible world, it is a cat in every possible world and therefore could not be a dog, and vice versa (see eg. Kripke (1972) and Putnam (1975) for an extensive justification). Hence there is again a parallelism between sense-relations and inference relations.

In fact, it is often assumed that we only have intuitive access to sense relations of this sort via inference relations between propositions. If certain inferences hold, this justifies the existence of certain sense relations.
This way, the intuitive evaluation of inferences may take priority over that of sense relations: the justification of certain sense relations proceeds via that of inferences, and these in turn rest on our intuitions about conceivable possible worlds (or at least conceivable situations; but after all, worlds are only large situations, and nothing prevents very small worlds (cf. www.bussongs.com/songs/its_a_small_world_after_all_short.php)).

7.7 Sentence Semantics and Lexical Semantics

The above remarks suggest that sentence semantics and the formal study of meaning in truth-functional semantics also provides the ground for the study of the meaning of words. In a sense, however, this conclusion is somewhat premature. Consider the case of cat and dog again. From sentence semantics we've learned that each extension is a set. So the intension of dog, eg., is a function that assigns to each possible world a certain set. Of course we have required that this be the set of dogs. But we also argued that we do not know the extension in a large enough situation, let alone in our world or even in a possible world. In fact we can only stipulate certain extensions for possible worlds. Therefore, in a formal semantics the interpretation functions are not by themselves restricted; it is normally assumed that any such function can do the job, and that restrictions must be stipulated.

But this also means that in a formal model, no special restrictions on the intensions of lexical items hold per se, except for the intensions of logical expressions like and, or, all, etc. This in turn means that without any further restrictions the semantic model tells us nothing about sense relations; rather, at the outset all lexical items are interpreted independently from each other and no such relations are automatically built into the model.

This is rather bad news. The formal models we develop cannot be claimed to be models for natural semantics unless we specify sense relations. Practically, this is done only if necessary for particular descriptive purposes, by stipulating so called meaning postulates. These are sentences of the language under investigation that are assumed to be valid in all models for natural language; the postulates exclude certain possible worlds from being conceivable, and they thereby restrict the interpretation function. For example,

(45) No cat is a dog

is a reasonable meaning postulate in every model for natural language. That is, (45) considered as a meaning postulate must be true in all possible worlds. This of course also restricts possible intensions: (45) excludes all extensions where the noun denotations overlap (in which case (45) would be false).
So the disappointing message is that at the end of the day sentence semantics tells us almost nothing about the meaning of individual words. In particular, it cannot in full answer the question “What Is Meaning?” (although the title of a semantics introduction by a prominent semanticist might suggest otherwise).

On the other hand, semanticists generally do not bewail this unfortunate state of affairs. Why not? For one thing, sentence semantics may still provide the tools for the analysis of interesting lexical items, for example modal verbs like *must, can, ought, might*, and others. (cf. eg. Lewis (1973), Kratzer (1977), or Kratzer (1981)). As a simple example for the kind of semantic analysis we have in mind, consider the verb *know*: we already analysed its extension as a relation between a subject and a proposition. Imagine one utters (46-a) truthfully. One could not at the same time deny the truth of (46-b)

\[\begin{align*}
\text{(46)} & \quad \text{a. Mary knows that Bill snores} \\
& \quad \text{b. Bill snores}
\end{align*}\]

It would thus be a contradiction to say

\[\text{(47)} \quad \#\text{Mary knows that Bill snores, but Bill doesn’t snore}\]

Therefore (46-a) entails (46-b). This is specific to the verb *know*; the inference does not hold with a verb like *believe*:

\[\begin{align*}
\text{(48)} & \quad \text{Mary believes that Bill snores} \\
& \quad \text{∗∗. Bill snores}
\end{align*}\]

It is fully consistent to say:

\[\begin{align*}
\text{(49)} & \quad \text{Mary believes that Bill snores, but (in fact) Bill doesn’t snore}
\end{align*}\]

We therefore seem to have missed something in our account of the meaning of *know*. One possible way to refine our analysis is to say that we have to restrict the interpretation function for *know* in such a way that the inference from (37) to (46-b) becomes valid. This could be done in the following way:

\[\begin{align*}
\text{(50)} & \quad \text{For all individuals } x, \text{ for all propositions } p, \text{ and for all possible worlds } w:} \\
& \quad \text{if } \langle x, p \rangle \in [\text{know}]_w, \text{ then } w \in p.
\end{align*}\]

This says that if p is known (by x) in a world w, then this world w is in p, which means that p is true in w. Therefore we can infer (that) p from x knows that p.

This is a crucial restriction on the interpretation function, otherwise the inferences would be invalid and we would have missed one aspect of the meaning.
of know. But this entailment is not due to any sense relations of the classical type. Rather it concerns the relation between sentences and lexical items, and it is for this reason that sense relations are only a very restricted and limited area of semantic relations. It is for this reason that sentence semantics is much more general than lexical semantics and that the methods developed for sense relations do not carry over to sentence semantics, let alone to a more general semantic theory.

Another reason why semanticist do not bother much about sense relations is this: even when restricting ourselves to the comparatively simple task of finding the correct type of extension for certain syntactic classes of expressions, there are all kinds of problems of a non-trivial nature that remain. To mention just one example, recall that we described the extension of adjectives like red as sets. Similarly, the denotation of nouns is sets and the modification of nouns by adjectives is set intersection.

Now upon closer inspection this seems to be too simplistic. There are two types of problems. One is exemplified by (51):

(51) Every alledged murderer is a murderer

This sentence turns out to be a tautology under the above assumptions. But clearly the sentence is wrong! So something must have gone wrong in our semantics of adjectives.

The second sort of problem is exemplified by

(52) a. Every big midget is a big entity  
    b. Every small elephant is a small animal

These also come out as tautologies in our semantics. But this time it’s not that our model contains too many possible worlds: restricting the set of possible worlds will not help as long as the semantics is pure intersection. Again, something has gone fundamentally wrong here, and it’s not always easy to fix the problem.40

In general, enough remains to be done, even with only moderate aims in mind, and this seems to be true still today, after almost 40 years of intensive research in truth-functional semantics.

39There are further conditions that one might impose; the pioneering work in this area has been done by Jaakko Hintikka (1962). These conditions on the interpretation functions all have the effect of rendering certain entailments valid. The study of the relation between these valid inferences and the conditions we can impose on the interpretation functions is sometimes called intensional logic.

40A locus classicus where these problems were discussed is Kamp (1975).
This note is of a more technical nature. It's concerned with a refinement of the principle of compositionality. Recall that the procedure is the following. First our model is abstract in that it assumes an interpretation function of an unspecified sort. Based on this function we compute the intensions of complex expressions. But from the example discussed above it becomes clear that we do this by calculating the extensions of a complex expression, that is, its truth conditions. Having done so, the intension of the expression results from having calculated each extension for all possible worlds.

This means that intensions are calculated world-by-world. Moreover, in order to calculate an extension, we sometimes have to take into account intensions, as was the case with the complement of know. In a formal semantics this means that we must recursively define extensions for complex expressions, and at the same time intensions. Now, concerning the Principle of Compositionality, the statement that intensions are made up of intensions seems to be a little bit of an overstatement. Given the recursive mechanism informally described above, the main work is done by stating a definition of truth, and therefore by computing the extensions. It would therefore be possible to sharpen the Principle of Intensional Compositionality in a way that makes it more transparent that intensions are calculated from extensions:

\[(53) \text{Compositionality of Denotation and Truth Conditions:}\]
\[
\text{The extension of a compound expression is a function of the intensions of its immediate parts and the way they are composed.}\]

Since intensions always depend on extensions, (53) implies that the intensional principle as stated in (37) also holds (trivially so). In fact, I do know an alternative explication of intensional compositionality that does not reduce to (53).

On the other hand, (53) is still a bit imprecise in that intensions do not always play a role in determining extensions. They only do so in “intensional contexts”. In the terminology of Frege, these contexts are called ungerade (uneven), and Frege maintains that in such contexts, the extension (Bedeutung) of an expression is its intension (Sinn). This terminological manoeuvre—though not inconsistent (as has been demonstrated in so-called intensional logic and the work of Richard Montague) would allow us to dispense with intensional compositionality altogether in

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Footnotes:

41 Technically, we need a “double recursion” on the expressions of our language that simultaneously defines extensions and intensions. Neither notion can be defined independently from the other.

42 As far as I know, there is no official name for this principle, it’s just a mixture between extensional and intensional compositionality.
favor of our original extensional principle. The price to pay, however, is a much more complicated notion of extension. Personally, I find Frege’s terminology still a little bit confusing (from a pedagogical point of view); in favor of a more straightforward and transparent terminology, I would rather prefer to give up extensional compositionality, replacing it by some version of intensional compositionality.

7.9 Tense, Time and Logic*

In this section we will briefly comment on an additional component of propositions that has hitherto be ignored. Suppose I utter

(54) I have a beard

And twenty minutes later, I say

(55) I don’t have a beard

This sounds like a contradiction, but in fact it isn’t. For suppose I shaved between the utterances. Then both sentences should be true. But of course they are true only at the time of utterance. This reveals that time is an important issue in our reasoning with sentences. In fact, even Frege, who was mostly concerned with mathematical, hence eternal truths, acknowledged that reference to time should be part of any proposition.\(^{43}\)

Technically this is usually achieved by making interpretations time-dependent. Previously, interpretations assigned an extension to every possible world, now interpretations assign extensions to every pair \(⟨w,t⟩\) consisting of a possible world \(w\) and a moment of time \(t\) (or sometimes a time interval). We thus assimilate moments of time (or time intervals) to possible worlds in that assignment functions (which determine the extensions) not only depend on a world \(w\) but also on a time \(t\). This is needed in order to express something like:

(56) \[ \begin{align*}
&\text{a. } \texttt{It will be the case that } p \texttt{ is true } \iff \text{ there is a moment of time } t' \text{ after } t, \text{ such that } \llbracket p \rrbracket_{w,t'} = T. \\
&\text{b. } \texttt{It was the case that } p \texttt{ is true } \iff \text{ there is a moment of time } t' \text{ before } t, \text{ such that } \llbracket p \rrbracket_{w,t'} = T. 
\end{align*} \]

In what follows we will interpret a maximally simple formal language that contains two operators that can be read as ‘Past’ and ‘Future’. We will then discuss some of the problems that come up when interpreting tenses of natural language. Readers

\(^{43}\) “Alle Bestimmungen des Orts, der Zeit, u.s.w. gehören zu dem Gedanken, um dessen Wahrheit es sich handelt.” From Frege (1893), p. xvii.
not interested in this development can skip this section without harm. We also require some acquaintance with the formal language introduced in section 6.4.

7.9.1 An Elementary Tense Logic

Recall that the recursive definition of the formal language discussed there contained the following condition (see p. 107):

**Syntax:**

\[ ... \]

2. If \( p \) is in \( L \), then \( \neg p \) is in \( L \)

\[ ... \]

Let us now extend clause 2. by adding to operators \( P \) and \( F \):

\[ \neg q, Fq, \text{ and } Pq \] are in \( L \).

Everything else remains as before.

In order to semantically interpret these operators we will adopt two simplifications. First of all, we will ignore possible worlds, just as we ignored moments of time previously. This implies that propositions are not sets of world-time pairs, but sets of moments of time simpliciter. In consequence, the interpretation function \( I \) assigns not a set of worlds to any propositional constant, but a set of points or moments on a time scale.

The second simplification concerns our definition of intensions and extensions. We previously assumed a system that implicitly relied on a double recursion by defining extensions and intensions in an intertwined way. Being concerned here with propositions only (and not with extensions of verbs, nouns, etc.), we can simplify things a bit in defining first the intensions of propositions in the following way (still without accounting for \( P \) and \( F \)):

**Semantics (1):**

1. If \( p \) is an atomic formula, then \( \llbracket p \rrbracket = I(p) \).
2. If \( p \) and \( q \) are expressions of \( L \), then \( \llbracket (p \land q) \rrbracket = \llbracket p \rrbracket \cap \llbracket q \rrbracket \).
3. If \( p \) and \( q \) are expressions of \( L \), then \( \llbracket (p \lor q) \rrbracket = \llbracket p \rrbracket \cup \llbracket q \rrbracket \).
4. If \( q \) is an expression of \( L \), then \( \llbracket \neg q \rrbracket = \{ t : t \notin \llbracket p \rrbracket \} \)

We may then say that a proposition \( p \) is true at a time \( t \) if and only if \( t \in \llbracket p \rrbracket \). Compare this with our previous theorem that a proposition \( p \) is true in a possible world \( w \) if and only if \( w \in \llbracket p \rrbracket \). The analogy should be obvious.

It only remains to interpret \( Fq \) and \( Pq \). For this to work properly, our model must specify a linear ordering on the set \( T \) of moments of time that puts the
elements of $\mathcal{T}$ in the relation “before” and “after”.

**Semantics (2):**

Assume that $\prec$ is a linear ordering on a set $\mathcal{T}$, the set of moments of time.\(^{44}\)

5. If $q$ is an expression of L, then $\llbracket Fq \rrbracket = \{ t : \text{there is a } t', t \prec t' \text{ and } t' \in \llbracket q \rrbracket \}$

6. If $q$ is an expression of L, then $\llbracket Pq \rrbracket = \{ t : \text{there is a } t', t' \prec t \text{ and } t' \in \llbracket q \rrbracket \}$

Now, if $t$ is in $\llbracket FA \rrbracket$, then there is a moment of time $t'$ that follows $t$ in the linear ordering, at which $A$ holds. This is what we express as: it will be the case that $A$. And likewise, if $t$ is in $\llbracket PA \rrbracket$, then there is a moment of time $t'$ that precedes $t$ in the linear ordering, at which $A$ holds. This is what we express as: it was the case that $A$.

### 7.9.2 Some Inferences

Having stated the truth conditions, we may now ask which formulas and which inferences are valid. An inference is valid in L iff every possible assignment and every linear ordering that makes the premise true, also makes the conclusion true. Here are some examples.

(57) $F(p \land q) \therefore (Fp \land Fq)$

For this inference to be valid we have to show that any $t$ in $F(p \land q)$ is also an element of $(Fp \land Fq)$. (For ease of exposition and reasons of laziness we omit the double brackets around the formulas, assuming that $p$ stands for $\llbracket A \rrbracket$ for some constant $A$ of our language.) If $t$ is $F(p \land q)$ there is a moment in the future $t'$ where $(p \land q)$ is true, hence $t'$ is an element of the intersection of $p$ and $q$, ie. it must be an element in both $p$ and $q$. It therefore must be the case that $p$ is true at $t'$ (and the same for $q$), but since $t'$ follows $t$ in the ordering, it holds that $Fp$ is true at $t$ (and the same for $Fq$). But from $Fp$ and $Fq$ we can conclude that $(Fp \land Fq)$, hence $t$ is in $(Fp \land Fq)$. Q.E.D.

Consider next:

(58) $(Fp \land Fq) \therefore F(p \land q)$

\(^{44}\)Depending on one’s cosmology, one might assume that the linear ordering specifies a first point, the time of the Big Bang (Urknnall), but no end-point. We did not include these conditions here, see the discussion below.
This inference is not valid. The reason is easy to see: the premise asserts that \( p \) is true at some time in the future, and also that \( q \) is true at some time in the future, but it does not say that these moments of time must be identical. Thus it is not implied that \( p \) and \( q \) hold together at the same moment \( t' \). But this would be required for the truth of the conclusion. If \( p \) and \( q \) do not hold at the same time, then the conclusion is false, but the premise could still be true. The inference is invalid.

Next consider:

\[(59) \quad p \quad \vdash \text{PF} p\]

This formula says that if \( p \) is true at \( t \), so must be \( \text{PF} p \) and there must be a point \( t' \) in the past such that \( p \) will be true. But since \( p \) is true at \( t \), \( t \) is in the future of \( t' \), so the inference seems valid. Intuitively, the following seems to be a reasonable conclusion: if it is raining (now), then it has been true (yesterday) that it will be raining (the day after yesterday = now).

Formally, however, this is not true in all models. Consider an ordering with a first point, the Big Bang. At that time \( t_{bb} \) it cannot be true that \( \text{P} \alpha \), because there is no past, so \( t_{bb} \) cannot be an element of \( \text{P} \alpha \). But then, \( p \) could be true at \( t_{bb} \), but the conclusion \( \text{PF} p \) is false. Hence the conclusion is invalid. On the other hand (59) is formally valid only in a model where time has no beginning. Likewise,

\[(60) \quad p \quad \vdash \text{FP} p\]

is valid only when time has no end point.

What about this formula:

\[(61) \quad \text{F} p \quad \vdash \text{FF} p\]

If this inference is to be valid, then between any two moments of time there must exist a third moment. Technically speaking, the ordering must be “dense”. In order to see this, assume that there are two points \( t \) and \( t' \) such that there is no point between them (the ordering is not dense). And assume an assignment where \( p \) is true at \( t' \) but at no other time. We will show that the situation described is a counterexample that refutes (61). Given that \( p \) is true at \( t' \), \( \text{F} p \) is true at \( t \). This is the premise. For the conclusion to be valid, it must also be true at \( t \). But \( \text{F} p \) states that \( \text{F} p \) is true at some time \( x \) later than \( t \), which means that \( p \) must be true at some time \( y \) later than \( x \). By hypothesis, \( y = t' \), since \( t' \) is the only moment where \( p \) is true. But this means that \( x \) is between \( t \) and \( t' \). This contradicts our
The initial assumption that there is no moment between $t$ and $t'$.

The above reasoning suggests that some formulas can be used to define certain properties of the ordering of an interpretation (or a model). Eg., a formula like

\[ Fp \rightarrow FFp \]

is valid only if all models in which (62) is a tautology, share a certain property, namely that time is dense. We say that (62) is an axiom that expresses the density of the ordering; this way (62) plays the same role with respect to the meaning of tense as do the meaning postulates with respect to lexical items of natural language.

As philosophers in the 1950th developed a more general interest in axiomatization, they asked which properties of time can be expressed by which formulas (and which could not so be expressed). For example can we express the science fiction idea that time is circular (it is finite but each event repeats itself endlessly)? For an early reference, see Prior (1957), Prior (1967).

7.9.3 Tense in Natural Language

Above we defined a formal language of tense logic and designed a semantics for this a language. Now, when it comes to describing the meaning of a natural language expression like (63),

\[ \text{(63) John will come} \]

most philosophers would be content to say that the meaning should be the same as that of a formula like

\[ \text{(64) } Fp, \text{ where } p = \text{John comes} \]

What they do, then, is to translate the sentence into some formula, presupposing that the meaning of the formula (its semantic interpretation) and that of the natural language expression coincide. For us linguists, however, this is not sufficient. For one thing, linguists would not be content to merely find some translation that seems to work on intuitive grounds alone. What they would like to see is an explicit translation procedure that starts off with natural language syntax, rigorously applies transformational processes, and finally ends up with the formulae

\[ \text{45} \]

Investigations into the logic of time were only part of a much larger scientific program that aimed at finding a logic for notions like knowledge, belief, obligations, possibilities and other concepts. The resulting axiomatic systems were called “Philosophical Logics”, see Stegmüller (1979) (Chapter II, Section 3 “Philosophische Logiken”, pp. 147-220 in the 6th edition) for a concise overview.
proposed by philosophers, logicians or mathematicians on intuitive grounds. Moreover, linguists often detect inadequacies due to a mismatch between the proposed formal system and natural language. Let us briefly discuss two examples.

(65) Everyone will win

In a formal system like the one introduced above, we would expect that this sentence is translated as

(66) \( Fq \) with \( q \) being the proposition that everyone wins

This, however, is not the most natural reading of (65). What (66) effectively says is that at some moment in the future it will be the case that everyone wins \textit{at that moment}. So there must be several winners simultaneously—a rather implausible situation. Hence the preferred reading is that for every person there will be a (different) time \( t' \) after \( t \) so that that person is \textit{the} winner at \( t' \).

In other words, in the preferred reading \textit{everyone} has wider scope than \textit{will}. This fact reveals that we have to analyse the internal structure of a sentence in order to capture the fact that \textit{will} has wide scope over \textit{win}, but not over \textit{everyone}. Likewise, a sentence like

(67) Everyone went to school

suggests that we even have to look into the morphology of words in order to pick out the parts that are relevant for semantic interpretation.

As a second example, imagine a family going on a holiday; they were just leaving town in their car when Mary says:

(68) I didn't turn the stove off.

The information conveyed by the sentence should make Arthur turn around and drive back home again. Arthur, a famous logician, quickly translates (68) into his newly developed tense logic. This yields:

(69) \( \neg P(\text{Mary turns the stove off}) \)

Computing the truth conditions, (69) amounts to the following:

(70) It is not the case that at some moment in the past it holds that Mary turns the stove off

But this amounts to saying that Mary \textit{never} turned off the stove before now. This is obviously false and therefore cannot be the intended meaning of the sentence. Arthur then concludes that he might have misconstrued the scope of negation, so
the next formula he tries is this:

\[(71) \quad P \neg (\text{Mary turns the stove off})\]

This looks much better: At some moment in the past it is not the case that Mary turns the stove off. However, thinking about the literal meaning of (71) it turns out that this is trivially true: it is obvious that there might be indefinitely many moments where Mary did other things than turning the stove off. So either the sentence is trivially false, or it is trivially true. In neither interpretation would the content of the sentence have the intended effect of making the logician drive home.

The example reveals that natural language does not work the way the logician would have predicted. This is an important insight. It tells us something about natural language that we might not have found out without any attempt of formalization. The example shows that something is going wrong in the way we conceive of tense, and that the truth conditions we formulated when making up the formal language are too simplistic.

However, it is not obvious which lesson is to be drawn from the example. An educated proposal for an intuitively correct paraphrase is this:

\[(72) \quad \text{At some relevant time interval before the utterance time (immediately before leaving the house) it’s not the case at any moment within that interval that Mary turns the stove off.}\]

The relevant interval mentioned in (72) (the relatively short interval before leaving) has been called the reference time (Betrachtzeit) by the German philosopher Hans Reichenbach (1891-1953). He distinguishes between reference time, utterance time (Äußerungszeit) and event time (Ereigniszeit). The reference time in the above example is partly determined by pragmatics, so the basic insight here is that tensed sentences can only be dealt with in a system that leaves some room for pragmatic considerations. However, there is a restriction on which time intervals can be relevant which is expressed by the grammatical tenses: The future tense expresses that the reference time is located anywhere on the time line after the utterance time, and the past tense expresses that the reference time lies anywhere before the utterance time. The event time then still has to be related to the reference time, this is done by a grammatical system called aspect. A sentence like

\[(73) \quad \text{John had slept for three hours when Jill came in}\]

then expresses that the event time of sleeping is an interval (of three hours) that occurred before the reference time of Jill’s entering. In (68), however, aspect plays
no role, so it follows that the event time is located somewhere within the reference time.

Reichenbach’s analysis suggests that any system that treats tense operators as simple operators on propositions (as we did in our formal language above) cannot account for tense and aspect in natural language. He proposes that tense expresses a relation between moments of time and time intervals (rather than a relation between propositions, as in our formal language). The reference time is often pragmatically determined, but can also made explicit by the use of adverbials, as did the adverbial *when*-sentence in (73). This also explains the awkwardness of (74):

(74) *John slept tomorrow

The adverbial *tomorrow* says that the time of reference is the day after the time of utterance; on the other hand, the past tense contained in *slept* says that the reference time is before the utterance. This is a plain contradiction, which explains the unacceptability of (74). Cf. Reichenbach (1947) and Rathert (2004) for more on tense and aspect in logic and grammar.

8 Presuppositions (Präsuppositionen)

This section deals with a phenomenon that has been dealt with in philosophy for centuries; it has already been touched upon in the discussion of examples like (1) (from Russell (1905)).

(1) The present king of France is bald

The problem is this: suppose the expression *the present king of France* has no denotation in our present world. What, then, is the denotation of the entire sentence?

8.1 The Definite Article

The trouble seems to be located in the semantics of the definite article *the*. Two analysis have been proposed. Frege is a famous proponent of one of them. According to him, an expression like *the king* has no denotation in case there is no king. Therefore, the function that determines the intension of the expression *the king* is partial: it does not have a value in case there is no such king. In other words, the semantics of *the* introduces a gap in the denotation of *the king* in case the extension of the predicate *king* is the empty set. But now, by the principle of compositionality, Frege is led to conclude that due to the lack of an extension of
a part, the whole sentence also lacks an extension, that is, it does not have a truth value. This way the gap in the extension of a certain expression propagates or is inherited to the extension of the entire expression.

Truth-valueless sentences cannot be used to make assertions. Asserting a sentence like (1) therefore requires that the predicate King of France is not empty; this requirement is called the **existential presupposition** (Existenzpräsupposition) of the definite article. The term **presupposition** is a translation of Frege’s notion Voraussetzung; the re-translation of presupposition into German as used by linguistics is not Voraussetzung, but Präsupposition.

Another type of presupposition arises if the extension of the predicate king has more than one element in a particular world. Suppose France has two kings. Then Frege would claim that the expression *the king of France* does not denote, and that for this reason, (1) also lacks a truth value in that kind of situation. We thus arrive at the two presuppositions in (2) which can be combined into (3):

(2)  
   a. There is at least one King of France  
   b. There is no more than one King of France

(3)  
   There is exactly one king of France

(2-b) is called the **uniqueness presupposition** (Einzigkeitspräsupposition); both the uniqueness and the existential presupposition in (3) are the combined presuppositions of *the*.

Note that according to the principle of compositionality, both sentences in (4) lack a truth value when uttered today:

(4)  
   a. Yesterday I met the King of France  
   b. I never met the King of France

The alternative to this is to treat the definite article as a kind of quantifier on a par with other determiners. Recall that we already treated determiners as relations between two sets. In the case of (1) these are the extensions of *king* and of *bald*. The relation corresponding to *the* holds if and only if the denotation of *king* contains exactly one element that is also an element of *bald*. The determiner can accordingly be defined as:

(5)  
   \[ [\text{the}]_w = \{ (X, Y) : \text{there is an } a \text{ such that } X = \{a\} \text{ and } a \in Y \} \]

Assuming that *the* functions exactly the same way as the other determiners, closer reflection will reveal that sentence (1) is true only if (3) is, but otherwise (1) is false. This follows since we did not allow for truth value gaps. In consequence, each sentence has a denotation.

Likewise, (4-a) is false if there is no King. On the other hand, (4-b) can be
paraphrased as:

(6) It is not the case that I ever met the King of France

The embedded sentence has a false presupposition, hence it comes out false, but then the negation of it is true, so that (4-b) is true.

This sort of analysis is in line with Russell (1905) who strongly objected against Frege’s analysis. He definitely wanted to avoid the additional complexities induced by truth value gaps. Discussing (7),

(7) The King of France is not bald

Russell argues that the sentence is ambiguous. Its preferred reading can be paraphrased as:

(8) There is exactly one King of France and that king is not bald

In this case the sentence still implies its presupposition, because the presupposition is contained in (8) as one of the conjuncts. If the presupposition is false, the entire sentence also is.

But there might also be a reading where (7) is understood as the negation of the false sentence (1) and therefore should come out true when the presupposition is false. This second reading can perhaps be highlighted in a context like this:

(9) Of course the king of France isn't bald because there is no such king

He proposed that the ambiguity arises as an ambiguity in the scope of negation. Negation may negate the predicate bald or the entire clause. In the latter case the paraphrase is this:

(10) It is not the case that there is exactly one king and he is bald.

Suppose no king. Then the conjunction is false and the negation is true. Hence, Russell was effectively one of the first to propose that sentences when properly understood may have different “logical forms” although he didn't call them so.

This analysis has not been accepted by linguists, partly because they were unwilling to share the intuition that a sentence is false when its presupposition is false, partly because they were influenced by Sir Peter Frederick Strawson (1919-2006) who in Strawson (1950) insisted that definite descriptions are unlike quantifiers in that they function like names: When using definite descriptions and names we refer to individuals, but we do not when using quantifiers like every or exactly one. The claim is that the logical type of the king is that of an individual, not that of a quantifier.
Lord Russel himself, still alive at the time of Strawson’s criticism, was unimpressed by the argument because he maintained that the logical analysis of expressions is one thing, but what we as the users of language do with these expressions, in another. In Russell (1957) he therefore accused Strawson for having confused semantic and pragmatic aspects of language.

Russell’s reply had no effect on the linguistic community, but instead Strawson was celebrated as one of the founders of pragmatics. The debate is still ongoing, but for the present purpose there is no need to take sides. To summarize, the important property of the presupposition triggered by the is the following: In both the positive and the negative sentence, I am entitled to draw a certain conclusion, namely (3). That is, if the sentence is true, the presupposition must be, and for the negated sentence, there is an analysis that also supports the truth of the presupposition. Let us now investigate how this diagnostics can be applied to other linguistic phenomena.

8.2 Entailments and Truth Value Gaps

The purpose of this section is to illustrate other cases of presupposition and some further aspects of the phenomenon that turned out to be relevant from a linguistic point of view. As a point of departure, recall our analysis of the verb know in section 7.7. The important fact to be accounted for there was the entailment shown in (11):

(11) John knows that Berta is sick.
    \[ \therefore \text{Berta is sick.} \]

The validity of the inference was guaranteed by a condition on the interpretation of know: For any world \( w \) the following must be true in any model:

(12) If \( x \) knows \( p \) in \( w \), then \( w \) is an element of \( p \) (ie. \( p \) is true in \( w \)).

This is a property of the interpretation function \( I(\text{know}) \) which assigns an intension to the verb know. It cannot be a property of verbs like believe because the inference in (13) is invalid.

(13) John believes that Berta is sick.
    \[ * \therefore \text{Berta is sick.} \]

Verbs like know:

(14) John remembers that Berta is sick.
    \[ \therefore \text{Berta is sick.} \]
(15) Berta managed to become sick.  
∴ Berta became sick.

Verbs unlike *know*:

(16) Berta tried to become sick.  
*∴ Berta became sick

(17) It seems that Berta is sick  
*∴ Berta is sick

Verbs like *know, remember, manage, ...* are called “veridical” or “factive”: they *entail* their complements. Verbs like *believe, seem, and try* are not, they leave open the truth of their complements. We say that the inference is licensed by the lexical item *know (remember, manage, ... but not by believe, try, seem ...)*; it is *lexically triggered* because it is the meaning of the verb that licenses the conclusion.

Curiously, in some cases of factivity the meaning of the verb might not be the only relevant factor. Compare the following inferences:

(18) John has forgotten that Berta wanted to come  
∴ Berta wanted to come

(19) Berta has forgotten to come  
*∴ Berta came

The verb *forget* is factive only when it takes a *that*-complement, but it is not factive when taking an infinitival complement; on the contrary, it implies that the complement is false:

(20) Berta forgot to come  
∴ Berta didn't come

Likewise, the verbs *prevent* and *stopp* imply the negation of their complements:

(21) John prevented Berta from becoming sick  
∴ Berta didn't become sick

(22) John stopps smoking  
∴ John does not smoke (any more)

All these inferences must be licensed by spelling out the semantics of the verbs in a detailed way, so as to justify the conclusion by making it logically valid. How this can be done technically in each case is irrelevant at the moment; it suffices to observe that these inferences depend on the meaning of the verbs and the kind
of syntactic construction; both must be incorporated into the truth conditions of sentences containing them.

Now, the most relevant observation that will motivate the remainder of this section is the following:

(23) John didn't know that Berta was sick.
    \[ \therefore \text{Berta was sick.} \]

This seems to be a valid inference. If I truthfully utter that John didn't know \( p \), I must believe that \( p \) is true. Even when I say

(24) I didn't know that Berta is sick

it still must be the case that I assume now that Berta is sick. Therefore, using the present tense in (25) is rather weird.

(25) #I don't know that Berta is sick

HOMEWORK:

(26) a. John regrets that Berta is sick
    b. John does not regret that Berta is sick

The verb \textit{regret} has the same presupposition as \textit{know}. Explain why it is not weird to say

(27) I don't regret that Berta is sick

These observations, obvious and unproblematic as they are, nonetheless induce a severe problem. Suppose \( x \) knows \( p \) is \( T \), then \( p \) must be \( T \). Next suppose that \( x \) knows \( p \) is \( F \), then again \( p \) should be \( T \). But since \( T \) and \( F \) are the only truth values we have, it should be the case that \( p \) is always \( T \), so that \( p \) should be a tautology. But surely the sentence \textit{Berta is sick} is not a tautology. What went wrong?

\section*{8.2.1 The Fregean Tradition}

One possible answer that seems to be agreed upon by linguists in the Fregean tradition is the following: If in fact Berta is not sick, then neither the sentence \textit{John knows that Berta is sick}, nor the sentence \textit{John doesn't know that Berta is sick} can truthfully be uttered. In that case, neither sentence can be true nor false, the sentences cannot have a truth value. That is, many linguists would allow for situations where sentences lack a truth value, they admit for \textbf{truth value gaps}.

As another illustration, consider the following inferences:

(28) a. Berta managed to get sick
    \[ \therefore \text{Berta got sick} \]
b. Berta didn’t manage to get sick
∴ Berta didn’t get sick

The inferences seem to be valid, but this now gets us into the same trouble as before: If there are only two truth values, and if A implies B, and not A implies not B, then both A and B must be synonymous. But this is clearly wrong for the sentences above. The difference between *Berta is sick* and *Berta managed to get sick* (literally understood, i.e. without any trace of irony) is that Berta took some effort to reach her aim of getting sick. And this must also be the case if she didn’t manage to get sick. So both premises have truth value gaps in case Berta does not make any effort to get sick.

The admittance of truth value gaps therefore allows us to solve the problem: the presupposition is not a tautology, because it can be false when its trigger sentence lacks a truth value. Furthermore, the premises and the conclusions in (28) are not equivalent, because the premises might lack a truth value while the conclusions do not, other truth values being equal. Moreover, truth value gaps allows us to define the notion of presupposition in the following way:

\[
(29) \text{Definition:}
\]

If a sentence A entails a sentence B, and if the negated sentence not A also entails B, and if B is not a tautology, then B is called a presupposition of A.

According to this definition, *Berta took some effort to get sick* is a presupposition of (28). Likewise, *that Berta is sick* is a presupposition of *John knows/doesn’t know that Berta is sick*.

\[
(30) \text{Theorem 1:}
\]

If B is a presupposition of A, then \([A] \subseteq [B]\) and \([\neg A] \subseteq [B]\).

\[
(31) \text{Theorem 2:}
\]

If B is a presupposition of A, and if \([B]_w \neq T\), then \([A]_w\) is neither true nor false.

\[
(32) \text{Theorem 3:}
\]

If A and \(\neg A\) have the same presuppositions, then A lacks a truth value if and only of \(\neg A\) does.

Truth value gaps imply an additional complication in our semantics. Recall that above we claimed that propositions are functions from possible worlds to truth values and then we somewhat simplified our semantics by saying that propositions can be identified with the sets of possible worlds in which they are true. This simplification is no more possible as soon as presuppositions enter the scene.
In order to describe that B is a presupposition of A in the way we did a minute ago, we have to say (a) that B is true in all A-words, (b) that B is true in all not-A worlds, and that (c) B is false in all worlds where A has no truth value. This last step divides propositions into three different parts: positive extensions, where A is true, negative extensions where A is false, and a remaining set of worlds where A has no truth value. This threefold distinction would get lost if we identify a proposition with only its positive extension.

We therefore have to return to the idea that propositions are functions from worlds into truth values, but we now add the possibility that such functions could be undefined for a given world. Being undefined of course means that the proposition has no truth value in that world, which in turn should imply that some presupposition of the proposition is violated in that world. Violation of presupposition therefore leads to undefinedness, and it is partial functions that can be undefined for some of their arguments.

Unfortunately, the above modification is not restricted to propositions, as the phenomenon of presupposition is ubiquitous in semantics. For example, when I say either I like him or I don’t like him, in any case whatever refers to the pronoun him will, in normal circumstances be an animate male being, whereas the pronoun her is confined to females (in English). This seems to be part of the meaning of these pronouns, yet the meaning is only a presupposition. When saying I don’t like him, I cannot thereby deny that the denotation of he is male. Similarly, when saying

(33) #The table knows me

something very very odd seems to happen, and you would have great difficulties in telling (or finding out) whether or not (33) is true. It’s simply non-sensical, because knowing requires an conscious subject. Again this is a presupposition, philosophers also called violations of presupposition of this kind category mistakes. In linguistics, the analogous terminology would be that a selectional restriction of the verb know is violated.

In sum then, all selectional restrictions can count as presuppositions. This considerably complicates our semantics, since almost any predicate comes along with selectional restrictions and therefore leaves room for undefinedness. Technically, this means that a predicate like, say glatzköpfig (= bold), should divide the entities of the universe into three domains: one is the set of people who can be meaningfully asserted to be glatzköpfig, one set of people who can be meaningfully denied to be glatzköpfig, and the remainder who do not have a head at all. This means that for each world and almost any predicate we have to assume a positive extension yielding the value T when applied to an x, a negative extension yielding the value F when applied to an x, and finally yielding undefined when
applied to the remaining x’s.

By the same reasoning, suppose we want to describe the meaning of

(34) John is hungry

in a world where I happened to call my TV-set John. Then sentence (34) does not make much sense, unless we try to understand hungry in a metaphorical sense. Confining ourselves to literal meaning, we would nonetheless say that hungry requires that the subject is animate. If it is not, the literal extension should be undefined. But if we were to stick to the old truth conditions, this would not come out correctly, the sentence would come out false.

This situation requires the truth conditions to be duplicated as conditions for truth and for falsity, with respect to a positive and a negative extension. (34) is T iff John is an element of the positive extension of the VP, it is F iff John is an element of the negative extension of the VP, and it is undefined in all other cases. As interpretation functions are partial functions, and VP-denotation can themselves be complex, being composed from smaller building blocks, truth conditions become more complicated in intriguing ways, as evidenced by some of the examples to be discussed below.

Summarizing so far, almost any expression carries along certain presuppositions, if only in the form of selectional constraints.

8.2.2 The Projection Problem

In order to decide whether a sentence should be evaluated as false or meaningless, linguists and philosophers developed a particular understanding of the term presupposition that is independent of a particular truth theoretical analysis. Normally, the term presupposition is understood in such a way that both the speaker when uttering, and also the hearer when understanding a sentence like

(35) John doesn’t regret that Bill died

somehow take the truth of Bill died for granted. Hearer and speaker take part in a conversation that is based on common knowledge of the participants. This common knowledge is presupposed in a conversation, and some expressions (like regret) require that certain propositions are part of the common ground. If not, the sentence is weird, or misleading, provoking a reaction that goes beyond simply negating its content. For example, if I hear (35) and if I think that Bill didn’t die, the proposition that Bill died is not part of the common ground. It does not suffice then to say “that’s wrong”, because that would imply that John does regret that Bill died. That’s not what I want to say, rather I must say something that makes it clear that the participants in the conversation do not share a common ground.
And conversely, if I can utter A successfully (without provoking weird consequences) and if \( p \) is not part of the common ground before and after the utterance of A, then \( p \) is not a presupposition of A. Let us now consider:

(36) John is in town and my best friend met him yesterday

This sentence implies that I do have a best friend, and it claims that this person. But now suppose I do not have any friends. Is the entire sentence false or truth-valueless? Now, if my having a (best) friend is a presupposition of the term \( my \) best friend, as seems reasonable, Frege’s theory would predict that the sentence

(37) My best friend met John yesterday

is truth-valueless, and so would be the complex sentence (36) as a whole. This is because the lack of a denotation of some part always leads to a lack of denotation of the entire expression. An in fact, if I do not have a friend, there is something wrong with uttering (36), as the fact that I have friends seems to be part of the common ground when uttering (36).\(^{46}\)

But now what about:

(38) Either John has no sister or he is visiting his sister

Formally, since \( John’s/his \) sister implies that John has a sister, the entire sentence should lack a truth value in case he has no sister, but intuitively, the sentence is true in that situation because the first conjunct of the disjunction is. Intuitively, the entire sentence has no presupposition at all, because the common ground does not contain the information that John has a sister. Otherwise it would be weird and uninformative to utter the first part of the disjunction. In contrast,

(39) Either John is at home or he is visiting his sister

still presupposes that he has a sister, and this should make the entire sentence truth-valueless.

The same considerations apply to

(40) If Berta is sick, John will know that Berta is sick

Here again, the entire sentence does not intuitively presuppose that we know already that Berta is sick; on the contrary, the presupposition is introduced as part of the proposition and it is therefore not a presupposition of the entire sentence.

\(^{46}\)Perhaps you as the hearer might not have known before hearing (36) that I have a friend, but being informed by (36), the hearer tacitly accommodates the common ground by adjusting his background assumptions to that of the speaker.
On the other hand,

(41) If Berta is at home, John will know that Berta is sick

still presupposes that Berta is sick.

This problem of determining the presuppositions of complex sentences is called the projection problem for presupposition. The question is this: If p is a presupposition of A, and if A is a proper part of B, is p still a presupposition of B? Frege didn’t discuss the problem and did not provide any means for solving it. The main difficulty here is that in view of projection problem the sentential connectives seem to lose their property of being truth-functional. This is clear from looking at (38) and (39). Combining T with undefined in (38) should yield T, but combining T with undefined in (39) should yield T.

8.2.3 The Russellian Tradition

Although neither Frege nor Russell explicitly discussed the projection problem, the kind of a solution Russell proposed for the King of France problem also seems to be applicable to the problem at hand.

Suppose that we represent

(42) John knows that Berta is ill

by the combination of two propositions:

(43) ⟨Berta is ill⟩ and John knows that Berta is ill

The part in angled brackets is assumed to be the presupposition induced by the verb know. That is, we assume that this proposition is part of the common ground. Let us now look at

(44) John doesn’t know that Berta is ill

Initially, the sort of representation we get when interpreting know is one where both know and the presupposition triggered by know is in the scope of the negation:

(45) It is not the case that ⟨Berta is ill⟩ and John knows that Berta is ill

But now assume that there is a kind of syntactic transformation which, at the level of LF, brings it about that the part in angled brackets has to be moved into a position where it outscopes negation:

(46) ⟨Berta is ill⟩ and it is not the case that John knows that Berta is ill
This operation is a syntactic manipulation and in more recent theories this is also part and parcel of a solution to the projection problem. In such a framework, the problem can be restated in the following way: If A has a presupposition p, and if B contains A, is it possible to move p out of A into B? If p lands on the top level of a sentence, it is presupposed and should belong to the common ground; if not, the presupposition is cancelled and does not arise.

The solution then rests on assumptions about when and why such an LF-movement is permitted. The conditions are semantic in nature: If, for example, a presupposition of A is already semantically entailed by other material in B, it cannot project but must be interpreted by anaphorically relating it to that material, as in

(47) If Berta is sick, John will know this

No such relation is possible as in (41), hence the presupposition must project.

The proposition marked ⟨.⟩ still is part of the semantic representation of a sentence, it cannot be separated from the rest of the sentence by saying that in ⟨p⟩ and A the p-part is the presupposition and the A part is the assertion. Consider again

(48) The King of France is bold

and ask yourself what p and A are. The p-part is easy:

(49) ⟨There is exactly one king of France⟩ and . . .

But in the B-part we cannot continue with “the King of France is bald” because now the presupposition would again be part of the assertion. The only way of getting rid of the presupposition seems to be by an anaphoric relation:

(50) ⟨There is exactly one king of France⟩ and he is bald

But now the A-part alone cannot be interpreted without the p-part. We cannot, therefore, separate the assertion neatly from the presupposition; in essence, the presupposition is asserted and expressed as an anaphoric relation to some proposition in the common ground.

Another obstacle for separating the presupposition from the assertion comes from the use of quantifiers. Consider first:

(51) a. John stopped smoking
    b. John didn’t stop smoking
    c. The period before the event time within the reference time is one where John didn’t smoke.
The assertion then seems to be:

(52)  a. John did not smoke after that time  
      b. John smoked (=continued smoking) after that time

with that time referring to the time described in (51-c). But now consider:

(53) Someone stopped smoking

The presupposition seems to be that someone had been smoking within the reference time up to some time $t$. But the assertion cannot be that someone did not smoke after that time, because there is no guarantee that the “someone” in the presupposition and the “someone” in the assertion pick out the same person. This problem is avoided in a representation like the following:

(54) For someone it holds that $\langle$ he has been smoking up to time $t$ $\rangle$ and he did not smoke after $t$.  

Another obvious advantage of this method is that the logic does not need truth value gaps. It would of course be possible to introduce such gaps if the topmost proposition $\langle p \rangle$ is unembedded and false, but technically this would be irrelevant for the underlying logic we are working with. In terms of truth conditions, then, presuppositions do not play a distinctive role in the computation of a sentence, given its Logical Form. On the other hand, the sort of information conveyed by a sentence in a certain situation is sensitive to presuppositions: The latter normally are not part of the new information conveyed by a sentence, as the name suggests, they are assumed to be not asserted, but presupposed.

This way, a certain division of labor suggests itself: The fact that a sentence is felt inappropriate (and therefore lacking a truth value) need not be captured as a proper part of its truth conditions in isolation; rather it should be interpreted as a fact about the common ground of the conversation, indicating that $\langle p \rangle$ is not part of the common ground in some discourse. Presuppositions are then dealt with in discourse semantics, depending on the context of utterance, which according to traditional philosophical terminology would also be called “pragmatics”. A certain terminological confusion arises from the fact that in philosophy any kind of context dependence has been called pragmatics, whereas in linguistics, all sorts of context dependence that are triggered by the conventionalized rules for the interpretation of certain lexical items still belong to semantics.

An obvious disadvantage of the proposed method is that the mechanism of projection is neither purely semantic nor purely syntactic, and that any recourse to the level of LF threatens to undermine the idea of a compositional semantic interpretation of what there is.
We cannot do justice here to the vast amount of literature on the topic, cf. eg. wikis on presupposition triggers. The present outline of the Russellian method was inspired by Sand (1992), cf. also the Further Reading Section. We should stress at this point, however, that due to Strawson’s influence presuppositions in the literature are predominantly described as a pragmatic phenomenon, whereas we tried to make it clear that presuppositions are lexically triggered and therefore belong to the lexical meaning of expressions, that is, its semantics.

HOMEWORK: Determine, and try to explicitly account for, the presuppositions in (55):

(55)  
    a. John is smoking again
    b. If I were rich, I could retire
References and Further Reading


The further reading part still has to be written. I recommend Hodges (1977) as an easily accessible introduction to logic. Other proposals will be added.