Common Grounds, Questions and Relevance

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1 Introduction

Conventional wisdom says that questions (as semantic entities) have a different type than propositions. Nonetheless, assertions and direct questions share the ability of bringing about a change in the common ground of the conversants. Under this perspective, they should uniformly be analyzed as update functions over common ground (a property they share with all kinds of speech acts). Leaving aside the issue of whether the meaning of a sentences should be *identified* with its update potential, this paper aims at investigating the dynamic behavior of different kinds of questions in comparison mainly to assertions. The present approach relies heavily on Zeevat's work on common ground updates (Zeevat 1997), and it extends the latter by some concepts from Groenendijk and Stokhof 1984.

2 Zeevat's system

Zeevat starts with the observation that updating the common ground in conversation is a non-monotonic process, i.e. a successful assertion - for instance - gives rise to a proper belief revision and not only expansion. Suppose speaker A asks whether it rains and speaker B answers with yes. This answer gives rise to a transition from a common ground where its holds that A does not know whether it rains to one where he does know. This observation excludes the option of modeling information states as sets of possible worlds and updates as eliminative functions. This simple picture has to be supplemented with two ingredients:

1. Information states do not only contain information about the world, but also about the epistemic states of the individual discourse participants,

their information about each other etc. Multimodal Kripke models lend themselves naturally for this purpose.

2. In the course of an (idealized) conversation, the participants acquire information about the world, and therefore they get rid of information about each other's ignorance (since it is no longer true).

The latter fact proves that the accessibility relations cannot be considered as fixed, i.e. as part of the model, but each information state has its own accessibility relations. In other words, information states in Zeevat's sense can be considered as a pair consisting of a set of possible worlds s and a set R_i of binary relations on the set W of all possible worlds, one for each discourse participant i[1]. Common grounds are a special kind of information states, namely those where $\bigcup_{i \in DP} R_i$ is reflexive (DP being the set of discourse participants), and where the domain of each R_i is a subset of s. This has the consequence that exactly those facts are in a common ground that are commonly believed according to this common ground. Updating a common ground with a formula φ is performed in two consecutive steps: First s is restricted to those worlds where φ is true - which may lead to a state that is not a common ground - and second the resulting state is minimally changed to become a common ground by restricting the R_is to s.

3 Relevance

Zeevat's information states represent two kinds of information: information about the world and information about the knowledge states of the subjects. The kind of change of the common ground induced by a question does not affect these informational aspects[2]. Rather, the purpose of a question is to raise an issue which wasn't subject of general attention before. To put it in other terms, something which wasn't relevant before is relevant now. Hence to analyze questions, we have to incorporate a third aspect: relevance. Following the basic ideas of Groenendijk and Stokhof 1984, this can be done by assuming that the epistemic state of a subject shouldn't be represented by a set of worlds, but by a set of epistemic alternatives, i.e. mutually exclusive sets of possible worlds. This is equivalent to an equivalence relation on a subset of W. Its intuitive content is the following: u and v stand in this relation iff the subject considers the difference between these

[1] Zeevat uses Aczel's AFA set theory (Aczel 1988) which leads to definitions which look quite different. I translate his concepts into well-founded set theory since this strikes me less elegant but more transparent. To make the definitions fully equivalent, we have to make the inessential assumption that the union of the R_is form a connected graph with a domain overlapping with s.

[2] Zeevat also discusses questions in his paper, but to this end he makes use of a relation *wants* that isn't further analysed. The present paper can be seen as an attempt to do this.

two worlds as irrelevant. The issue *whether* φ is considered relevant iff no worlds that differ wrt. φ stand in this relation.

Since the "relevance state" of an agent may change from world to world as well as his knowledge state, there should be a function that assigns each agent in each world a set of epistemic alternatives. This can be captured by a three-place relation Q_i on worlds which has the property that $\lambda x \lambda y. Q_i(w, x, y)$ is an equivalence relation for arbitrary i and w.

It is plausible to assume that the epistemic alternatives of an agent jointly exhaust his belief space. A second assumption I'd like to make is that an agent considers everything he knows as relevant (which is a pretty strong idealization, but it simplifies matters considerably). These assumptions together entail that the belief space of an agent is just the union of his epistemic alternatives. This has the pleasant consequence that belief as well as relevance is represented by the Qs and the Rs become redundant ($R_i(u, v)$ iff $Q_i(u, v, v)$).

The following definitions summarize these considerations:

Definition 1 (Information States): An information state σ is a tuple $\langle s, Q_1, \ldots, Q_n \rangle$, where

- s is a subset of W
- $1, \ldots, n$ are discourse participants, and
- Q_i is a three-place relation on W, such that for all $w \in W$, $\{\langle u, v\rangle | Q(w, u, v)\}$ is an equivalence relation on a subset of W.

$\langle s, Q_1, \ldots, Q_n \rangle$ is a *common ground* iff $\forall w \in s : \langle w, w, w \rangle \in \bigcup_{1 \leq i \leq n} Q_i$ and $Q_i \subseteq s \times s \times s$

For simplicity, we sometimes write $x \in \sigma$ or $x \subseteq \sigma$ instead of $x \in / \subseteq s$.

Next we define a simple multimodal language that enables us to reason about belief and relevance. $\mathbf{K}_i\varphi$ should be read as "agent i believes that φ" and $\mathbf{R}_i\varphi$ as "i considers φ relevant", and F is a function that assigns to each possible world a propositional valuation function.

Definition 2 (Semantics):

(i) $\|p\|_\sigma = \{w | p \in F_w\}$

(ii) $\|\varphi \wedge \psi\|_\sigma = \|\varphi\|_\sigma \cap \|\psi\|_\sigma$

(iii) $\|\neg\varphi\|_\sigma = W - \|\varphi\|_\sigma$

(iv) $\|\mathbf{K}_i\varphi\|_\sigma = \{w|\forall v(Q_i(w,v,v) \Rightarrow v \in \|\varphi\|_\sigma)\}$

(v) $\|\mathbf{R}_i\varphi\|_\sigma = \{w|\forall u, v(Q_i(w,u,v) \Rightarrow (u \in \|\varphi\|_\sigma \Leftrightarrow v \in \|\varphi\|_\sigma))\}$

Validity of a formula in an information state is defined in the obvious way:

$$\begin{array}{rcl} \sigma \models \varphi & \text{iff} & \sigma \subseteq \|\varphi\|_\sigma \\ \models \varphi & \text{iff} & \forall \sigma : \sigma \models \varphi \end{array}$$

We use $\Box\varphi$ as abbreviation $\mathbf{K}_1\varphi \wedge \ldots \wedge \mathbf{K}_n\varphi$ and $\mathbf{R}\varphi$ for $\mathbf{R}_1\varphi \wedge \ldots \wedge \mathbf{R}_n\varphi$. The individual accessibility relation can be merged into a new one, Q^*:

$$Q^*(u,v,w) \text{ iff } \langle v,w\rangle \in TC(\bigcup_{1\leq i\leq n} \{\langle x,y\rangle|Q_i(u,x,y)\}$$

where $TC(R)$ is the transitive closure of the relation R. This definition is useful since it holds that

$$\begin{array}{rcl} \|\Box\varphi\|_\sigma & = & \{w|\forall u(Q^*(w,u,u) \Rightarrow u \in \|\varphi\|_\sigma)\} \\ \|\mathbf{R}\varphi\|_\sigma & = & \{w|\forall u, v(Q^*(w,u,v) \Rightarrow (u \in \|\mathbf{R}\varphi\|_\sigma \Leftrightarrow v \in \|\mathbf{R}\varphi\|_\sigma))\} \end{array}$$

From the definitions it immediately follows that all substitution instances of $\mathbf{K}_i\varphi \rightarrow \mathbf{R}_i\varphi$ and $\Box\varphi \rightarrow \mathbf{R}\varphi$ are valid.

Dynamics As was mentioned above, updating a common ground with a formula consists of two separate steps. First the content of the formula is integrated into the factual component of the common ground:

(1) $\langle s, Q_1, \ldots, Q_n\rangle[\varphi] = \langle s \cap \|\varphi\|_{\langle s,Q_1,\ldots Q_n\rangle}, Q_1, \ldots Q_n\rangle$

In the second stage, the Qs are restricted appropriately such that again a common ground results.

(2) $\langle s, Q_1, \ldots, Q_n \rangle * = \langle s, Q_1 \restriction s \ldots, Q_n \restriction s \rangle$

(where $Q \restriction s$ is the restriction of Q to s)

4 Speech acts

4.1 Assertions

Different kinds of speech acts are characterized by pre- and postconditions. For the sake of simplicity, we only consider dialogues, i.e. there are only two participants, H(earer) and S(peaker). In the case of the assertion of a sentence φ, Zeevat proposes the following:

Preconditions:

$$\begin{array}{lcl} \sigma & \not\models & \mathbf{K}_S\varphi \\ \sigma & \not\models & \mathbf{K}_S\neg\varphi \\ \sigma & \not\models & \mathbf{K}_H\varphi \\ \sigma & \not\models & \mathbf{K}_H\neg\varphi \end{array}$$

We extend this by the requirement that the content of the assertion is commonly considered to be relevant:

$$\sigma \models \mathbf{R}\varphi$$

A felicitous assertion changes such a common ground into one where the subsequent conditions hold:

Postconditions:

$$\sigma' \models \mathbf{K}_S\varphi$$

This effect is achieved by performing the update $\mathbf{K}_S\varphi$, since for any common ground σ where the preconditions hold, $\sigma[\mathbf{K}_s\varphi]* \models \mathbf{K}_S\varphi$.

Possible reactions:

- Assent: φ. The assertion was successfull. The ultimate output state $\sigma[\mathbf{K}_S\varphi] * [\varphi]*$ supports φ, i.e. φ has now become part of the common ground.
- Dissent: $\mathbf{K}_H\neg\varphi$. The conversants now agree not to agree on whether φ. This can happen if the speaker had raised the issue of *whether* φ himself before making his assertion, or if the hearer raised it as an examination question.
- Doubt: $\mathbf{K}_H\neg\mathbf{K}_H\varphi$

4.2 Questions

4.2.1 Informative Question

This strategy of analyzing speech act can be extended to questions in the present framework. We limit attention to informative and examination questions here. Since we are dealing with a purely propositional system, we can only handle *yes-no* questions. In the case of a proper information seeking question *whether* φ, the following seems appropriate

Preconditions:

$$\begin{aligned} \sigma &\not\models \mathbf{R}_S\varphi \\ \sigma &\not\models \mathbf{K}_H\varphi \\ \sigma &\not\models \mathbf{K}_H\neg\varphi \\ \sigma &\not\models \neg(\mathbf{K}_H\varphi \vee \mathbf{K}_H\neg\varphi) \end{aligned}$$

The first condition requires that the question hasn't already been raised before. As an additional consequence, it entails that the speaker doesn't know the answer yet:

$$\sigma \not\models \mathbf{K}_S\varphi \vee \mathbf{K}_S\neg\varphi$$

since it holds that $\models \mathbf{K}_i\varphi \vee \mathbf{K}_i\neg\varphi \to \mathbf{R}_i\varphi$. Note that we do not exclude that $\sigma \models \mathbf{R}_H\varphi$, since it is very well possible that the hearer is known to know the answer, as long it is still open which one he knows.

Postconditions:

$$\sigma' \models \mathbf{R}_S\varphi$$

which is achieved by updating with $\mathbf{R}_s\varphi$. This does not immediately lead to a common ground where the assertion of φ or $\neg\varphi$ is licensed. Before, the hearer has the options either to accept or to refuse the question (cf. Ginzburg 1995):

Possible reactions:

- Acceptance: $\mathbf{R}_H\varphi$. Since $\sigma[\mathbf{R}_S\varphi]*[\mathbf{R}_H\varphi]* \models \mathbf{R}\varphi$, this results in a common ground where the question has been raised to salience. Since informative questions should be accompanied by an obligation for turn taking for the hearer (which cannot be expressed here), it is now up to the hearer to answer or to express his ignorance.
- Refutation: $\neg\mathbf{R}_H\varphi$. See subsubsection 4.2.3 for discussion.

4.2.2 Examination Questions

A particularly intricate kind of questions are those where the first precondition of informative questions does not hold, but the speaker is known to know an answer:

(3)

$$\sigma \models \mathbf{K}_s\varphi \vee \mathbf{K}_s\neg\varphi$$

There are at least two kinds of situations where this can happen. The most obvious one are examinations in a broad sense. But it also frequently occurs that the speaker raises a question as part of a monologue for rhetorical reasons (this is nevertheless to be distinguished from rhetorical questions where one particular answer is already part of the common ground).

If (3) holds, it also holds that

(4)

$$\sigma[\mathbf{R}_s\varphi]* = \sigma$$

In other words, such a question does not change anything in the common ground. Nevertheless it does have a communicative effect, since the expected grounding move of the hearer, $\mathbf{R}_H\varphi$, raises the issue *whether* φ into salience. This suffices to elicit an assertion on this topic by the conversant whose turn it is. In the case of an examination question, this is the hearer, otherwise the speaker. This can be seen as an indication of the fact that the common ground alone does not suffice to describe appropriately what is going on in dialogues, but that the conversation gameboard has to include a component like LATEST-MOVE in the sense of Ginzburg 1995.

4.2.3 Negations of Questions

The update corresponding to the question *whether* φ is $\mathbf{R}_S\varphi$. The negation of this update is well-defined. It is performed for instance when the hearer refutes to accept a question asked by the speaker. What is special about this kind of formula is that it is not persistent. There are common grounds σ and σ' with the following property:

$$\begin{array}{rcl} \sigma & \models & \neg\mathbf{R}\varphi \\ \sigma & \leq & \sigma' \\ \sigma' & \models & \mathbf{R}\varphi \end{array}$$

This is welcome, it is possible to convince someone of the relevance of an issue. The sample dialogue in (5) may illustrate this.

(5) Sergeant Is Mr. Smith left-handed?

Inspector What the hell does this matter?

Sergeant The murder was committed by a left-hander.

Inspector Okay, we should investigate this.

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