Deconstructing Jacobson's Z

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1 Introduction

One crucial intuition behind the enterprise of model theoretic semantics can be summarized by the slogan "Language is one thing, and meanings are something different". In more technical terms, interpretation is seen as a mapping from an algebra of syntactic forms to an algebra of meanings, and these two algebras are sharply distinguished conceptually. (This does not entail that they are disjoint—expressions may refer to other expressions or even to themselves.) Variables, as they are commonly used, do not fit into this picture. On the one hand, variables are expressions, i.e. atomic elements of the syntactic algebra. On the other hand, under a Fregean conception of interpretation, meanings are functions from variable assignments to other objects. So variables are set-theoretic building blocks of the semantic algebra as well. Thus a variable free approach to the syntax/semantics interface has an *a priori* conceptual appeal.
At a first glance, the greatest obstacle to a variable free semantics for natural language is the phenomenon of anaphora. The Categorial literature contains a series of proposals to overcome this problem. Generally there are two possible routes here. By definition, anaphoric expressions re-use the meaning of another expression, their antecedent. So the interpretation of anaphors involves a multiplication of semantic resources. So it has to be decided whether this is due to the lexical meaning of the anaphoric expression or whether meaning multiplication is done in syntax. The former route is taken by Szabolcsi 1989 and Moortgat 1996a, while Hepple 1990, Jacobson 1999 and Jäger 1998 opt for the latter alternative.
The present paper offers a unifying perspective. Working within the framework of multi-modal Type Logical Grammar (cf. Kurtonina and Moortgat 1995; Moortgat 1997), anaphora resolution is broken into two aspects. Duplication of meaning is due to the lexical meaning of anaphors, while the non-local character of this process is taken care of in syntax and controlled by multi-modal techniques. So the present approach falls into the first group of theories mentioned above. However, it is shown that Jacobson's 1999 and Jäger's 1998 system each can be embedded into one version of the multimodal system. So the two approaches 'resolution in lexicon' and 'resolution in syntax' should be regarded as complementary rather than mutually exclusive.

2 Jacobson 1999

2.1 Combinatory presentation

According to Jacobson, the meaning of an anaphoric expression is a function from the meaning of its antecedent to its meaning in context. Applied to anaphoric pronouns, this comes down to the claim that they denote the identity function over individuals. Anaphoric expressions are coded as such in their syntactic category too. To this end, Jacobson introduces a novel type-forming connective. A sign of category A^B is an anaphoric expression which may be transformed into a sign of category B provided it finds a suitable antecedent of category B. For typographic reasons and to stress the similarity with the other two categorial slashes we use the notation $A|B$ instead of A^B. So an anaphoric pronoun receives category $N|N$. The semantic impact of "|" is similar to the other slashes; it creates a functor category. Jacobson's proposal is framed in the general setup of Combinatory Categorial Grammar. The behavior of the third slash is governed by two combinatory schemata.

First, an anaphoric "gap" may percolate upward in complex structures. This is formalized by a mixed version of the Geach rule plus a monotonicity scheme.[1] (The arrows "→ is used as a meta-variable over "\" and /.)

$$\frac{}{x : A \to B \Rightarrow \lambda yz.x(yz) : (A|C) \to (B|C)} \mathbf{G}$$

$$\frac{x : A \Rightarrow M : B}{y : A|C \Rightarrow \lambda z.M[(yz)/x] : B|C} M$$

Second, anaphoric dependencies are established by means of the scheme **Z**.[2]

$$x : A \to B_1 \to \cdots \to B_n \to C \to D \Rightarrow$$
$$\lambda yz_1 \cdots z_n w.xwz_n \cdots z_1(yw) : (A|C) \to B_1 \to \cdots \to B_n \to C \to D \quad (\mathbf{Z})$$

2.2 Multimodal decomposition

In this subsection, I will present an embedding of Jacobson's system into the general theory of structural control proposed in Kurtonina and Moortgat 1995. For reasons of space, a presentation of this framework has to be left out here. The interested reader is referred to the cited work and Moortgat 1997.

We assume the base logic to be **NL**, the non-associative Lambek Calculus. To start with, it is easy to see that in the presence of the residuation laws, the combinator **G** and Jacobson's monotonicity rule are jointly equivalent to the following inference rule:

$$\frac{X[x : A] \Rightarrow M : B}{X[y : A|C] \Rightarrow \lambda z.M[(yz)/x] : B|C} \mathbf{G'}$$

Likewise, **Z** is equivalent to the combination of the following rules:

$$\frac{X[x : A \circ Y[y : B]] \Rightarrow M : C}{X[x : A \circ Y[z : B|A]] \Rightarrow M[(zx)/y] : C} \mathbf{Z'}_1$$

$$\frac{X[Y[y : B] \circ x : A] \Rightarrow M : C}{X[Y[z : B|A] \circ x : A] \Rightarrow M[(zx)/y] : C} \mathbf{Z'}_2$$

In words, an anaphoric slot can travel up in a resource tree, and an anaphora-slash can be introduced on the right hand side of a sequent if it is c-commanded by an antecedent of the appropriate category.

Let us suppose for a moment that our base logic is not **NL** but **LP**, i.e. we have unlimited access to associativity and permutation. Then—as the reader may check herself—all three rules become derivable iff we expand $A|B$ to $B\backslash(B \bullet A)$. This makes sense intuitively; if we ignore linear order and hierarchical structure, a pronoun may be considered as something which consumes it antecedent, makes a copy of it, and returns it to its original position.[3] Under the strict resource management regime of **NL**, such a treatment fails due to the non-local character of the rules given above. To make it work, we have to add structural rules which give access to associativity and permutation, and we have to restrict these rules by means of modal control devices to avoid a collapse of the base logic into **LP**. This is achieved by the interaction postulates given in figure 1.

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[1] Jacobson's original formulation is somewhat more restrictive, limiting the premise to results of the application of **G**.

[2] Jacobson limits the type C to N, but there doesn't seem to be a special motivation for this restriction.

[3] This intuition is at the bottom of the Linear treatment of anaphora in Dalrymple et al. 1997, who assign a pronoun the lexical entry $\lambda x.\langle x, x\rangle : N \multimap (N \otimes N)$.

Let us look at each postulate separately. We assume that all anaphoric expressions are lexically locked by some unary modality $\Diamond$. Postulate (P0) replaces such a modally marked resource by a place holder constant $\mathbf{t}$, while the unlocked anaphoric resource is attached to $\mathbf{t}$ by a binary product $\bullet_1$.[4]

$$
\begin{array}{rcll}
\Diamond A & \longleftrightarrow & \mathbf{t} \bullet_1 A & (P0) \\
A \bullet (B \bullet_1 C) & \longleftrightarrow & \langle\nwarrow\rangle(A \bullet B) \bullet_1 C & (P1) \\
(A \bullet_1 B) \bullet C & \longleftrightarrow & \langle\nearrow\rangle(A \bullet C) \bullet_1 B & (P2) \\
A \bullet (B \bullet_1 C) & \longleftrightarrow & \langle\leftarrow\rangle((A \bullet_2 C) \bullet B) & (P3) \\
(A \bullet_1 B) \bullet C & \longleftrightarrow & \langle\rightarrow\rangle(A \bullet (C \bullet_2 B)) & (P4) \\
A \bullet_2 (B \bullet_1 C) & \longleftrightarrow & (A \bullet_2 C) \bullet_2 B & (P5) \\
A \bullet_2 \Diamond B & \longleftrightarrow & \Diamond_1 (A \bullet_2 B) & (P6)
\end{array}
$$

Figure 1: Interaction postulates

The postulates (P1) and (P2) move the second argument of $\bullet_1$ up in the tree, marking the nodes on its path with the unary modalities $\langle\nearrow\rangle$ and $\langle\nwarrow\rangle$, depending on whether the movement originated from the left or the right daughter. (P3) and (P4) move the second argument of $\bullet_1$ to the left (right) sister-node, changing the mode of combination from $\bullet_1$ to $\bullet_2$ and marking the path with $\langle\leftarrow\rangle$ ($\langle\rightarrow\rangle$). Thus (P0)–(P4) enable any resource marked with $\Diamond$ to enter a local $\bullet_2$-configuration with any c-commanding node, while its place of origin is marked with $\mathbf{t}$ and the path with appropriate arrow modalities. So an anaphor can approach any c-commanding antecedent. After resolution, this process can be reversed since all postulates work in both directions.

Anaphora resolution as such is modeled by Modus Ponens plus $\bullet_2$ elimination as in the **LP**-treatment sketched above. After putting back the resolved anaphor to its original location, the anaphora-modality $\Diamond$ is matched by a corresponding $\Box^\downarrow$ and thus cancelled. So the anaphora type $A|B$ is to be deconstructed as $\Diamond(B \backslash_2 (B \bullet_2 \Box^\downarrow A))$. A Curry-Howard term M with type $A|B$ is translated as $\lambda x.\langle x, Mx\rangle$. So under the present perspective, the meaning of a pronoun is $\lambda x.\langle x, x\rangle$, as in Dalrymple et al. 1997.

$$
\dfrac{\dfrac{A \Rightarrow A}{}\,id \quad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{X[A \circ Y[B]] \Rightarrow C}{X[A \circ Y[\langle\Box^\downarrow B\rangle]] \Rightarrow C}\,\Box^\downarrow L}{X[A \circ Y[\mathbf{t} \circ_1 \Box^\downarrow B]] \Rightarrow C}\,P0}{\cdots}\,P1/2}{X[A \circ (Y'[\mathbf{t}] \circ_1 \Box^\downarrow B)] \Rightarrow C}\,P1/2}{X[\langle\leftarrow\rangle((A \circ_2 \Box^\downarrow B) \circ Y'[\mathbf{t}])] \Rightarrow C}\,P3}{X[\langle\leftarrow\rangle(A \bullet_2 \Box^\downarrow B \circ Y'[\mathbf{t}])] \Rightarrow C}\,\bullet_2 L}
{\dfrac{\dfrac{\dfrac{\dfrac{X[\langle\leftarrow\rangle((A \circ_2 A \backslash_2 (A \bullet_2 \Box^\downarrow B)) \circ Y'[\mathbf{t}])] \Rightarrow C}{X[A \circ (Y'[\mathbf{t}] \circ_1 A \backslash_2 (A \bullet_2 \Box^\downarrow B))] \Rightarrow C}\,P3}{\cdots}\,P1/2}{X[A \circ Y[\mathbf{t} \circ_1 A \backslash_2 (A \bullet_2 \Box^\downarrow B)]] \Rightarrow C}\,P1/2}{X[A \circ Y[\Diamond(A \backslash_2 (A \bullet_2 \Box^\downarrow B))]] \Rightarrow C}\,P0}\,\backslash_2 L
$$

Figure 2: Derivation of **Z'**

These postulates are already sufficient to make both instances of **Z'** derivable. The derivation of $\mathbf{Z'}_1$ is given in figure 2. There $Y'[\mathbf{t}]$ is used as shorthand for the structure that is exactly like Y except that all nodes dominating $\mathbf{t}$ are marked either with $\langle\nearrow\rangle$ or with $\langle\nwarrow\rangle$, depending on whether its left or its right daughter contains $\mathbf{t}$.

The derivation of $\mathbf{Z'}_2$ is analogous except that we use (P4) instead of (P3).

To deconstruct the **G'**, a qualification concerning modal decoration is necessary. It was mentioned above that the unary modality $\Diamond$ serves to mark **lexical** types as being anaphoric. Anaphoric types that are constructed during derivation come without this kind of decoration. More generally, we assume that all negative occurrences of a type $A|B$ (in the sense of van Benthem 1991:75) are translated as $\Diamond(B \backslash_2 (B \bullet_2 \Box^\downarrow A))$—i.e. with modal marking—while positive occurrences of $A|B$ are mapped just to $B \backslash_2 (B \bullet_2 A)$. Since **Z'** only involved negative occurrences, the

[4] This device is taken from Moortgat 1996b.

derivation given above is not affected by this.
This taken in to account, **G'** is derivable as well provided anaphora resolution is able to deal with hypothetical antecedents. This is ensured by (P5). In the derivation in figure 3, we use the same notational convention as above and leave out obvious steps.
Since positive and negative occurrences of $A|B$ are translated differently, we have to make sure that the identity axiom for $A|B$ remains derivable. This is where postulate (P6) comes in (cf. figure 4).

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{C \Rightarrow C}\,id \qquad \dfrac{\dfrac{\dfrac{C \Rightarrow C \qquad X[A] \Rightarrow B}{C \circ_2 X[A] \Rightarrow C \bullet_2 B}\,\bullet_2 R}{\cdots}\,P0\cdots 5}{(C \circ_2 \Box^{\downarrow} A) \circ_2 X'[\mathbf{t}] \Rightarrow C \bullet_2 B}\,P0\cdots 5}{(C \circ_2 C \backslash_2 (C \bullet_2 \Box^{\downarrow} A)) \circ_2 X'[\mathbf{t}] \Rightarrow C \bullet_2 B}\,\backslash_2 L, \bullet_2 L}{C \circ_2 (X'[\mathbf{t}] \circ_1 C \backslash_2 (C \bullet_2 \Box^{\downarrow} A)) \Rightarrow C \bullet_2 B}\,P5}{C \circ_2 X[\Diamond(C \backslash_2 (C \bullet_2 \Box^{\downarrow} A))] \Rightarrow C \bullet_2 B}\,P0/1/2}{X[\Diamond(C \backslash_2 (C \bullet_2 \Box^{\downarrow} A))] \Rightarrow C \backslash_2 (C \bullet_2 B)}\,\backslash_2 R
$$

Figure 3: Derivation of **G'**

3 Jäger 1998

In Jäger 1998 I followed Jacobson in extending the inventory of type forming connectives with | and assigning pronouns the lexical entry $\lambda x.x : N|N$. The system is formulated in the type logical version of Categorial Grammar while Jacobson is working in the Combinatory tradition. Besides, Jäger 1998 differs from Jacobson's proposal in assuming that precedence rather than c-command is the necessary and sufficient to license anaphoric relationships. This is motivated by considerations concerning inverse linking constructions, weak crossover, VP ellipsis and cross-sentential anaphora.

$$
\dfrac{\dfrac{\dfrac{}{A \Rightarrow A}\,id \qquad \dfrac{\dfrac{\dfrac{}{A \Rightarrow A}\,id \quad \dfrac{}{B \Rightarrow B}\,id}{A \circ_2 \langle \Box^{\downarrow} B \rangle \Rightarrow A \bullet_2 B}\,\Box^{\downarrow} L, \bullet_2 R}{\langle_1 A \circ_2 \Box^{\downarrow} B \rangle_1 \Rightarrow A \bullet_2 B}\,P6}{\dfrac{\langle_1 A \circ_2 A \backslash_2 (A \bullet_2 \Box^{\downarrow} B) \rangle_1 \Rightarrow A \bullet_2 B}{A \circ_2 \langle A \backslash_2 (A \bullet_2 \Box^{\downarrow} B) \rangle \Rightarrow A \bullet_2 B}\,P6}\,\backslash_2 L, \bullet_2 L}{\Diamond(A \backslash_2 (A \bullet_2 \Box^{\downarrow} B)) \Rightarrow A \backslash_2 (A \bullet_2 B)}\,\backslash_2 R, \Diamond L
$$

Figure 4: Derivation of the identity axiom

Accordingly, the counterpart of Jacobson's **Z**—the rule of use for |—is (equivalent to) the following (where $X[A][B]$ is a structure containing the substructures A and B in that order):

$$
\frac{X[x : A][y : B] \Rightarrow M : C}{X[x : A][z : B|A] \Rightarrow M[(zx)/y] : C}\,|L
$$

Jacobson's **G** and the monotonicity rule are both covered by the rule of proof

$$
\frac{x : A \circ y : p \circ X \Rightarrow \langle x, y, M \rangle : A \bullet p \bullet B}{X \Rightarrow \lambda x.M : B|A}\,|R
$$

The resulting logic is called $\mathbf{L}_{|}$.
The rule of proof is not without problems. To start with, it imposes a restriction on the form of the Curry-Howard term of the premise. In other words, the Curry-Howard labeling is not just a book keeping device here but an intrinsic part of the proof theory. As a consequence, it proved to be difficult to develop an appropriate model theory for this logic. Furthermore, the system crucially relies on the unrestricted availability of associativity.

One might wonder whether these limitations are really unavoidable. A careful examination of the linguistic applications reveals that the rule of proof for $|$ can be replaced by the weaker rule **G'** given above without changing the linguistic impact in any way.

Given this, an suitable deconstruction of the the Jäger 1998 version of $A|B$ should make $|L$ and **G'** admissible rules. This can be done in a way very similar to the multimodal system given in the previous section. The only adjustment needed is to replace (P4), which admits forward binding, by the two postulates given in figure 5.

$$\begin{array}{lcll}
(A \bullet B) \bullet_2 C & \longleftrightarrow & \langle\swarrow\rangle((A \bullet_2 C) \bullet B & (P4.1) \\
(A \bullet B) \bullet_2 C & \longleftrightarrow & \langle\searrow\rangle(A \bullet (B \bullet_2 C)) & (P4.2)
\end{array}$$

Figure 5: Revised Interaction postulates

The derivation of **G'** given in figure 3 does not make reference to (P4), so it remains valid in the revised system. As for **Z**, observe that now, due to the absence of (P4), an anaphoric resource can only move up the c-commanding nodes that precede it. The extension with (P4.1) and (P4.2) furthermore admits to move this resource down to any node dominated by this c-commanding node (again accompanied by marking all traversed nodes with and arrow modality). This amounts to saying that an anaphoric resource may travel to any node that precedes its base position. Thus $|L$ becomes a derivable rule. The proof is schematically given in figure 6. There I adopt the notational convention that $X'[A][B]$ is exactly like $X[A][B]$ except that every node at the shortest path leading from A to B is marked by the appropriate arrow modality.

$$\begin{array}{cl}
X[A][B] \Rightarrow C & \\ \hline
X[A][\langle\Box^{\downarrow}B\rangle] \Rightarrow C & \Box^{\downarrow}L \\ \hline
X[A][\mathbf{t} \circ_1 \Box^{\downarrow}B] \Rightarrow C & P0 \\ \hline
\cdots & P1/2/3/4.1/4.2 \\ \hline
\dfrac{}{A \Rightarrow A}\,id \qquad X'[A \circ_2 \Box^{\downarrow}B][\mathbf{t}] \Rightarrow C & P1/2/3/4.1/4.2 \\ \hline
X'[A \circ_2 A \setminus (A \bullet_2 \Box^{\downarrow}B)][\mathbf{t}] \Rightarrow C & \setminus_2 L, \bullet_2 L \\ \hline
\cdots & P1/2/3/4.1/4.2 \\ \hline
X[A][\mathbf{t} \circ_1 A \setminus (A \bullet_2 \Box^{\downarrow}B)] \Rightarrow C & P1/2/3/4.1/4.2 \\ \hline
X[A][\Diamond(A \setminus (A \bullet_2 \Box^{\downarrow}B))] \Rightarrow C & P0
\end{array}$$

Figure 6: Derivation of $|L$

While all relevant sequents that are derivable in $\mathbf{L}_{|}$ remain derivable under the translation given above, the multimodal system is more liberal. Notably, it interacts with quantification in an interesting way. The interaction postulates closely resemble (and are inspired by) the ones used in Moortgat 1996b to deal with quantifier scope. We can in fact replace Moortgat's deconstruction of the *in situ* binder $q(A, B, C)$ by $\Diamond((B/_1\Box^{\downarrow}{}_A) \setminus_1 C)$ in the present system.

Moortgat 1996a proposes to assign pronouns the type $q(N, N \setminus S, N \setminus S$. Under the deconstruction of $|$ and q assumed here, the translation of the following sequent is derivable:

$$N|N \Rightarrow q(N, N \setminus S, N \setminus S)$$

So the present treatment of pronouns is not just a generalization of Jacobson's, but also of Moortgat's proposal. Likewise, the types $((N \setminus S)/N) \setminus (N \setminus S)$ and $(((N \setminus S)/S) \setminus (N \setminus S))/(N \setminus S)$, which are assigned to reflexives and pronouns respectively in Szabolcsi 1989, are derivable from the translation of $N|N$.

The multimodal reformulation of $\mathbf{L}_{|}$ presented in this section avoids all three shortcomings mentioned above. First, none of the logical rules or interaction postulates imposes any constraints on the Curry-Howard terms of the premises. So the applicability of sequent rules solely depends on the types. Second, no special requirements

on the structural properties of $\bullet$, the default mode of combination, are made. No special reference to associativity or other structural rules are made.
Finally, the multimodal system, as well as the one presented in the previous section, is easily supplied with a sound and complete model theory. An obvious candidate is interpretation of binary operators in ternary frames in the sense of Došen 1992 that was extended to unary modalities in Moortgat 1995 and to 0-ary modalities as the constant **t** in Moortgat 1996b. Interaction postulates are compiled into frame conditions using the algorithm given in Kurtonina 1995. The soundness and completeness proofs are entirely standard; the latter is done by constructing canonical models from the set of types.

4 Conclusion

To sum up, the multimodal treatment of anaphora proposed here offers a unifying perspective on previous proposals in two respects. First, the two non-local phenomena of quantifier scope and anaphora are treated by means of the very same modal licensing and control devices. Second, it generalizes several seemingly incompatible categorial treatments of anaphora from the literature. In particular, even though the binding force of anaphors is essentially due to their lexical meaning there, it also covers accounts like Jacobson's where binding is done in syntax.

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