Vagueness, Signaling & Bounded Rationality

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Logic and Engineering of Natural Language Semantics
Tokyo, November 19, 2010
Overview

- Strategic communication
- Why vagueness is not rational
- Reinforcement learning with limited memory
- Quantal Best Response
Strategic communication: signaling games

- sequential game:
  1. **nature** chooses a type $T$
     - out of a pool of possible types $T$
     - according to a certain probability distribution $P$
  2. nature shows $w$ to sender $S$
  3. $S$ chooses a message $m$ out of a set of possible signals $M$
  4. $S$ transmits $m$ to the receiver $R$
  5. $R$ chooses an action $a$, based on the sent message.

- Both $S$ and $R$ have preferences regarding $R$’s action, depending on $t$.
- $S$ might also have preferences regarding the choice of $m$ (to minimize signaling costs).
Basic example

<table>
<thead>
<tr>
<th>types</th>
<th>messages</th>
<th>actions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="types" /></td>
<td><img src="image2.png" alt="messages" /></td>
<td><img src="image3.png" alt="actions" /></td>
</tr>
</tbody>
</table>

sender ➔ receiver

utility matrix

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>1,1</td>
<td>0,0</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>
Basic example: Equilibrium 1

utility matrix

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<td>0, 0</td>
<td>1, 1</td>
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</table>
Basic example: Equilibrium 2

- Types
- Messages
- Actions

Sender → Receiver

Utility matrix:

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two strict Nash equilibria
these are the only ‘reasonable’ equilibria:
- they are evolutionarily stable (self-reinforcing under iteration)
- they are Pareto optimal (cannot be outperformed)
Euclidean meaning space

- types
- messages
- actions

sender
receiver
Utility function

**General format**

\[ u_{s/r}(m, f, m') = \text{sim}(m, m') \]

- \( \text{sim}(x, y) \) is strictly monotonically decreasing in Euclidean distance \( \|x - y\| \)

In this talk, we assume a **Gaussian** similarity function

\[ \text{sim}(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma}\right). \]
Euclidean meaning space: equilibrium

types

messages

actions

sender

receiver
Simulations

(↗ my LENLS talk 2007)

- two-dimensional circular meaning space
- finitely many pixels (meanings)
- uniform distribution over meanings
Vagueness

- many evolutionarily stable/Pareto optimal equilibria
- all are strict (except for a null set at category boundaries)
- a *vague* language would be one where the sender plays a mixed strategy

Vagueness is not rational

Rational players will never prefer a vague language over a precise one in a signaling game. (Lipman 2009)

- similar claim can be made with regard to evolutionary stability (as corollary to a more general theorem by Reinhard Selten)

Vagueness is not evolutionarily stable

In a signaling game, a vague language can never be evolutionarily stable.
Vagueness and bounded rationality

- Lipman’s result depends on assumption of perfect rationality
- We present two deviations from perfect rationality that support vagueness:
  - Learning: players have to make decisions on basis of limited experience
  - Stochastic decision: players are imperfect/non-deterministic decision makers
Learning and vagueness

Fictitious play

- model of learning in games
- indefinitely iterated game
- player memorize game history
- decision rule:
  - assume that other player plays a stationary strategy
  - make a maximum likelihood estimate of this strategy
  - play a best response to this strategy
- always converges against some Nash equilibrium
Limited memory

- more realistic assumption: players only memorize last $k$ rounds (for fixed, finite $k$)
- consequence: usually no convergence
- long-term behavior depends on number of states — in relation to $k$
Formal definitions

\[\sigma(m|w) = \begin{cases} \left| \{ k | \bar{s}(k) = \langle w, m \rangle \} \right| & \text{if divisor } \neq 0 \\ \frac{1}{|M|} & \text{otherwise} \end{cases}\]

\[\rho(w|m) = \begin{cases} \left| \{ k | \bar{r}(k) = \langle m, w \rangle \} \right| & \text{if divisor } \neq 0 \\ \frac{1}{|W|} & \text{otherwise} \end{cases}\]
A simulation

**Game**
- signaling game
- 500 possible worlds, evenly spaced in unit interval \([0, 1]\)
- 3 distinct messages
- Gaussian utility function \((\sigma = 0.1)\)

**Fictitious play with limited memory**
- \(k = 200\)
- simulation ran over 20,000 rounds
A simulation

average over 10,000 rounds:
Signaling games + fictitious play with limited memory:
- predicts sharp category boundaries/unique prototypes for each agent at every point in time
- strategies undergo minor changes over time though
- in multi-agent simulations, we also expect minor inter-speaker variation
- vagueness emerges if we average over several interactions
- captures some aspect of vagueness (may provide solution for some instances Sorites paradox)
- still: even at this very moment, I do not know the exact boundary between red and orange ⇒ vagueness also applies to single agents
Stochastic choice

- real people are not perfect utility maximizers
- they make mistakes $\sim$ sub-optimal choices
- still, high utility choices are more likely than low-utility ones

Rational choice: best response

$$P(a_i) = \begin{cases} \frac{1}{|\arg_j \max u_i|} & \text{if } u_i = \max_j u_j \\ 0 & \text{else} \end{cases}$$

Stochastic choice: (logit) quantal response

$$P(a_i) = \frac{\exp(\lambda u_i)}{\sum_j (\lambda \exp u_j)}$$

Franke, Jäger & van Rooij (UTü/UvA)
Quantal response

- $\lambda$ measures degree of rationality
- $\lambda = 0$:
  - completely irrational behavior
  - all actions are equally likely, regardless of expected utility
- $\lambda \to \infty$
  - convergence towards behavior of rational choice
  - probability mass of sub-optimal actions converges to 0
- if everybody plays a quantal response (for fixed $\lambda$), play is in quantal response equilibrium (QRE)
- as $\lambda \to \infty$, QREs converge towards Nash equilibria
Quantal response

- Suppose there are two choices, $a_1$ and $a_2$, with the utilities
  - $u_1 = 1$
  - $u_2 = 2$
- probabilities of $a_1$ and $a_2$:
Quantal Response Equilibrium of $2 \times 2$ signaling game

- for $\lambda \leq 2$: only babbling equilibrium
- for $\lambda > 2$: three (quantal response) equilibria:
  - babbling
  - two informative equilibria
QRE and vagueness

- similarity game
- 500 possible worlds, evenly spaced in unit interval $[0, 1]$
- 3 distinct messages
- Gaussian utility function ($\sigma = 0.2$)
$\lambda \leq 4$

- only babbling equilibrium

**Graphs:**

1. Sender strategy vs. type

2. Receiver strategy vs. type
QRE and vagueness

$\lambda > 4$

- separating equilibria
- smooth category boundaries
- prototype locations follow bell-shaped distribution