

The empirical base of game theoretic modeling in linguistics

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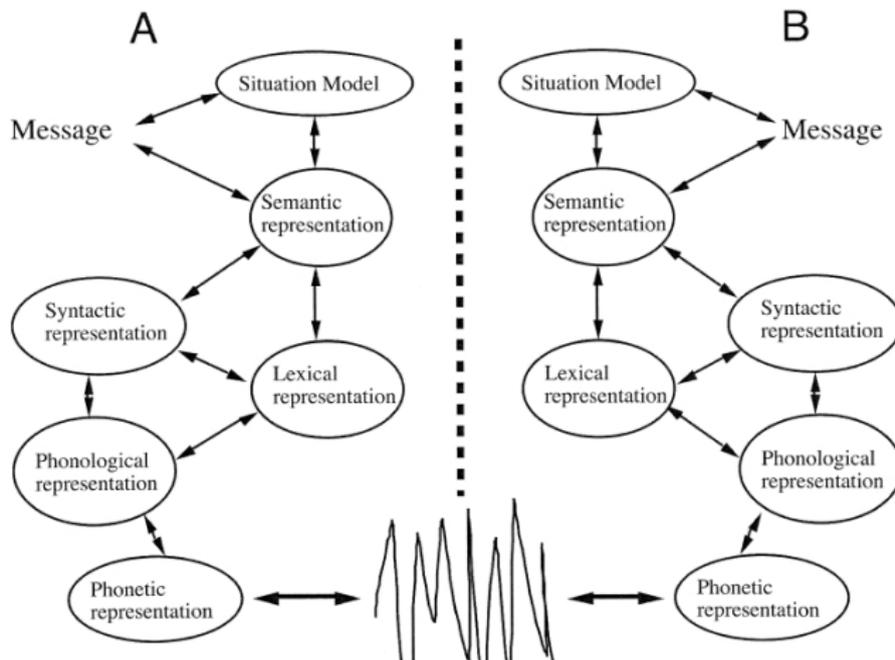
Structure of the talk

- Alignment in communication
- Exemplar dynamics
- George Price's General Theory of Selection
- Applying Price's framework to the exemplar dynamics
- Exemplar dynamics = replicator dynamics?
- Conclusion



Alignment in communication

■ naïve view on linguistic communication



Alignment in Communication

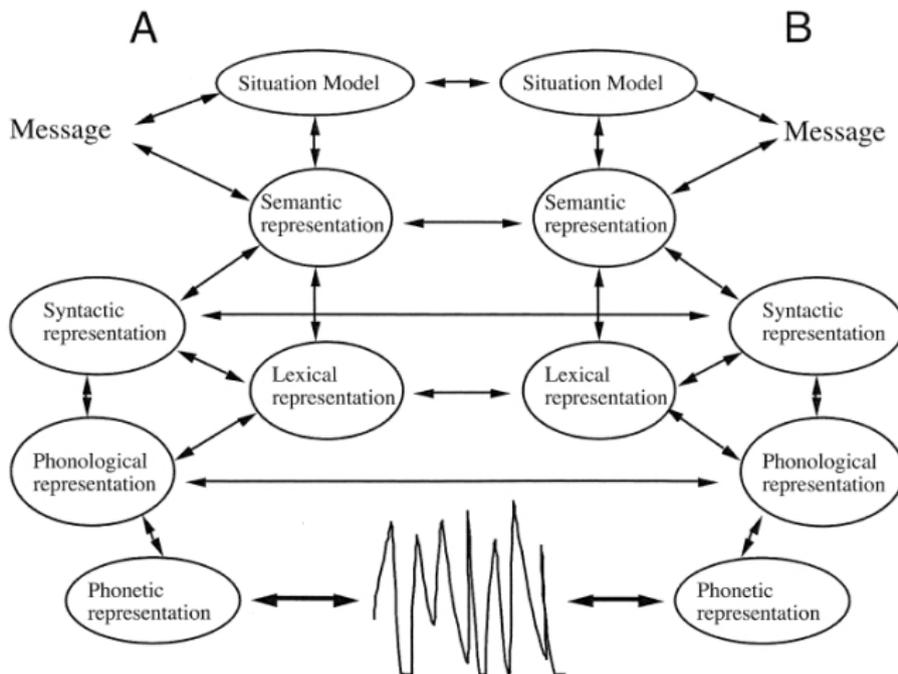
- empirical evidence for synchronization between communication partners *on all levels of linguistic representation*

1—B: . . . Tell me where you are?
2—A: Ehm : Oh God (*laughs*)
3—B: (*laughs*)
4—A: Right : **two along from the bottom one up**:*
5—B: Two along from the bottom, which side?
6—A: The left : going from left to right in the second box.
7—B: You're in the second box.
8—A: One up (*1 sec.*) I take it we've got identical mazes?
9—B: Yeah well : right, starting from the left, **you're one along**:
10—A: Uh-huh:
11—B: **and one up**?
12—A: Yeah, and I'm trying to get to . . .
[*28 utterances later*]
41—B: You are starting from the left, **you're one along, one up**? (*2 sec.*)
42—A: **Two along** : I'm not in the first box, I'm in the second box:
43—B: You're **two along**:
44—A: **Two up** (*1 sec.*) counting the : if you take : the first box as being one up:
45—B: (*2 sec.*) Uh-huh:
46—A: Well : I'm **two along, two up** (*1.5 sec.*)
47—B: Two up ? :
48—A: Yeah (*1 sec.*) so I can move down one:
49—B: Yeah I see where you are:

* The position being described in the utterances shown in bold is identified with an arrow in Figure 1. Colons mark noticeable pauses of less than 1 second.

Alignment in communication

- Alignment model according to Pickering and Garrod 2004



Alignment in communication

General notion of alignment

- an increase of the similarity in structure
- between two interacting dynamic systems
- in a largely automatic and non-reflexive fashion,
- without an explicit exchange of information on system states

Alignment in linguistic communication

- an increase in the disposition to use linguistic items,
- at all levels of linguistic structure,
- between communicating language users
- during discourse,
- without an explicit exchange of metalinguistic information about these items



Priming

- linguistic alignment is achieved mainly via **priming**
- priming:
 - tendency of language users to re-use linguistic items they just encountered
 - works bidirectionally:
 - self-priming = repetition
 - other-priming = imitation



Exemplar dynamics

- empiricist view on language processing/language structure
- popular in functional linguistics (esp. phonology and morphology) and in computational linguistics (aka. “memory-based”)

Basic idea

- large amounts of previously encountered instances (“exemplars”) of linguistic structures are stored in memory
- very detailed representation of exemplars
- little abstract categorization
- similarity metric between exemplars
- new items are processed by analogy to exemplars that are stored in memory



Exemplar dynamics \Rightarrow priming \Rightarrow alignment

exemplar dynamics \Rightarrow priming

- exemplars undergo decay – recent exemplars are more likely to be re-used than older ones
- exemplars are bidirectional: both received and produced exemplars are stored
- micro-scopic exemplar dynamics leads to meso-scopic priming effects: recently used items are likely to be re-used because its exemplar representations are still fresh



Exemplar dynamics \Rightarrow priming \Rightarrow alignment

priming \Rightarrow alignment

- other-priming leads to replication of highly active exemplars between interlocutors
- meso-scopic priming thus leads to macro-scopic synchronization of exemplar profiles between interlocutors



evolutionary exemplar dynamics

- exemplars form populations
- bidirectionality of exemplars and priming lead to replication of exemplars
- replication may be unfaithful \Rightarrow linguistic variation
- differential replication \Rightarrow evolutionary dynamics

How can this dynamics be modeled mathematically?



- 1922–1975
- studied chemistry; briefly involved in Manhattan project; lecturer at Harvard
- during the fifties: application of game theory to strategic planning of U.S. policy against communism
 - proposal to buy each Soviet citizen two pair of shoes in exchange for the liberation of Hungary
- tried to write a book about the proper strategy to fight the cold war, but *“the world kept changing faster than I could write about it”*, so he gave up the project
- 1961–1967: IBM consultant on graphic data processing



- 1967: emigration to London (with insurance money he received for medical mistreatment that left his shoulder paralyzed)
- 1967/1968: freelance biomathematician;
 - discovery of the **Price equation**
 - leads to an immediate elegant **Fisher's fundamental theorem**
 - invention of **Evolutionary Game Theory**
 - Manuscript *Antlers, Intraspecific Combat, and Altruism* submitted to *Nature* in 1968; contained the idea of a mixed ESS in the Hawk-and-Dove game
 - accepted under the condition that it is shortened
 - reviewer: John Maynard Smith
 - Price never resubmitted the manuscript, and he asked Maynard Smith not to cite it
 - 1972: Maynard Smith and Price: *The Logic of Animal Conflict*
 - Price to Maynard Smith: *"I think this the happiest and best outcome of refereeing I've ever had: to become co-author with the referee of a much better paper than I could have written by myself."*



- 1968–1974: honorary appointment at the Galton Labs in London
- 1970: conversion to Christianity
- around 1971: *The Nature of Selection* (published posthumously in 1995 in *Journal of Theoretical Biology*)
- around 1974: plans to turn attention to economy
- early 1975: suicide



The Nature of Selection

“A model that unifies all types of selection (chemical, sociological, genetical, and every other kind of selection) may open the way to develop a general ‘Mathematical Theory of Selection’ analogous to communication theory.”



The Nature of Selection

“Selection has been studied mainly in genetics, but of course there is much more to selection than just genetical selection. In psychology, for example, trial-and-error learning is simply learning by selection. In chemistry, selection operates in a recrystallisation under equilibrium conditions, with impure and irregular crystals dissolving and pure, well-formed crystals growing. In palaeontology and archaeology, selection especially favours stones, pottery, and teeth, and greatly increases the frequency of mandibles among the bones of the hominid skeleton. In linguistics, selection unceasingly shapes and reshapes phonetics, grammar, and vocabulary. In history we see political selection in the rise of Macedonia, Rome, and Muscovy. Similarly, economic selection in private enterprise systems causes the rise and fall of firms and products. And science itself is shaped in part by selection, with experimental tests and other criteria selecting among rival hypotheses.”



The Nature of Selection

Concepts of selection

- subset selection
- Darwinian selection

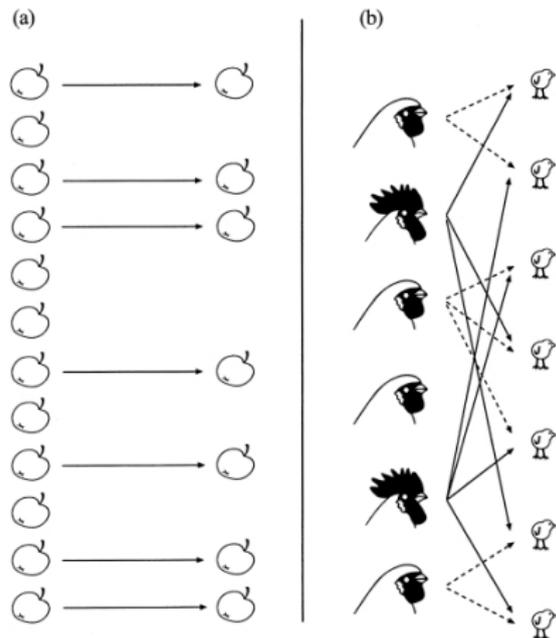


FIG. 1. Conventional concepts of selection. (a) Subset selection. (b) Darwinian selection.



The Nature of Selection

Concepts of selection

- common theme:
 - two time points
 - t : population before selection
 - t' : population after selection

- partition of populations into N bins
- parameters
 - abundance w_i/w'_i of bin i before/after selection
 - quantitative character x_i/x'_i of each bin

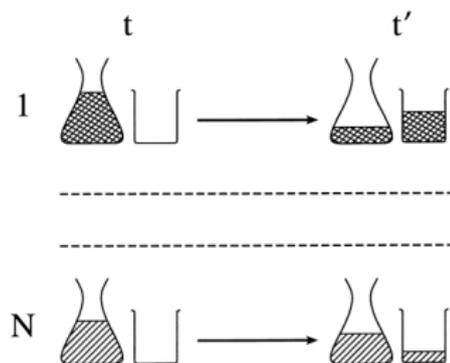


FIG. 2. A solution selection example.

The Nature of Selection

Each bin i at t is assigned a subset of the population at t' — the offspring of the i -individuals, if you like

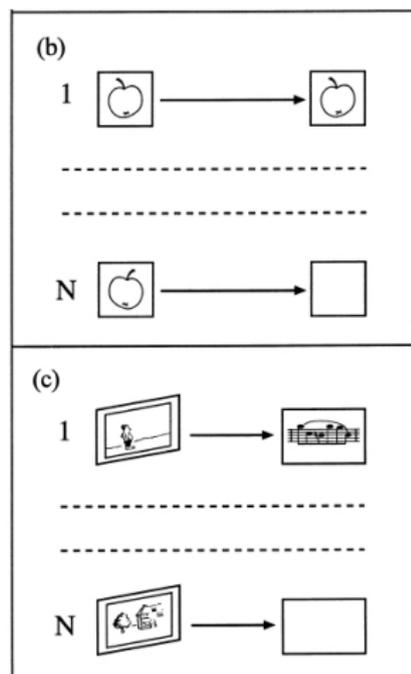
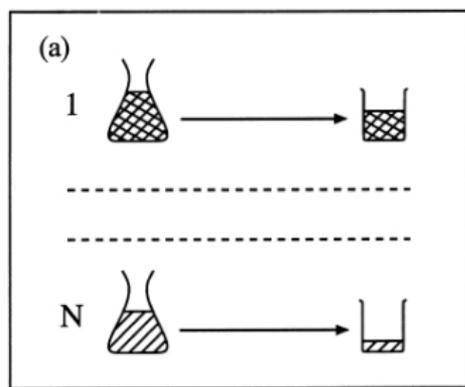


FIG. 3. Three selection examples arranged in the pattern of the general selection model. (a) The essential elements of the Fig. 2

The Nature of Selection

property change

- quantitative character x may be different between parent and offspring
- $\Delta x_i = x'_i - x_i$ need not equal 0
- models unfaithful replication (e.g. mutations in biology)

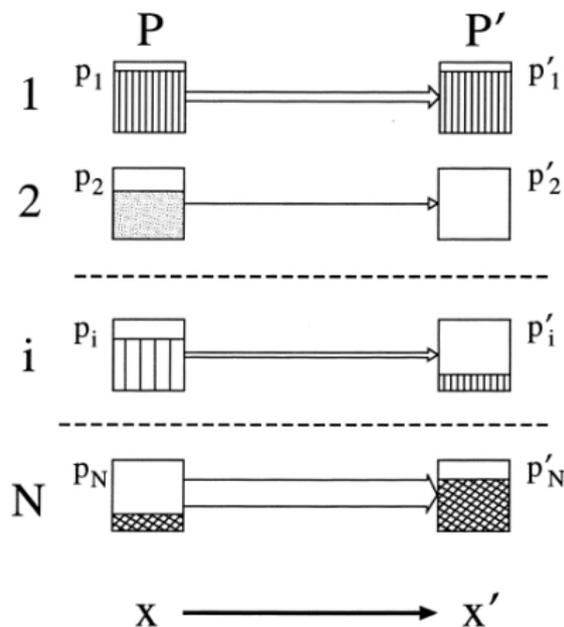


FIG. 4. The general selection model.

The Nature of Selection

genetical selection:

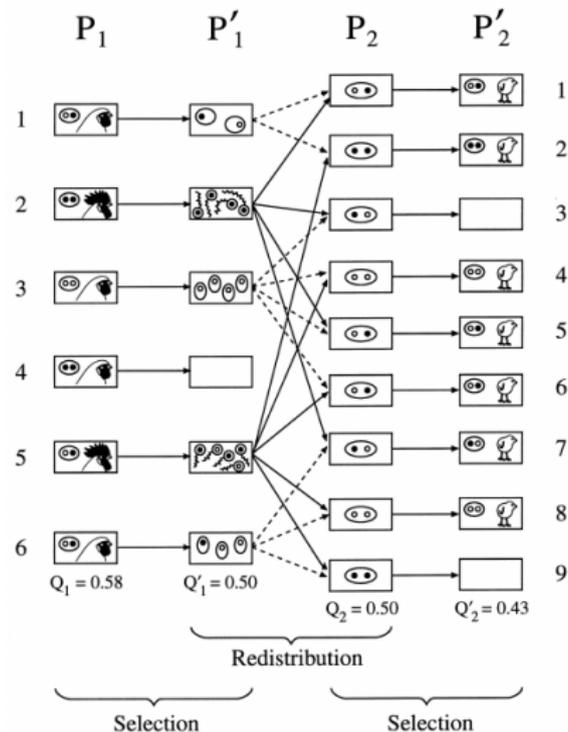


FIG. 5. A genetical selection example [showing how the Fig. 1(b) example is fitted to the general selection model].

The Price equation

Parameters

- w_i : abundance of bin i in old population
- w'_i : abundance of descendants of bin i in new population
- $f_i = w'_i/w_i$: fitness of type- i individuals
- $f = \frac{\sum_i w'_i}{\sum_i w_i}$: fitness of entire population
- x_i : average value of x within i -bin
- x'_i : average value of x within descendants of i -bin
- $\Delta x_i = x'_i - x_i$: change of x_i
- $x = \sum_i \frac{w_i}{w} x_i$: average value of x in old population
- $x' = \sum_i \frac{w'_i}{w} x'_i$: average value of x in new population
- $\Delta x = x' - x$: change of expected value of x



The Price equation

Discrete time version

$$f\Delta x = Cov(f_i, x_i) + E(f_i\Delta x_i)$$

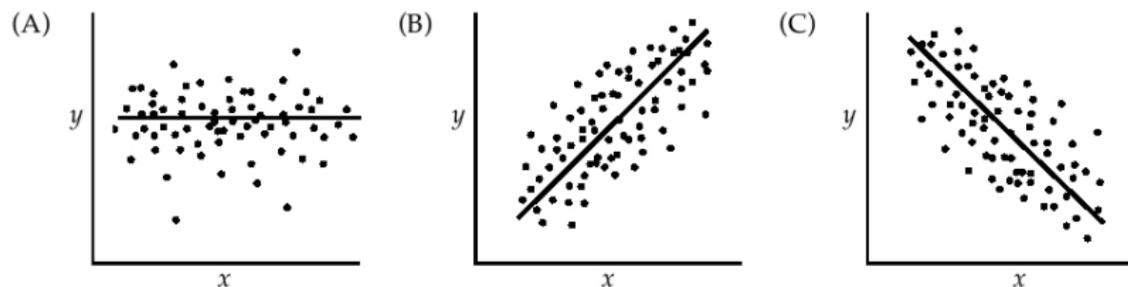
- $Cov(f_i, x_i)$: change of x due to natural selection
- $E(f_i\Delta x_i)$: change of x due to unfaithful replication

Continuous time version

$$\dot{E}(x) = Cov(f_i, x_i) + E(\dot{x}_i)$$



The Price equation



- Covariance \approx slope of linear approximation
 - (A) $= 0$: no dependency between x and y
 - (B) > 0 : high values of x correspond, on average, to high values of y and vice versa
 - (C) < 0 : high values of x correspond, on average, to low values of y and vice versa



The Price equation

- important: the equation is a tautology
- follows directly from the definitions of the parameters involved
- very general; no specific assumptions about the nature of the replication relation, the partition of population into bins, the choice of the quantitative parameter under investigation
- many applications, for instance in investigation of group selection



Applications of the Price equation

Fisher's Theorem

- x can be any quantitative character, including fitness
- for $x = f$, we have

$$\dot{E}(f) = Var(f) + E(\dot{f})$$

- $Var(f)$: increase in average fitness due to natural selection
- $E(\dot{f})$: decrease in average fitness due to deterioration of the environment



Applications of the Price equation

$$\dot{E}(x) = Cov(f_i, x_i) + E(\dot{x}_i)$$

Group selection

- population of groups that each consists of individuals
- bins = groups
- first term:
 - covariance between a certain trait x and group fitness
 - corresponds to natural selection at the group level
- second term:
 - average change of x **within** group
 - corresponds to natural selection at the individual level
- for “altruistic” traits, first term would be positive but second term negative



Applications of the Price equation

The replicator dynamics

- Suppose fitness is frequency dependent and replication is completely faithful
- f_i is a function F_i of the probability vector $\frac{\vec{w}_j}{w}$
- Let x be the indicator function δ_k of bin k

$$\begin{aligned}\dot{E}(\delta_k) &= \text{Cov}(F_i(\frac{\vec{w}_j}{w}), \delta_k(i)) \\ \frac{\dot{w}_k}{w} &= \frac{w_k}{w}(f_k - f)\end{aligned}$$

The replicator dynamics is a special case of the Price equation!



Consequences of Price's approach

- no single “correct” way to model language evolution
- prerequisites for applying Price's approach:
 - two populations at different time points
 - natural assignment of items of the new population to items in the old population
- it is up to the model builder
 - what populations consist of (any measurable set would do)
 - the evolution of which character is studied (as long as it is quantitative in nature)
 - what the nature of the “replication” relation is — any function from the new population to the old one will do
 - how populations are partitioned into bins



Exemplar dynamics of sender–receiver games

elementary sender–receiver games

- two players, S and R
- finite set of events E and finite set of signals S
- extensive form:
 - 1 nature picks an event $E_i \in E$ according to probability distribution e and shows it to S
 - 2 S picks signal $S_i \in S$ and shows it to R
 - 3 R guesses event E_j
- if $E_i = E_j$, both players receive utility 1, otherwise 0



Exemplar dynamics of sender–receiver games

exemplar modeling

- S and R are not agents, but multi-sets of exemplars
 - S : multi-set of event-signal pairs
 - R : multi-set of signal-event pairs
- if number of exemplars is high enough:
 - S can be conceived as probability distribution over $E \times S$
 - R can be conceived as probability distribution over $S \times E$



Exemplar dynamics of sender–receiver games

exemplar modeling

- “decision” of S if nature picks event E_i : pick an exemplar $\langle E_i, S_j \rangle$ according to $S(\langle E_k, S_j \rangle | k = i)$ and send signal S_j
- “decision” of R : pick an exemplar $\langle S_j, E_k \rangle$ according to $R(\langle S_l, E_k \rangle | l = j)$
- if $i = k$:
 - a copy of $\langle E_i, S_j \rangle$ is added to the exemplar pool S
 - a copy of $\langle S_j, E_i \rangle$ is added to the exemplar pool R
- otherwise S and R remain unchanged



Exemplar dynamics of sender–receiver games

some notation

- \bar{P} : event-signal matrix
- normalization of S :

$$\bar{P}(i, j) \doteq \frac{S(i, j)}{\sum_k S(k, j)}$$

- \bar{Q} : signal-event matrix
- normalization of R :

$$\bar{Q}(i, j) \doteq \frac{R(i, j)}{\sum_k R(k, j)}$$



Applying the Price equation: 1

- individuals: exemplars
- family of populations/parameters:

Populations

- probability distribution $\bar{P}(i)$,
for some i ; or
- probability distribution $\bar{Q}(j)$,
for some j

Bins

- equivalence classes: two exemplars are identical if both components are identical



Applying the Price equation: 1

Character x to be studied

- indicator function δ_{ij} for some event E_i and some signal S_j , or
- indicator function δ_{ij} for some signal S_i and some event E_j

Fitness

probability of an exemplar (from a given bin) to be replicated



Applying the Price equation: 1

- replication is always faithful
- second term of Price equation can be dropped

Family of continuous time Price equations

$$\dot{E}(\delta_{ij}) = \text{Cov}(\bar{Q}'(j), \delta_{ij})$$

$$\dot{\bar{P}}(i, j) = \bar{P}(i, j)(\bar{Q}(j, i) - (\overline{PQ})(i, i))$$

$$\dot{E}(\delta_{ij}) = \text{Cov}(\bar{P}'(j), \delta_{ij})$$

$$\dot{\bar{Q}}(i, j) = \bar{Q}(i, j)(e_j \bar{P}(j, i) - \sum_k e_k \bar{P}(k, i) \bar{Q}(i, k))$$

This could be called the extensive form replicator dynamics.

Applying the Price equation: 2

- individuals: tuples of exemplars – one exemplar for each event (sender mode) or one exemplar for each signal (receiver mode)
- formally: sender functions (matrix) P /receiver functions (matrix) Q



Applying the Price equation: 2

Populations

- probability distribution
 $\bar{P}(P) = \prod_i \prod_j P(i, j) \bar{P}(i, j)$, or
- probability distribution
 $\bar{Q}(Q) = \prod_i \prod_j Q(i, j) \bar{Q}(i, j)$

Bins

- functions
 - $P : E \mapsto S$
 - $Q : S \mapsto E$

Character x to be studied

- indicator functions δ_P or δ_Q

Fitness

probability that an element of the function in question is replicated



Applying the Price equation: 2

- replication is always faithful
- second term of Price equation can be dropped

Family of continuous time Price equations

$$\begin{aligned}\dot{E}(\delta_P) &= \text{Cov}(\lambda P^* \cdot \langle e, \text{diag}(P^* \bar{Q}) \rangle, \delta_P) \\ \dot{\bar{P}}(P) &= \bar{P}(P) (\langle e, \text{diag}(P \bar{Q}) \rangle - \langle e, \text{diag}(\bar{P} \bar{Q}) \rangle) \\ \\ \dot{E}(\delta_Q) &= \text{Cov}(\lambda Q^* \cdot \langle e, \text{diag}(\bar{P} Q^*) \rangle, \delta_Q) \\ \dot{\bar{Q}}(Q) &= \bar{Q}(Q) (\langle e, \text{diag}(\bar{P} Q) \rangle - \langle e, \text{diag}(\bar{P} \bar{Q}) \rangle)\end{aligned}$$

This is the asymmetric replicator dynamics of the sender–receiver game.



Applying the Price equation: 3

- individuals: symmetric strategies (pair of sender and receiver strategy)
- formally: pair $\langle P, Q \rangle$ of sender functions (matrix) P and receiver functions (matrix) Q



Applying the Price equation: 3

Populations

- probability distribution
 $s(P, Q) = \overline{P(P)\overline{Q}(Q)}$

Character x to be studied

- indicator functions
 $\delta_{P,Q}$

Bins

- pure symmetric strategies
 - $\langle P, Q \rangle : (E \mapsto S) \times (S \mapsto E)$

Fitness

probability that an element of the one of the function in question is replicated



Applying the Price equation: 3

- replication is always faithful
- second term of Price equation can be dropped

Family of continuous time Price equations

$$\begin{aligned}\dot{E}(\delta_{P,Q}) &= \text{Cov}(\lambda P^* \lambda Q^* \cdot \langle e, \text{diag}(P^* \bar{Q}) + \text{diag}(\bar{P}, Q^*) \rangle, \delta_P) \\ \dot{s}(P, Q) &= s(P, Q) (\langle e, \text{diag}(P \bar{Q}) + \text{diag}(\bar{P} Q) \rangle - 2 \langle e, \text{diag}(\bar{P} \bar{Q}) \rangle)\end{aligned}$$

This is the symmetric replicator dynamics of the sender–receiver game.



take home messages

- various versions of replicator dynamics can be derived via Price's formula
- simple exemplar dynamics can be described via replicator equation



Methodological points #1

- there is not one single correct conceptualization of cultural evolution, even if the underlying process is well-understood
- Price's approach gives general recipe to develop conceptualization of evolutionary process
- different conceptualizations of the same process may lead to different, “complementary” descriptions



Methodological points #2

- same formula — the replicator dynamics — can describe very different levels of cultural evolution
 - deliberate learning from experience
 - semi-automatic imitation of successful strategies
 - fully automatic stimulus-response behavior like priming



Future work

- more realistic exemplar dynamics involves similarity based error bias
- can be modeled via the second term of the Price equation
- \approx shift from replication dynamics to replication-mutation dynamics (cf. Martin Nowak's work)
- exemplars are nested - constructions contain words that contain morphemes that contain sounds that contain gestures...
- perhaps analogy to group selection

