Exemplar dynamics and George Price's General Theory of Selection

Gerhard Jäger
Gerhard.Jaeger@uni-bielefeld.de

December 8, 2006

University of Groningen

Overview

Structure of the talk

- Exemplar dynamics
- George Price's General Theory of Selection
- Applying Price's framework to the exemplar dynamics
- Conclusion

Exemplar dynamics

- empiricist view on language processing/language structure
- popular in functional linguistics (esp. phonology and morphology) and in computational linguistics (aka. "memory-based")

Basic idea

- large amounts of previously encountered instances ("exemplars") of linguistic structures are stored in memory
- very detailed representation of exemplars
- little abstract categorization
- similarity metric between exemplars
- new items are processed by analogy to exemplars that are stored in memory

Exemplars and evolution

evolutionary exemplar dynamics

- exemplars form populations
- exemplars get replicated
- replication may be unfaithful $\Rightarrow$ linguistic variation
- differential replication $\Rightarrow$ evolutionary dynamics

How can this dynamics be modeled mathematically?

George R. Price

- 1922–1975
- studied chemistry; briefly involved in Manhattan project; lecturer at Harvard
- during the fifties: application of game theory to strategic planning of U.S. policy against communism
  - proposal to buy each Soviet citizen two pair of shoes in exchange for the liberation of Hungary
- tried to write a book about the proper strategy to fight the cold war, but *"the world kept changing faster than I could write about it"*, so he gave up the project
- 1961–1967: IBM consultant on graphic data processing

George R. Price

- 1967: emigration to London (with insurance money he received for medical mistreatment that left his shoulder paralyzed)
- 1967/1968: freelance biomathematician

George R. Price

- discovery of the **Price equation**
- leads to an immediate elegant proof of **Fisher's fundamental theorem**
- invention of **Evolutionary Game Theory**
  - Manuscript *Antlers, Intraspecific Combat, and Altruism* submitted to *Nature* in 1968; contained the idea of a mixed ESS in the Hawk-and-Dove game
  - accepted under the condition that it is shortened
  - reviewer: John Maynard Smith
  - Price never resubmitted the manuscript, and he asked Maynard Smith not to cite it
  - 1972: Maynard Smith and Price: *The Logic of Animal Conflict*
  - Price to Maynard Smith: *"I think this the happiest and best outcome of refereeing I've ever had: to become co-author with the referee of a much better paper than I could have written by myself."*

George R. Price

- 1968–1974: honorary appointment at the Galton Labs in London
- 1970: conversion to Christianity
- around 1971: *The Nature of Selection* (published posthumously in 1995 in Journal of Theoretical Biology)
- around 1974: plans to turn attention to economics
- early 1975: suicide

The Nature of Selection

"A model that unifies all types of selection (chemical, sociological, genetical, and every other kind of selection) may open the way to develop a general 'Mathematical Theory of Selection' analogous to communication theory."

The Nature of Selection

"Selection has been studied mainly in genetics, but of course there is much more to selection than just genetical selection. In psychology, for example, trial-and-error learning is simply learning by selection. In chemistry, selection operates in a recrystallisation under equilibrium conditions, with impure and irregular crystals dissolving and pure, well-formed crystals growing. In palaeontology and archaeology, selection especially favours stones, pottery, and teeth, and greatly increases the frequency of mandibles among the bones of the hominid skeleton. In linguistics, selection unceasingly shapes and reshapes phonetics, grammar, and vocabulary. In history we see political selection in the rise of Macedonia, Rome, and Muscovy. Similarly, economic selection in private enterprise systems causes the rise and fall of firms and products. And science itself is shaped in part by selection, with experimental tests and other criteria selecting among rival hypotheses."

The Nature of Selection

Concepts of selection

- subset selection
- Darwinian selection

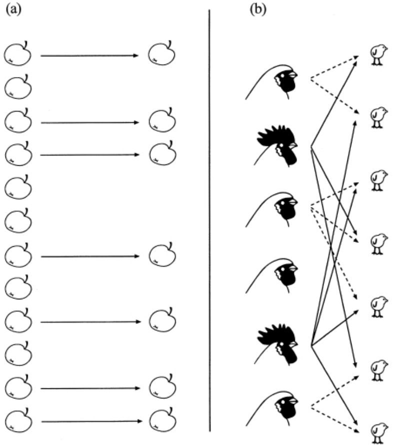


FIG. 1. Conventional concepts of selection. (a) Subset selection. (b) Darwinian selection.

The Nature of Selection

Concepts of selection

- common theme:
  - two time points
    - t: population before selection
    - t': population after selection

- partition of populations into N bins
- parameters
  - abundance w_i/w'_i of bin i before/after selection
  - quantitative character x_i/x'_i of each bin

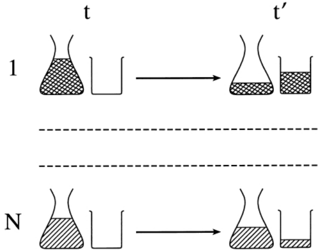
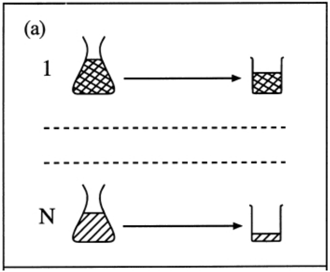


FIG. 2. A solution selection example.

The Nature of Selection

Each bin i at t is assigned a subset of the population at t' — the offspring of the i-indviduals, if you like



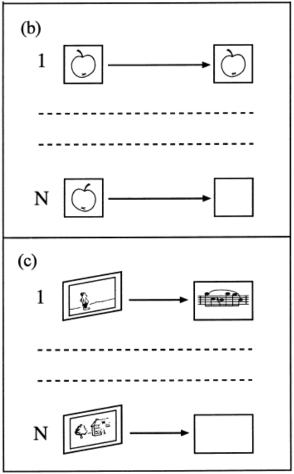


FIG. 3. Three selection examples arranged in the pattern of the general selection model. (a) The essential elements of the Fig. 2

The Nature of Selection

property change

- quantitative character x may be different between parent and offspring
- $\Delta x_i = x'_i - x_i$ need not equal 0
- models unfaithful replication (e.g. mutations in biology)

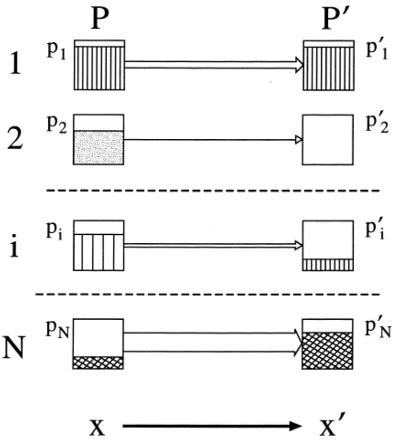


FIG. 4. The general selection model.

The Nature of Selection

genetical selection:

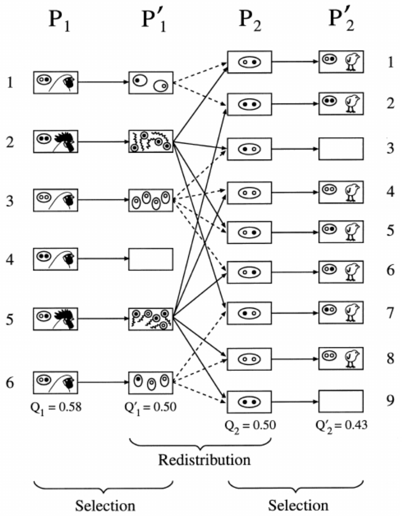


FIG. 5. A genetical selection example [showing how the Fig. 1(b) example is fitted to the general selection model].

The Price equation

Parameters

- w_i: abundance of bin i in old population
- w'_i: abundance of descendants of bin i in new population
- $f_i = w'_i/w_i$: fitness of type-i individuals
- $f = \frac{\sum_i w'_i}{\sum_i w_i}$: fitness of entire population
- x_i: average value of x within i-bin
- x'_i: average value of x within descendants of i-bin
- $\Delta x_i = x'_i - x_i$: change of x_i
- $x = \sum_i \frac{w_i}{w} x_i$: average value of x in old population
- $x' = \sum_i \frac{w'_i}{w} x'_i$: average value of x in new population
- $\Delta x = x' - x$: change of expected value of x

The Price equation

Discrete time version

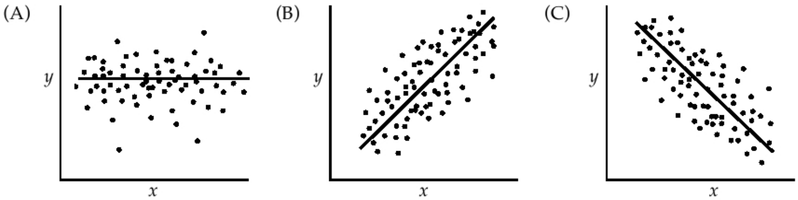
$$f\Delta x = Cov(f_i, x_i) + E(f_i \Delta x_i)$$

- $Cov(f_i, x_i)$: change of x due to natural selection
- $E(f_i \Delta x_i)$: change of x due to unfaithful replication

Continous time version

$$\dot{E}(x) = Cov(f_i, x_i) + E(\dot{x}_i)$$

The Price equation



- Covariance $\approx$ slope of linear approximation
  - (A) $= 0$: no dependency between x and y
  - (B) > 0: high values of x correspond, on average, to high values of y and vice versa
  - (C) < 0: high values of x correspond, on average, to low values of y and vice versa

The Price equation

- important: the equation is a tautology
- follows directly from the definitions of the parameters involved
- very general; no specific assumptions about the nature of the replication relation, the partition of population into bins, the choice of the quantitative parameter under investigation
- many applications, for instance in investigation of group selection

Consequences of Price's approach

- no single "correct" way to model language evolution
- prerequisites for applying Price's approach:
  - two populations at different time points
  - natural assignment of items of the new population to items in the old population
- it is up to the model builder
  - what populations consist of (any measurable set would do)
  - the evolution of which character is studied (as long as it is quantitative in nature)
  - what the nature of the "replication" relation is — any function from the new population to the old one will do
  - how populations are partitioned into bins

Exemplar dynamics and blending inheritance

Exemplars and categoricity

- algebraic descriptions of language have trouble describing variability of language use
- exemplar models are explicitly designed to deal with this
- big question for exemplar people: how come language does behave categorically in many respects?

Exemplar dynamics and blending inheritance

Exemplars and categoricity

- Wedel (2004): **reversion to the mean** in production
- newly produced exemplars are result of averaging over a sample of old exemplars

Exemplar dynamics and blending inheritance

Model architecture (inspired by Wedel)

- exemplars are n-dimensional vectors ($n = 2$ in the sample simulation)
- exemplar pool is initialized with random set
- creation of new exemplars:
  - draw a sample S of s exemplars at random from the exemplar pool
  - find the mean m of S

$$m = \frac{1}{s} \sum_{v \in S} v$$

  - add m to exemplar pool and forget oldest exemplar

Modeling via the Price equation

Assumptions

- population of exemplars is practically infinite
- continuous distribution over some finite vector space
- all exemplars are equally likely to be picked out as part of S

Modeling decisions

- ancestor population: old exemplar pool
- successor population: new exemplar pool, including the newly created exemplar
- all elements of S are "parents" of the newly added exemplar
- each exemplar forms its own bin

Modeling via the Price equation

Consequences

- all bins have identical fitness
- first term of the Price equation can be ignored
- continous population $\rightarrow$ continuous time dynamics

$$\dot{E}(x) \quad = \quad E(\dot{x}_i)$$

Modeling via the Price equation

First application: evolution of the population average

- let g be the center of gravitation of the population
- character to be studies: v_i, i.e. position of the i-the exemplar
- then

$$\dot{v}_i \quad = \quad g - v_i$$

- hence:

$$\dot{E}(v_i) = \dot{g} \quad = \quad 0$$

- in words: the center of gravitation remains constant

Modeling via the Price equation

Second application: evolution of variance

- character to be studied: variance of the population

$$
\begin{aligned}
Var(v_i) &= E[(v_i - g)^2] \\
\dot{V}ar(v_i) &= E[(v_i \dot{-} g)^2] \\
\dot{V}ar(v_i) &= -Var(v_i) \\
Var(v_i)(t) &= k\exp(-t)
\end{aligned}
$$

- in words: the variance decreases at exponential rate

Conclusion

Future work

- more realistic exemplar dynamics involves fitness differences between exemplars
- can be modeled via the first term of the Price equation
- $\approx$ shift from replication dynamics to replication-mutation dynamics (cf. Martin Nowak's work)
- exemplars are nested - constructions contain words that contain morphemes that contain sounds that contain gestures...

Wedel, A. (2004). *Self-Organization and Categorical Behavior in Phonology*. Ph.D. thesis, University of California at Santa Cruz.