Presuppositions, games, and bounded rationality

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Overview

- signaling games
- from semantics to pragmatics via iterated best response
- implicatures
- presuppositions: binding vs. accommodation
- preference orderings and bounded rationality
- conclusions

Signaling games

- sequential game:
  1. **nature** chooses a world w
     - out of a pool of possible worlds W
     - according to a certain probability distribution P
  2. nature shows w to sender **S**
  3. S chooses a signal/form f out of a set of possible signals F
  4. S transmits f to the receiver **R**
  5. R guesses a meaning $m \in M$

Signaling games

- utility of either player depends both on w and on m
- *cheap talk*: utility does not depend on f
- interests of S and R need not coincide

Signaling games: an example

Example (from Stalnaker 2005):

	m_1	m_2	m_3	m_4
w_1	5 / 5	10 / 10	0 / 0	0 / 0
w_2	5 / 5	0 / 0	6 / 0	8 / 1
w_3	5 / 5	0 / 0	6 / 6	0 / 0

(each cell: top right / bottom left)

rows: *worlds*
columns: *meanings*
bottom left: *S's utility*
top right: R's utility

Stalnaker's example (cont.)

Suppose

- $p(w_1) = P(w_2) = P(w_3) = \frac{1}{3}$
- there are four signals
- signals have the "conventional meanings" $\{w_1\}, \{w_2\}, \{w_3\}$, and $\{w_1, w_2, w_3\}$

Stalnaker's example (cont.)

naïve R:

$$R : \begin{bmatrix} \{w_1\} & \to & m_2 \\ \{w_2\} & \to & m_4 \\ \{w_3\} & \to & m_3 \\ W & \to & m_1 \end{bmatrix}$$

Stalnaker's example (cont.)

best response of S:

$$R : \begin{bmatrix} \{w_1\} & \rightarrow & m_2 \\ \{w_2\} & \rightarrow & m_4 \\ \{w_3\} & \rightarrow & m_3 \\ W & \rightarrow & m_1 \end{bmatrix}$$

$$S : \begin{bmatrix} w_1 & \rightarrow & \{w_1\} \\ w_2 & \rightarrow & W \\ w_3 & \rightarrow & \{w_3\} \end{bmatrix}$$

Stalnaker's example (cont.)

best response of R:

$$S : \begin{bmatrix} w_1 & \rightarrow & \{w_1\} \\ w_2 & \rightarrow & W \\ w_3 & \rightarrow & \{w_3\} \end{bmatrix} \qquad R : \begin{bmatrix} \{w_1\} & \rightarrow & m_2 \\ \{w_2\} & \rightarrow & ? \\ \{w_3\} & \rightarrow & m_3 \\ W & \rightarrow & m_4 \end{bmatrix}$$

Stalnaker's example (cont.)

best response of S:

$$S: \begin{bmatrix} w_1 & \rightarrow & \{w_1\} \\ w_2 & \rightarrow & W \\ w_3 & \rightarrow & \{w_3\} \end{bmatrix} \longleftarrow R: \begin{bmatrix} \{w_1\} & \rightarrow & m_2 \\ \{w_2\} & \rightarrow & ? \\ \{w_3\} & \rightarrow & m_3 \\ W & \rightarrow & m_4 \end{bmatrix}$$

Some observations

- fixed point of *iterated best response* is Nash equilibrium
- R effectively interprets the signal with the literal meaning W—the tautology—as $\{w_2\}$
- strengthening from W to $\{w_2\}$ can be considered an **implicature**
- schematically:
  - starting point: **semantics**
  - fixed point of iterated best response: **pragmatics**

Cooperative games

- games where interests of S and R coincide:

$$u_S = u_R$$

- common goal is the efficient transmission of information:

$$M = POW(W)$$

- "nature's" probability distribution P is assumed to be common knowledge

Cooperative games

- utility can thus be defined as

$$u(w, m) = P(w|m)$$

- captures Gricean maxims of
  - **quality**: lying leads to utility 0
  - **quantity** (at least partially): the more informative the message, the higher the utility for both (provided the message is true)

Costly signaling

- talk is not cheap
  - complexity of signals are costs ($=$ negative utility)
  - signals differ in complexity
- $c(f)$: costs (positive real number)
- utility in world w of signal f which is interpreted as meaning m:

$$P(w|m) - c(f)$$

- captures maxim of manner

Utility of strategies

- overall utility is determined by **strategies**
  - sender strategy: function $S : W \mapsto F$
  - receiver strategy: function $R : F \mapsto POW(W)$
  - average utility (depends on nature's probability function):

$$u_P(S,R) = \sum_{w \in W} P(w) \cdot (P(w|R(S(w))) - c(S(w)))$$

Implicatures

The Q-Heuristics

“What isn’t said, isn’t.”

- related to Grice’s Maxim of Quantity
- accounts for scalar and clausal implicatures

(1) a. Some boys came in. $\rightsquigarrow$ Not all boys came in.
b. Three boys came in. $\rightsquigarrow$ Exactly three boys came in.

(2) b. If John comes, I will leave. $\rightsquigarrow$ It is open whether John comes.
c. John tried to reach the summit. $\rightsquigarrow$ John did not reach the summit.

Q-implicatures

(B = boy, C = come in)

- worlds
  - $w_1 : \exists x.Bx \wedge \forall y.By \rightarrow Cy$
  - $w_2 : \exists x.Bx \wedge Cx \wedge \exists y.By \wedge \neg Cy$
  - $w_3 : \exists x.Bx \wedge \neg \exists y.By \wedge Cy$
- probabilities

$$P_i(t_1) = P_i(t_2) = P_i(t_3) = \frac{1}{3}$$

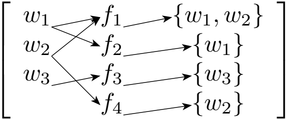
Q-implicatures

- signals:
  - f_1: "Some boys came in."
  - f_2: "All boys came in."
  - f_3: "No boys came in."
  - f_4: "Some, but not all boys came in."
- costs:

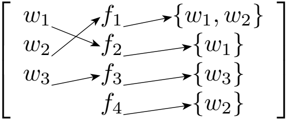
$$c(f_1) = c(f_2) = c(f_3) < c(f_4) - 0.5$$

Q-implicatures

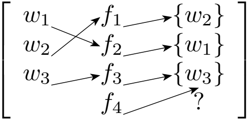
1. semantic convention:



2. *Best response* of S:



3. *Best response* von R:



Q-implicatures

- one round of best response on each side leads to a fixed point
- justifies the (Q-)implicature

  "Some boys came in." *implicates* $\exists x.Bx \wedge \neg Cx$

Q-implicatures

- essentially by Gricean reasoning:
  - there are two competing expressions of similar complexity
  - the literal meaning of the first expression entails the literal meaning of the second expression
  - the speaker wants the hearer to be as well-informed as possible
  - hence the weaker expression can only be used if the stronger one is false
  - hence the stronger expression implicates that the weaker expression is false

I-implicatures

The I-Heuristics

"What is expressed simply is stereotypically exemplified."

- related to Maxim of Manner
- accounts for
  - pragmatic strengthening

(3) a. John's book is good. $\rightsquigarrow$ The book that John is reading or that he has written is good.
b. a secretary $\rightsquigarrow$ a female secretary
c. road $\rightsquigarrow$ hard-surfaced road

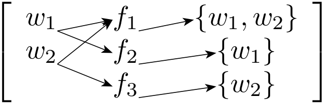
  - ...

I-implicatures

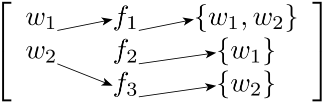
- worlds:
  - w_1: hard-surfaced road
  - w_2: soft-surfaced road
- probabilities
  - $P(w_1) \gg P(w_2)$
  - lets say: $P(w_1) = 9 \cdot P(w_2)$
- signals:
  - f_1: "road"
  - f_2: "hard-surfaced road"
  - f_3: "soft-surfaced road"
- costs:
  - $c(f_1) = 0.10$
  - $c(f_2) = 0.25$
  - $c(f_3) = 0.25$

I-implicatures

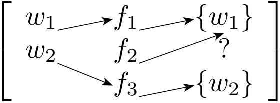
1. semantic convention:



2. *Best response* of S:



3. *Best response* of R:



I-implicatures

- conflicting interests for the speaker:
  - incentive to avoid costs (Manner): use f_1 in w_1
  - incentive to maximize information (Quantity): use f_2 in w_1
- depending on concrete probabilities and costs, either motivation may be stronger
- however: if Manner wins over Quantity, it will always be the more probable ("stereotypical") denotation that is implicated

Context dependency

- refinement: S and R have a joint common ground
- S's contribution should be informative against this background
- formally:
    - $cg(w)$ is the common ground of S and R in world w
    - refined utility function

$$u_P(S, R, w) = P(w|R(S(w)) \cap cg(w)) - c(S(w))$$

Iterated best response

- suppose R sends the signal f
- conventional meaning of f: $i(f)$
- R's strategies under successive cycles of iterated best response:

$$
\begin{aligned}
R_0(f) &= i(f) \\
R_{n+1}(f) &= \{w|\forall f'.P(w|R_n(f) \cap cg(w)) - c(f) \geq \\
&\quad P(w|R_n(f') \cap cg(w)) - c(f')\}
\end{aligned}
$$

- pragmatic strategy R_ω is the fixed point of this sequence (if there is one), i.e. $R_\omega = R_n$ if $R_n = R_{n+1}$

Iterated best response

Observation:

- suppose it is always possible to say nothing
- costs of silence are 0
- then it holds for all f:
  - $R_\omega(f) \subseteq i(f)$
  - $w \in R_\omega(f) \rightarrow cg(w) \not\subseteq R_\omega(f)$
- in words:
  - An utterance of f implicates that
    - f is true
    - $i(f)$ is contextually informative
- side conditions:
  - $\models \Box_r\phi \leftrightarrow \Box_s\Box_r\phi$
  - $\models \Box_s\Box_r\phi \leftrightarrow \Box_r\Box_s\Box_r\phi$

Presuppositions

The basic idea

(4) We regret that there is no bus service today.

- presupposed material is in the scope of $\Box_r$
- literal meaning of (4): $\phi \wedge \Box_r(\psi)$
  - ϕ: S regrets that there is no bus service today
  - ψ: There is no bus service today

Presuppositions

Presupposition binding

- $\forall w : cg(w) \models \Box_r(\psi)$
- effect of literal interpretation:

$$cg(w) \cap (\phi \wedge \Box_r(\psi)) = cg(w) \cap \phi$$

- presupposition has no impact on pragmatic interpretation

Presupposition accommodation

- suppose $\forall w : cg(w) \models \Box_r(\psi)$
- intuitive idea:
  - $R_0(f) : \phi \wedge \Box_r(\psi)$
  - $R_1(f) \models \phi \wedge \Box_s \Box_r(\psi)$
- in words: R should infer that there is no bus service, S believes that R already knew this, and S regrets that there is no bus service
- This is a contradiction, however, because S is assumed to be well-informed about R's epistemic state
- **presupposition failure leads to pragmatic deviance**

Presuppositions

Why accommodation works nevertheless

- suppose R doesn't care about what S thinks about R; he only cares about the facts
  1. literal interpretation: there is no bus service, R know this, and S regrets it
  2. relevant part of this interpretation: there is no bus service
  3. best response of S: "We regret that there is no bus service today" can be used to convey ϕ
  4. best response of R: interpret "We regret that there is no bus service today" as ϕ

Decision problems

- refinement: R does not value knowledge *per se* but wants to solve a **decision problem**
- decision problem: partition of W (i.e. a question denotation in the sense of Groenendijk & Stokhof)
- $[w]$: partition cell that contains w
- $[m] : \bigcup\{[w] | [w] \cap m \neq \emptyset\}$
- modified utility function :

$$u_P(S, R, w) = P([w] | [R(S(w)) \cap cg(w)]) - c(S(w))$$

Back to presupposition accommodation

- R wants to learn about facts, not about opinions
- $v \equiv w$ iff $\forall$ atomic $p : v \models p \Leftrightarrow w \models p$
- $[w]$: equivalence class of w under $\equiv$

Observation

$[\phi] = [\phi']$, where ϕ' is the strongest modality free formula that entails ϕ

Back to presupposition accommodation

- literal meaning of f: $\phi \wedge \Box_r \psi$
- if $cg \models \Box_r \psi : [cg(w) \cap (\phi \wedge \Box_r \psi)] = cg(w) \cap \phi$ (Binding)
- if $cg \not\models \Box_r \psi : [cg(w) \cap (\phi \wedge \Box_r \psi)] = cg(w) \cap (\phi \wedge \psi)$ (Accommodation)
- **If content of presupposition is not part of common ground, it is pragmatically asserted.**

Binding, accommodation, and bounded rationality

- van der Sandt: preference ordering between various ways of presupposition resolution:
  - binding is better than accommodation
  - high accommodation is better than low accommodation
  - accommodation must preserve consistency and informativeness
- **bounded rationality:**
  - game may have more than one reachable Nash equilibrium
  - preference for equilibria that incur *lowest inference costs* for the receiver
  - inference costs (tentative):
    - suppose $[\phi] = \phi'$
    - number of inference rules that have to be applied for the inference

$$\phi \vdash \phi'$$

Binding, accommodation, and bounded rationality

- **Binding**: no modal inferences required
- **Accommodation:** non-trivial inference, for instance

$$\phi \land \Box_r \psi \vdash \phi \land \psi$$

  - *high accommodation:*

$$\phi \land \Box_r \psi \vdash \phi \land \psi$$

  - *low accommodation:*

$$\xi \rightarrow \phi \land \Box_r \psi \vdash \xi \rightarrow \phi \land \psi$$

Binding, accommodation, and bounded rationality

- however, inconsistent or uninformative readings can never be Nash equilibria
  - strong constraints: only reachable Nash equilibrium are pragmatically licit
  - soft constraints: if there are several reachable Nash equilibria, choose the one that incurs the lowest inference costs

Summary

- semantic conventions may be sub-optimal
- fixed point of iterated best response, starting from the semantic convention, defines pragmatic information transfer
- computation of Nash equilibria incurs processing costs
- if there are several accessible Nash equilibria, minimizing inference costs guides equilibrium selection
- applications:
  - Gricean implicatures
  - presupposition resolution