Some notes on the formal properties of Bidirectional Optimality Theory

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December 20, 2000

Outline of talk

- OT and semantics: issues
- Blutner’s Bidirectional OT
- Alternative definition and naive algorithm
- Finite state implementation
Optimality Theory: The basic picture

- Three components:
  1. **GEN**: (very general) relation between input and output
  2. **CON**: set of ranked violable constraints on input-output pairs
  3. **EVAL**: Choice function that identifies optimal input-output pairs among a set of candidates (depending on CON)

- **CON** induces a (well-founded) ordering of i/o pairs
- **EVAL** picks out the minimal members of its argument wrt. this ordering

\[ \langle i, o \rangle \text{ is optimal iff } o \in \text{EVAL}_{\text{CON}}(\{o' \mid \text{GEN}(i, o') \}) \]
• Two types of constraints:
  1. Markedness constraints ⇒ refer to output only
     ○ “syllables have onsets”, “vowels are oral” ...
  2. Faithfulness constraints ⇒ refer to i/o pairing
     ○ “don’t delete material”, “don’t add material” ...

**Application to syntax/semantic**

• In phonology/morphology, OT takes the speaker perspective

• applied to syntax/semantics, this means:
  1. **GEN** is given by compositional (underspecified) semantics
  2. Markedness constraints only apply to forms, not to meanings
  3. A form/meaning pair may be blocked by a better form for the same meaning, but not the other way round
Competition/Blocking in semantics and pragmatics

- Competition between forms
  - Scalar implicatures
  - Clausal implicatures
- But: also competition between meanings
  - Presupposition resolution (cf. van der Sandt (1992))
  - Bridging inference
  - ...

Reconciling the perspectives

- Tension between “speaker economy” and “hearer economy” is often discussed, for instance Horn (1984):

  *Q-principle*: Say as much as you can (given I).

  *I-principle*: Say no more than you must (given Q).
Blutner’s formalization

Definition 1 (Blutner’s Bidirectional Optimality)

1. \( \langle f, m \rangle \) satisfies the Q-principle iff \( \langle f, m \rangle \in \text{GEN} \) and there is no other pair \( \langle f', m \rangle \) satisfying the I-principle such that \( \langle f', m \rangle < \langle f, m \rangle \).

2. \( \langle f, m \rangle \) satisfies the I-principle iff \( \langle f, m \rangle \in \text{GEN} \) and there is no other pair \( \langle f, m' \rangle \) satisfying the Q-principle such that \( \langle f, m' \rangle < \langle f, m \rangle \).

3. \( \langle f, m \rangle \) is z-optimal iff it satisfies both the Q-principle and the I-principle.


Application of Bidirectional OT to semantic/pragmatic issues include

- Iconicity effects: Blutner (2000)
- Syntax and semantics of German adverbs: Egg (1999); Jäger and Blutner (2000); von Stechow (2000)
- Anaphora resolution: Beaver (2000)
- Presupposition resolution: Zeevat (1999)
- …
**Alternative definition**

**Definition 2 (X-Optimality)** A form-meaning pair $\langle f, m \rangle$ is $x$-optimal iff

1. $\langle f, m \rangle \in \text{GEN}$,

2. there is no $x$-optimal $\langle f', m \rangle$ such that $\langle f', m \rangle < \langle f, m \rangle$.

3. there is no $x$-optimal $\langle f, m' \rangle$ such that $\langle f, m' \rangle < \langle f, m \rangle$.

**Theorem 1** If “$<$” is well-founded, then there is a unique $x$-optimality relation and a unique $z$-optimality relation

*Proof idea:* Recursion theorem

**Theorem 2** If “$<$” is transitive and well-founded, then $x$-optimality and $z$-optimality coincide.

*Proof:* see Jäger (2000)
Algorithm

\[ OPT = \emptyset; \]
\[ BLCKD = \emptyset; \]

\[ \textbf{while } (OPT \cup BLCKD \neq GEN) \{ \]
\[ \quad OPT = OPT \cup \{x \in GEN - BLCKD|\forall y < x : y \in OPT \cup BLCKD\}; \]
\[ \quad BLCKD = BLCKD \cup \{\langle f, m \rangle \in GEN - OPT | \]
\[ \quad \langle f', m \rangle \in OPT \lor \langle f, m' \rangle \in OPT \}; \]
\[ \} \]

\[ \textbf{return } (OPT); \]
OT and finite state techniques: Frank and Satta 1998

- Naive algorithms only work with finite candidate sets
- Bad news: Set of optimal candidates might be undecidable if candidate set is infinite
- Good news: Large subclass of OT systems can even be implemented by finite state techniques

Computational issues

- Set of optimal outputs might be undecidable, even if GEN and all constraints are decidable
  - Let $T$ be a Turing machine
  - $\text{GEN} = \mathbb{N} \times \mathbb{N}$
  - $c_1 = \{ n | T \text{ halts after less than } n \text{ steps} \}$
  - $c_2 = \{ 0 \}$
  - $0$ is an optimal output iff $T$ halts \Rightarrow undecidable in the general case
Finite State techniques

- FSA (Finite State Automaton): standard definition, each FSA defines a regular language
- FST (Finite State Transducer):
  - FSA that produces output
  - every state transition consumes one input sign or the empty string and produces an output sign or the empty string
  - every FST defines a rational relation

Figure 1: FST implementing the rational relation \( \{ (a^n, b^n c^*) | n \in \mathbb{N} \} \)
Some closure properties of regular languages and rational relations

- Every finite language is regular.
- If $L_1$ and $L_2$ are regular languages, then $L_1 \cap L_2, L_1 \cup L_2, L_1 - L_2$ are also regular languages.
- If $R_1$ and $R_2$ are rational relations, then $R_1 \cup R_2, R_1 \circ R_2$ and $R_1^\cup$ are also rational relations.
- If $R$ is a rational relation, then $Dom(R)$ and $Rg(R)$ (the domain $\{x \mid \exists y . xRy\}$ and the range $\{y \mid \exists x . xRy\}$ of $R$) are regular languages.
- If $L_1$ and $L_2$ are regular languages, then $L_1 \times L_2$ and $I_{L_1}$ are rational relations.

- Frank and Satta: If
  - $\textbf{GEN}$ is a rational relation,
  - there are no faithfulness constraints
  - all constraints are binary (i.e. they don’t count violations) and
  - all constraints can be represented by a regular language,

then the set of optimal input-output pairs is a rational relation.
Conditional Intersection

Definition 3
Let $R$ be a relation and $L \subseteq Rg(R)$. The conditional intersection $R \uparrow L$ of $R$ with $L$ is defined as

$$R \uparrow L \triangleq (R \circ I_L) \cup (I_{Dom(R)} - Dom(R \circ I_L) \circ R)$$

Theorem 3 (Frank and Satta)
Let $\mathcal{O} = \langle \text{GEN}, C \rangle$ with $C = \langle c_1, \ldots, c_p \rangle$ be an OT-system such that $C$ solely consists of binary output markedness constraints. Then $\langle i, o \rangle$ is unidirectionally optimal iff $\langle i, o \rangle \in \text{GEN} \uparrow c_1 \cdots \uparrow c_p$. 
Extension to Bidirectionality

- Bidirectional OT: competition both between different inputs and different outputs
- Thus both input markedness constraints and output markedness constraints
- So we also need backward conditional intersection:

\[ R \downarrow L \equiv (R^u \uparrow L)^u \]

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Definition 4 (Bidirectional Conditional Intersection)

Let \( \mathcal{O} = \langle \text{GEN}, C \rangle \) be an OT-system and \( c_i \) be a binary markedness constraint.

\[ R \uparrow c_i \equiv \begin{cases} 
R \circ I_{Rg((\{\varepsilon\} \times Rg(R))^c_i)} & \text{if } c_i \text{ is an output markedness constraint} \\
I_{Dom((Dom(R) \times \{\varepsilon\})^c_i)} \circ R & \text{else} 
\end{cases} \]
Lemma 1
Let $\mathcal{O} = \langle \text{GEN}, C \rangle$ be an OT-system (with binary markedness constraints only), where $C = \langle c_1, \ldots, c_p \rangle$. Then
\[
\langle i, o \rangle \in \text{GEN} \uparrow c_1 \cdots \uparrow c_p
\]
iff $\langle i, o \rangle \in \text{GEN}$, and there are no $i', o'$ with $\langle i', o' \rangle \in \text{GEN}$ and $\langle i', o' \rangle < \langle i, o \rangle$.

- **Notation:** $R^C \doteq R \uparrow c_1 \cdots \uparrow c_n$ (where $C = c_1, \ldots, c_p$)

Definition 5
Let $\mathcal{O} = \langle \text{GEN}, C \rangle$ be an OT-system.

\[
\begin{align*}
OPT_0 & = \emptyset \\
OPT_{\alpha+1} & = OPT_\alpha \cup \\
& \quad \left( \text{I}_{\text{Dom}(\text{GEN}) - \text{Dom}(OPT_\alpha)} \circ \text{GEN} \circ \text{I}_{\text{Rg}(\text{GEN}) - \text{Rg}(OPT_\alpha)} \right)^C \\
OPT_\beta & = \bigcup_{\alpha < \beta} OPT_\alpha \quad (\beta \text{ a limit ordinal}) \\
OPT & = \bigcup OPT_\alpha
\end{align*}
\]

Lemma 2
Let $\mathcal{O} = \langle \text{GEN}, C \rangle$ be an OT-system. Then $\langle i, o \rangle \in OPT$ iff $\langle i, o \rangle$ is $x$-optimal.
**Lemma 3**
Let $\mathcal{O} = \langle \text{GEN}, C \rangle$ be an OT-system with $C = c_1, \ldots, c_p$, where all $c_i$ are binary markedness constraints. Then $OPT = OPT_{2p}$.

**Theorem 4**
Let $\mathcal{O} = \langle \text{GEN}, C \rangle$ be an OT-system with $C = \langle c_1, \ldots, c_p \rangle$, where all $c_i$ are binary markedness constraints. Furthermore, let $\text{GEN}$ be a rational relation and let all $c_i$ be regular languages. Then the set of $x$-optimal elements of $\text{GEN}$ is a rational relation.

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**Outlook: Extension to faithfulness constraints**

- Generalization to faithfulness constraints requires closure of relations under intersection:

\[
R \uparrow S \triangleq (R \cap S) \cup (R \circ I_{\text{Rg}(R)} \uparrow \text{Rg}(\text{Dom}(R) \cap S) \times \text{Rg}(R))
\]

<table>
<thead>
<tr>
<th>Relations</th>
<th>Languages</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Dom}$, $\text{Rg}$</td>
<td>$\cap$, $\cup$, $\circ$, $^{-}$</td>
</tr>
<tr>
<td>$\times$, $I$</td>
<td>$\cap$, $\cup$</td>
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Figure 2: Closure conditions needed for $x$-optimality
References
