Residuation, Structural Rules and Context Freeness

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Venice

May 22, 2002

Outline of talk

- generative capacity of Categorial Grammars
- **NL**
- adding modalities
- context freeness of **NL**$\Diamond$-grammars
- generalizations
- comparison to Kandulski

Generative capacity of Categorial Grammars

Basic systems

	AB	CF	Bar-Hillel *et al.* (1960)
+ residuation	**NL**	CF	Kandulski (1988)
+ associativity	**L**	CF	Pentus (1993)
+ further structural rules	**LP, LC**, ...	$\subset$ CF	van Benthem (1991)

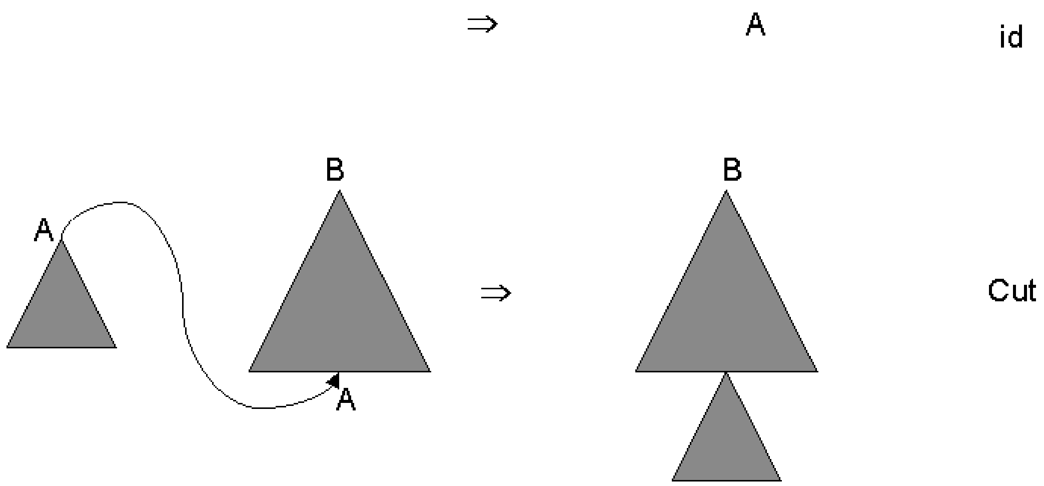
Generative capacity of Categorial Grammars

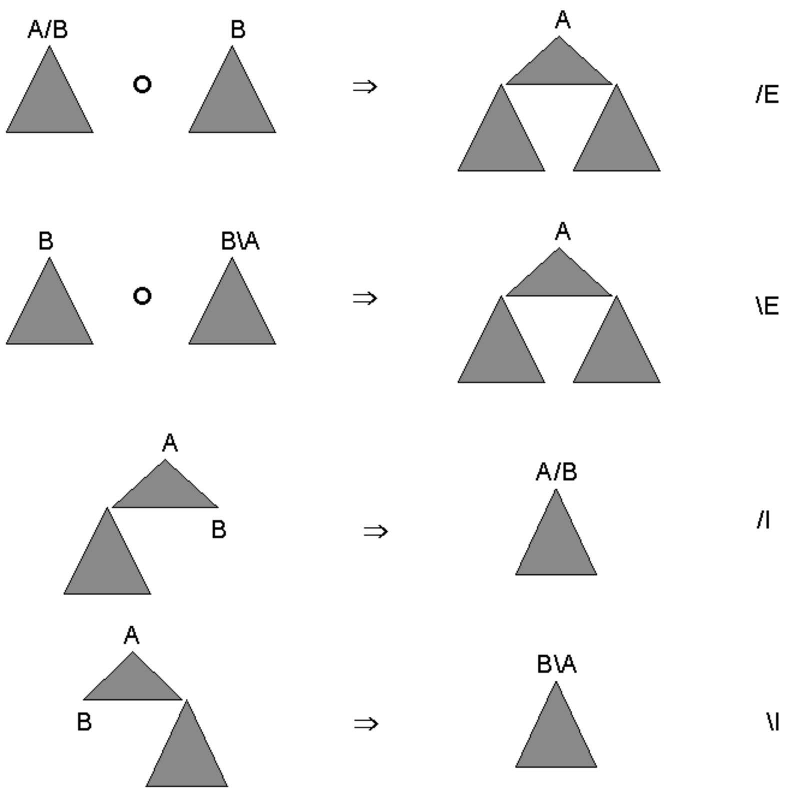
Multimodal systems

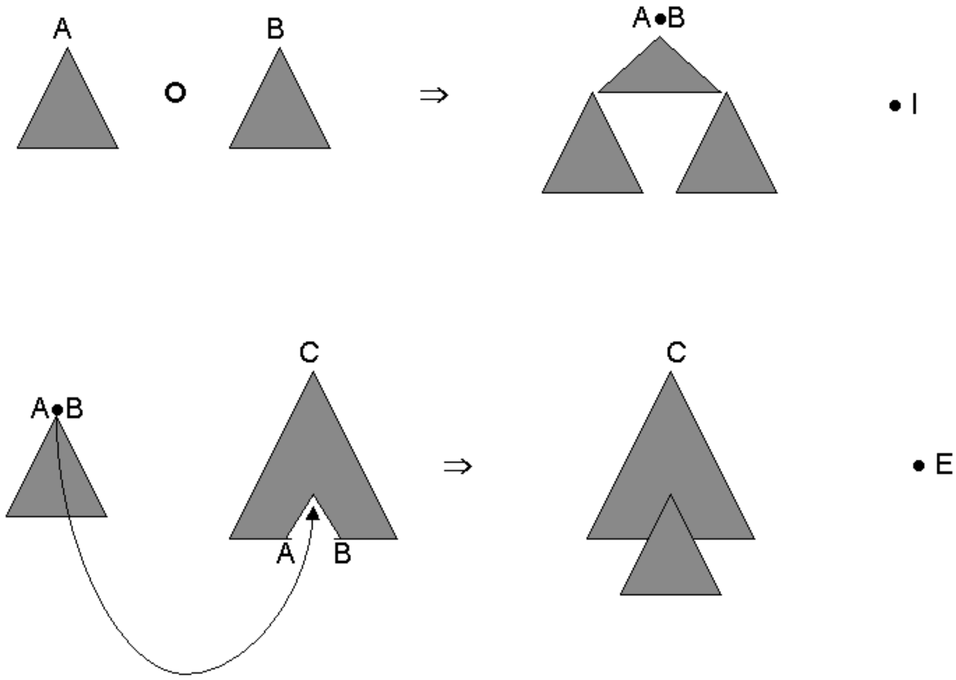
	NL $+ \Diamond_i, \Box^{\downarrow}{}_i$	CF	
+ associativity	**L** $+ \Diamond_i, \Box^{\downarrow}{}_i$	CF	Jäger (2001)
+ further structural rules	**NL** + postulates	Turing-complete	Carpenter (1999)

The non-associative Lambek Calculus

- Reasoning over binary trees
- root and leafs are labeled with formulas over $\backslash, \bullet, /$
- Natural Deduction format:

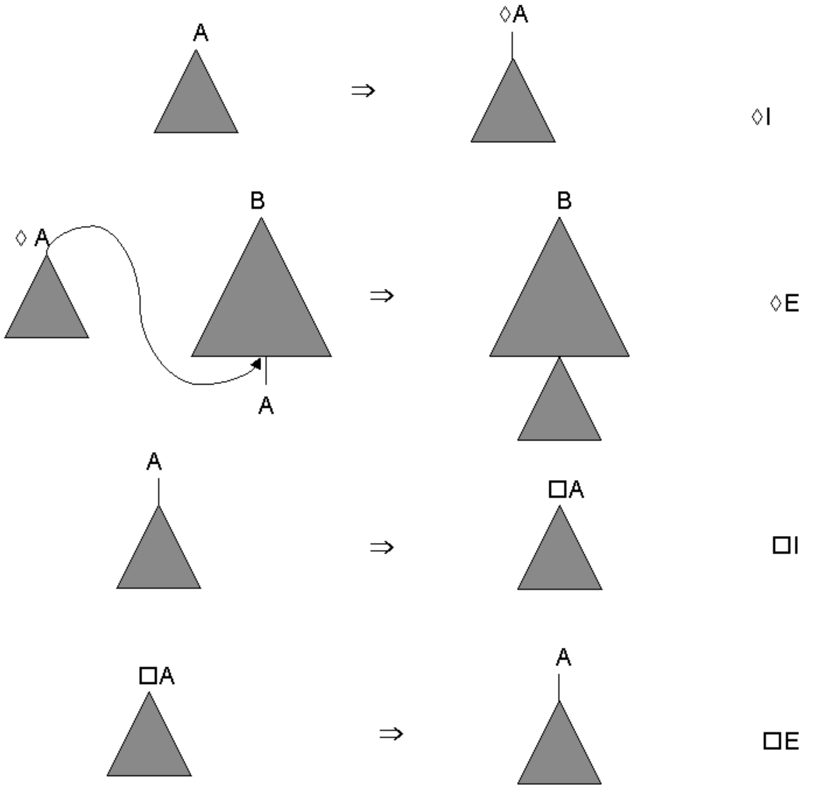






NL$\diamond$

- conservative extension of **NL**
- logical vocabulary: additionally two unary modal operators, $\diamond$ and $\Box^{\downarrow}$
- reasoning over unary/binary trees



From Logic to Grammar

- **NL**$\Diamond$-grammars consist of
  - Lexicon: finite set of elementary non-branching trees with category on top and terminal item on bottom: $\begin{array}{c} Cat \\ | \\ lex \end{array}$
  - finite set of designated categories
- A string of terminals is recognized by the grammar iff it is the yield of a tree with a designated category as root label.

Generative Capacity

Lemma 1 *Every context free language L is recognized by some* **NL**$\diamond$*-grammar.*

Proof:

- Bar-Hillel *et al.* (1960): Every context free language is recognized by some first order **AB**-grammar
- for first order grammars, the difference between **AB** and **NL**$\diamond$ doesn't matter
- basically Cohen's 1967 proof for inclusion of cfl in **L**-languages

$\dashv$

Sequent presentation of NL$\diamond$

$$\frac{}{A \Rightarrow A}\, id \qquad \frac{X \Rightarrow A \qquad Y[A] \Rightarrow B}{Y[X] \Rightarrow B}\, Cut$$

$$\frac{X[A \circ B] \Rightarrow C}{X[A \bullet B] \Rightarrow C}\, \bullet L \qquad \frac{X \Rightarrow A \qquad Y \Rightarrow B}{X \circ Y \Rightarrow A \bullet B}\, \bullet R$$

$$\frac{X \Rightarrow A \qquad Y[B] \Rightarrow C}{Y[B/A \circ X] \Rightarrow C}\, /L \qquad \frac{X \circ A \Rightarrow B}{X \Rightarrow B/A}\, /R$$

$$\frac{X \Rightarrow A \qquad Y[B] \Rightarrow C}{Y[X \circ A\backslash B] \Rightarrow C}\backslash L \qquad \frac{A \circ X \Rightarrow B}{X \Rightarrow A\backslash B}\backslash R$$

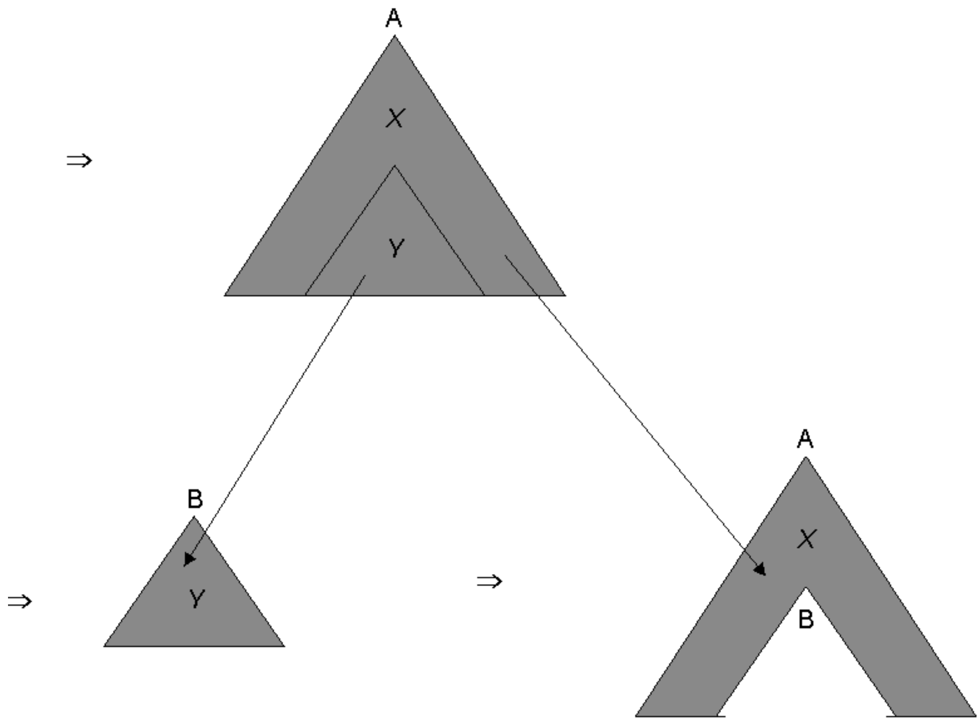
$$\frac{X \Rightarrow A}{\langle X \rangle \Rightarrow \Diamond A}\Diamond L \qquad \frac{X[\langle A \rangle] \Rightarrow B}{X[\Diamond A] \Rightarrow B}\Diamond R$$

$$\frac{X[A] \Rightarrow B}{X[\langle \Box^{\downarrow} A \rangle] \Rightarrow B}\Box^{\downarrow} R \qquad \frac{\langle X \rangle \Rightarrow A}{X \Rightarrow \Box^{\downarrow} A}\Box^{\downarrow} R$$

Lemma 2 *Let* $X[Y] \Rightarrow A$ *be a theorem of* **NL**$\Diamond$. *Then there is a type* B *such that*

1. **NL**$\Diamond \vdash Y \Rightarrow B$
2. **NL**$\Diamond \vdash X[B] \Rightarrow A$
3. *There is a type occurring in* $X[Y] \Rightarrow A$ *which contains at least as many connectives as* B.

Proof: Induction over sequent derivations. $\dashv$



Constructing a CFG from an NL◇-grammar

- **NL**◇(n): fragment of **NL**◇ where each formula contains at most n connectives
- Let n be the maximal number of connectives occurring in an **NL**◇-grammar G.
- Equivalent cfg (construction inspired by Pentus 1993):

$$\mathbf{NL}\Diamond(n) \vdash A \circ B \Rightarrow C \quad \leadsto \quad C \rightarrow A, B$$

$$\mathbf{NL}\Diamond(n) \vdash \langle A \rangle \Rightarrow B \quad \leadsto \quad B \rightarrow A$$

$$\mathbf{NL}\Diamond(n) \vdash A \Rightarrow B \quad \leadsto \quad B \rightarrow A$$

$$\begin{array}{c} Cat \\ | \\ lex \end{array} \quad \leadsto \quad Cat \rightarrow lex$$

$$A \in \mathcal{D} \quad \leadsto \quad A \rightarrow S$$

Lemma 3 *Every* **NL**$\diamond$*-recognizable language is context free.*

Proof: By lemma 2 and the construction above. $\dashv$

Theorem 1 **NL**$\diamond$*-grammars recognize exactly the context free languages.*

Proof: Immediate. $\dashv$

Generalization I: Connectives with arbitrary arity

- Both $\backslash, \bullet, /$ and $\diamond, \Box^{\downarrow}$ form families of residuated operators
- Can be generalized to products/implications of arbitrary arity
- Logic of Pure Residuation (**LPR**): Generalization of **NL** to infinite number of families of residuated connectives of any arity
- Lemma 2 holds for **LPR**

Theorem 2 **LPR***-grammars recognize exactly the context free languages.*

Generalization II: Structural Rules

- **LPR** can be strengthened by adding structural rules
- Lemma 2 remains valid if Permutation and/or Expansion are added
- Thus grammars based on these logics only recognize context free languages

$$\frac{X[Y \circ Z] \Rightarrow A}{X[Z \circ Y] \Rightarrow A}P \qquad \frac{X[Y] \Rightarrow A}{X[Y \circ Y] \Rightarrow A}E$$

- Associativity, Contraction and Weakening do not preserve lemma 2

Relation to Kandulski's work

- Kandulski (1995): Proof of context-freeness of **NLP**
- Kandulski (2002): Proof of context-freeness of **LPR**
- Differences:
  - Kandulski restricts designated categories to atomic categories
  - entirely different proof strategy: based on proof normalization in axiomatic version of **LPR**

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