1 A Closer Look at Order-Sensitive Implication

- we consider enriched modal models:

$$\forall W = \langle W, R, V, \{ \leq \}_w \rangle$$

where $\langle W, R, V \rangle$ is a modal model and $\leq_w$ is an ordering on $R(w)$ for each $w \in W$

- to facilitate notation we assume that all $\leq_w$ are well-founded: (Lewis 1973, “limit assumption”)

  - for all $w \in W$, for all $X \subseteq R(w)$, $X \neq \emptyset$, there is a $\leq_w$-minimal element in $X$:

$$\{u \in X \mid \neg \exists v \in X : v <_w u \} \neq \emptyset$$

- define for $A \subseteq W$:

$$\text{Min}_w(A) = \{v \in R(w) \cap A \mid \neg \exists v' \in R(w) \cap A : v' <_w v\}$$

- this gives an analysis of conditionals in terms of order-sensitive implication:

$$\llbracket A > \text{must}(C) \rrbracket = \{w \in W \mid \text{Min}_w(A) \subseteq C\}$$

$$\llbracket A > \text{might}(C) \rrbracket = \{w \in W \mid \text{Min}_w(A) \cap C \neq \emptyset\}$$

1.1 Varieties of Order-Sensitive Implication

- inferential properties of this semantics depend on further properties of the ordering $\leq_w$

- some relevant properties have for all $w \in W$, $A \subseteq W$:

  CENTERING: $w \in \text{Min}_w(\top)$

  COMPARABILITY: for all $u, v \in R(w)$: $u \leq_w v$ or $v \leq_w u$

  TOTALITY: for all $u, v \in R(w)$: if $u \neq v$, then $u <_w v$ or $v <_w u$
1.1.1 Lewis

- see especially: Lewis (1973)
- central assumptions: centering and comparability
- we obtain a system of spheres
- check some properties:
  - strengthening the antecedent?
    \[ \mathcal{A} > C \Rightarrow (\mathcal{A} \text{ and } \mathcal{B}) > C \]
  - simplification of disjunctive antecedents?
    \[ (\mathcal{A} \text{ or } \mathcal{B}) > C \Leftrightarrow \mathcal{A} > C, \ \mathcal{B} > C \]
  - disjunction-to-conditional?
    \[ \mathcal{A} \text{ or } C \Rightarrow \neg \mathcal{A} > C \]

1.1.2 Stalnaker

- see especially: Stalnaker (1968, 1981)
- central assumptions:

  **Uniqueness:** for every world \( w \) and proposition \( A \neq \emptyset \), there is a unique closest \( A \)-world \( v \) with \( \text{Min}_w(A) = \{v\} \)

  **Reflexivity:** if \( w \in A \), then \( \text{Min}_w(A) = \{w\} \)

  **Linear Consistency:** if \( \text{Min}_w(A) = \{w_A\} \) and \( \text{Min}_w(B) = \{w_B\} \), and if \( w_A \in B \) and \( w_B \in A \), then \( w_A = w_B \)

- NB: these properties result when we assume that \( \preceq \) is a total order centered on \( w \)
- NB: Stalnaker uses (equivalently) a selection function: \( f : W \times \mathcal{P}(W) \rightarrow W \) (with corresponding properties)
- special properties (due to Uniqueness): (defended in Stalnaker 1981)
  \[ [\mathcal{A} > \text{must}(C)] = [\mathcal{A} > \text{might}(C)] \]
  conditional excluded middle (CEM): \( \mathcal{A} > C \) or \( \mathcal{A} > \neg C \)

1.1.3 Excursion: Conditional Excluded Middle

- Lewis’ example against CEM: (see Lewis (1973), example due to Quine)
  
  (1) a. If Bizet and Verdi had been compatriots, Bizet would have been Italian.
  b. If Bizet and Verdi had been compatriots, Bizet would have been French.

- Lewis: both (1a) and (1b) are false
- Stalnaker: both (1a) and (1b) are neither true nor false
  - selection function is underspecified
  - supervaluation account (cf. van Fraassen 1966, 1974)
1.1.4 Excursion: ‘Might’-Counterfactuals

- Stalnaker (1981): $\mathcal{A} > \text{might}(C)$ to be analyzed as $\text{might} (\mathcal{A} > \text{must}(C))$

- Lewis’ argument against this: (Lewis 1973)

  - suppose I don’t have a penny in my pocket, but I don’t know whether I have
  - then:

    (2) If I had looked, I might have found a penny.

  should be true according to Stalnaker, but it seems false (but see also Stalnaker 1981)

2 Premisse Semantics

- see especially: Veltman (1976); Kratzer (1977, 1981)

- intuition:

  “The truth of counterfactuals depends on everything which is the case in the world under consideration: in assessing them, we have to consider all the possibilities of adding as many facts to the antecedent as consistency permits. If the consequent follows from every such possibility, then (and only then), the whole counterfactual is true.”

  (Kratzer 1981, p. 210, emphasis added)

- formally:

  - let $f : W \to \mathcal{P}(\mathcal{P}(W))$ associate each world $w$ with the relevant facts
  - let $\text{Con}(w, f, A)$ be the set of all subsets of $f(w)$ such that $f(w) \cap \{A\}$ is consistent:

    $$\text{Con}(w, f, A) = \{X \in f(w) | \bigcup X \cup A \neq \emptyset\}$$

  - let $\text{Con}^*(w, f, A)$ be the set of maximal sets in $\text{Con}(w, f, A)$:

    $$\text{Con}^*(w, f, A) = \{X \in \text{Con}(w, f, A) | \exists Y \subseteq X \subseteq \{X \in \text{Con}(w, f, A) | X \subseteq Y\}$$

    - NB: we assume that $\text{Con}^*(w, f, A)$ is not empty ($\equiv$ limit assumption)

  - truth-conditions for $\mathcal{A} > \text{must}(C)$ according to premisse semantics:

    $$\llbracket \mathcal{A} > \text{must}(C) \rrbracket = \{w \in W | \forall X \in \text{Con}^*(w, f, A) : X \subseteq C\}$$

2.1 Lewis’ Equivalence Result

- ordering semantics and premisse semantics are equivalent (Lewis 1981)

  - for every premisse function $f$ we can define an enriched model model with, for all worlds $w$, $R(w) = W$ and ordering:

    $$v \leq_w u \iff \{X \in f(w) | u \in X\} \subseteq \{X \in f(w) | v \in X\}$$

  - conversely, every enriched modal model can construct from some premisse function $f$

  - constructions yield equivalent truth-conditions under ordering- and premisse-based semantics for any conditional
2.2 Challenge for Premisse Semantics

- if $f(w)$ contains all propositions that are true in $w$, we don’t get intuitive results: (Kratzer 1981)
  - in that case $\mathcal{A} >$ must($C$) has truth-conditions:
    1. if $w \in A$, then $w \in [\mathcal{A} > $ must($C$)] if $w \in C$
    2. if $w \notin A$, then $w \in [\mathcal{A} > $ must($C$)] if $A \subseteq C$
  - to make premisse semantics work we need to specify $f(w)$ in non-trivial ways (Kratzer 1989, 2002)

References


— (2010). “If you’d wiggled A, then B would’ve changed” — Causality and Counterfactual Conditionals”. In: Synthese.

