Models of Language Evolution
Session 4: Introduction to Game Theory

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2014/11/19
Organizational Matters

- 22.10 Language Evolution - Overview
- 29.10 Language Evolution - Protolanguage
- 12.11 Introduction to Models of Language Evolution
- 19.11 **Introduction to Game Theory**
- 26.11 Evolutionary Game Theory
- 03.12 Games of Communication (*literature list*)
- 10.12 The Iterated Learning Model
- 17.12 Further Models (*project sketch/preliminary slides*)
- 07.01 Students’ Presentations
- 14.01 Students’ Presentations
- 21.01 Students’ Presentations
- 28.01 Students’ Presentations
- 04.02 Recent Work
- 11.02 Recent Work
Game Theory

- models strategic decisions of rational actors (players, agents)
- involves often situations of conflict or cooperation

Famous example: prisoner’s dilemma

Settings: benefit=2, costs=1, idle=1

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<td>2;2</td>
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<td>¬C</td>
<td>3;0</td>
<td>1;1</td>
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Table: Prisoner’s Dilemma
Public Goods Game

Standard settings

- $n$ players
- each player has initially $p$ Euro
- each player can pay into a public fund
- the total amount of the fund will be multiplied by factor $f$ and payed out to all players to an equal share

Example

- 10 players, initially 1 Euro, factor 2
- c.f. everybody pays in → win per player: 1 Euro
- c.f. nobody pays in → win per player: 0 Euro
Public Goods Game

Let’s play:

- Initially: everybody has a coin and a letter with his player name
- Investment: everybody put the money to invest (into the public font) into the letter
- Payout: after counting and multiplying, everybody gets her/his payout back
- Publication I: the whole investment amount
- Publication II: the private money ranking
Public Goods Game

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<th>9C</th>
<th>8C</th>
<th>7C</th>
<th>6C</th>
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<th>3C</th>
<th>2C</th>
<th>1C</th>
<th>0C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>1</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>0.2</td>
<td>0</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-0.6</td>
<td>-0.8</td>
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<tr>
<td><strong>D</strong></td>
<td>1.8</td>
<td>1.6</td>
<td>1.4</td>
<td>1.2</td>
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<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>0.2</td>
<td>0</td>
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</tbody>
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Table: 10 players ’Public Goods Game’ with \( p = 1 \) and \( f = 2 \).

- in a 10 players ’Public Goods Game’ with \( p = 1 \) and \( f = 2 \), not to cooperate (no payment) is always better by 0.8 points than to cooperate (full payment).
- in general: in a \( n \) players ’Public Goods Game’, not to cooperate (no payment) is better by \( p - (p \times \frac{f}{n}) \) point than to cooperate (full payment).
- note: if \( f < n \), not to cooperate is always better.
2 players public goods game

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<tbody>
<tr>
<td>C</td>
<td>1;1</td>
<td>-0.5;1.5</td>
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<tr>
<td>D</td>
<td>1.5;-0.5</td>
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**Table:** 2 players public goods game with $p = 2$ and $f = 1.5$.

- note: a 2 players public goods game with $p > f$ and ’all or nothing’-investment is a prisoner’s dilemma.
- to put it in another way: the prisoner’s dilemma is a special case of the public goods game.
The Game of Cooperation

The essential game of cooperation: $V_{gh} > C_h > 0$

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<tr>
<td>$C$</td>
<td>$V_{gh} - C_h; V_{gh} - C_h$</td>
<td>$-C_h; V_{gh}$</td>
</tr>
<tr>
<td>$D$</td>
<td>$V_{gh} - C_h$</td>
<td>$0; 0$</td>
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Table: The essential game of cooperation

To fill it with values: $V_{gh} = 1.5$, $C_h = 0.5$

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<tr>
<td>$C$</td>
<td>$1; 1$</td>
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<td>$D$</td>
<td>$1.5; -0.5$</td>
<td>$0; 0$</td>
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Table: The essential game of cooperation

- note: the essential game of cooperation is a prisoner’s dilemma and therefore a particular public goods game.
Why do we cooperate?

- Why do we cooperate, if not cooperate is always the better alternative?
- Which reasons/scenarios make cooperation the better alternative?
  - kin selection
  - group selection
  - reciprocity: ”I’ll scratch your bag, you scratch mine.”
    - direct reciprocity
    - indirect reciprocity
    - network Reciprocity

The Evolution of Cooperation

Robert Axelrod’s Computer Turnier (1979):

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Table: Prisoner’s Dilemma

- find the best strategy for the *repeated prisoner’s dilemma* (RPD)
- academics/scientists were invited to send in a strategy (decision rule)
- all sent in strategies played 200 rounds against each other
- the strategy with the highest average score won the tournament
Exemplary tournament:

1. Always Defect (AD): play always ‘defect’

2. Tit-For-Tat (TFT): start with ‘cooperate’, and then play what your opponent played last round

3. Good-Memory (GM): play ‘cooperate’, if opponent min. 50% of his play history was ‘cooperate’, else ‘defect’

<table>
<thead>
<tr>
<th>Round</th>
<th>AD</th>
<th>TFT</th>
<th>Round</th>
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<td>1.0</td>
<td>avg</td>
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Total result: GM (3.9), TFT (3.8), AD (2.4)
The Evolution of Cooperation

- TIT FOR TAT: Cooperate in the first round and then do what your opponent did last round
- FRIEDMAN: Cooperate until the opponent defects, then defect all the time
- DOWNING:
  - Estimate probabilities $p_1 = P(C_t | C_{t-1})$, $p_2 = P(C_t | D_{t-1})$
  - If $p_1 >> p_2$ the opponent is responsive: Cooperate
  - Else the opponent is not responsive: Defect
- TRANQUILIZER:
  - Cooperate the first moves and check the opponents response
  - If there arises a pattern of mutual cooperation: Defect from time to time
  - If opponent continues cooperating, defections become more frequent
- TIT FOR 2 TATS: Play TIT FOR TAT, but response with defect if the opponent defected on the previous two moves
- JOSS: Play TIT FOR TAT, but response with defects in 10% of opponent’s cooperation moves
The Evolution of Cooperation

Results:

1. the winner was TIT FOR TAT with 504 point per game (2.52 per encounter)

2. success in such a tournament correlates with the following properties:
   - be nice: cooperate, never be the first to defect.
   - be provicable: return defection for defection, cooperation for cooperation.
   - don’t be envious: be fair with your partner.
   - don’t be too clever: or, don’t try to be tricky.
Conclusion

- the prisoner’s dilemma is an ’essential game of cooperation’
- non-cooperation is in the general case (e.g. 1 encounter, 2 players, neutral context) the only rational decision
- but there are reasons for cooperation
  - kin selection
  - group selection
  - reciprocity (z.B. Tit-For-Tat)
- cooperation is an essential factor in communication!
- ”What are the reasons for giving away private (maybe valuable) information?”
What is a Game?

A game

- is a mathematical structure that depicts a decision situation between players/agents
- whereby the result of a player’s decision depend on the decision of other players
- is NOT a model of interactive decision finding (reasoning, choice), but depicts only the situation in which players can make decisions (the process of decision finding is called the ’solution concept’)

Solution Concepts

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Table: Stag hunt  Table: Pris Dilemma  Table: BoS

Which decision will players make?

▸ choose randomly
▸ choose the *dominant* strategy
▸ choose strategy with highest *expected utility*
▸ choose the *risk-dominant* strategy
▸ choose a *Nash equilibrium* (Pareto-optimal)
▸ choose by learning (update dynamics, repeated games)
▸ choose after communication
▸ choose the best response for a ’rational belief’
Evolutionary Game Theory: Basic Concept

- population of individuals (players, agents)
- individuals are (genetically) programmed for a specific behavior (strategy)
- individuals replicate and their strategy is inherited to offspring
- replication success (fitness) depends on the average utility of the strategy against the other strategies of the population (essence of game theory)
EGT-Setting and Replicator Dynamics

Given: a large (practically infinite) population \( P \) of agents, which play pairwise a game \( G = \langle S, U \rangle \) against each other, whereby:

- \( S = \{s_1, s_2, ..., s_n\} \) a set of strategies \( s_i \)
- \( U : S \times S \rightarrow \mathbb{R} \) a utility function over strategy pairs

Further definition:

- \( p(s_i) \): proportion of individuals that play \( s_i \)
- \( EU(s_i) = \sum_{s_j \in S} p(s_j)U(s_i, s_j) \): expected utility (fitness) for playing \( s_i \)
- \( AU = \sum_{s_i \in S} p(s_i)EU(s_i) \): average utility value of the whole population

replicator dynamics:
the replicator dynamics is defined by the following differential equation:

\[
\frac{dp(s_i)}{dt} = p(s_i)[EU(s_i) - AU]
\]
Replicator Dynamics

The replicator dynamics

\[
\frac{dp(s_i)}{dt} = p(s_i)[EU(s_i) - AU]
\]

realizes a simple dynamics:

- a strategy that is better than average increases in proportion of population
- a strategy that is worse than average decreases in proportion of population
- note: since a strategy represents a hard-coded behavior, it can be interpreted as type/species/breed
Replicator Dynamics

Example 1: The better survives

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<tr>
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<th>$s_B$</th>
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<tbody>
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<td>1,1</td>
</tr>
<tr>
<td>$s_B$</td>
<td>1,1</td>
<td>0,0</td>
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Table: A- & B-pigeon

**Figure:** replicator dynamics with mutation: proportion of A-pigeons $p(s_A)$ in the population for different initial proportions
Replikator Dynamik

Example 2: The ecological equilibrium

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<tr>
<td>$s_T$</td>
<td>2,7</td>
<td>3,3</td>
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Table: Hawk & Dove

Figure: replicator dynamics without mutation: proportion of eagles $p(s_A)$ in the population for different initial populations
Evolutionary Stable Strategy

Given a game $G = \langle S, U \rangle$. A strategy $s_i \in S$ is *evolutionary stable*, iff the following two conditions are fulfilled:

1. $U(s_i, s_i) \geq U(s_i, s_j)$ for all $s_j \in S \setminus \{s_i\}$
2. If $U(s_i, s_i) = U(s_i, s_j)$ for some $s_j$, then $U(s_i, s_j) > U(s_j, s_j)$

An ESS has the following properties:

- $\text{SNE} \subset \text{ESS} \subset \text{NE}$
- it has an *invasion barrier*
Timescale of Literature

1990 Pinker & Bloom: *language evolution theory*
1991
1992
1993
1994
1995 Bickerton: *PL-fossils in form of language behavior*
1996
1997
1998
1999 *Jackendoff: PL-fossils in instances of Human language*
   *Nowak & Krakauer: The Evolution of Language*
2000
2001 Simulating the Evolution of Language ← ← ← ← ←
2002 Hauser, Chomsky & Fitch: FLN = FLB + recursion
2003
2004
2005
2006
2007 *Bickerton: perspective from linguistics*
   *Kirby: perspective from LE-modelers*
2008 Jäger: Applications of Game Theory in Linguistics