Pragmatics & Game Theory: Branches of Game-Theoretic Linguistics

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Review

- conversational implicature
  - 4 maxims: quality, quantity, relevance, manner
  - particularized & generalized implicatures
- neo-Gricean pragmatics
  - scalar & clausal implicatures
  - Q- and R-based implicatures (Horn)
  - Horn’s division of pragmatic labor
  - Q-, I- and M-based implicatures (Levinson)
- game-theoretic pragmatics
  - situations: signaling games
  - possible behavior of production/perception: strategies (pure & behavioral)
  - actual behavior: rationalistic solution concepts (here: IBR)
Overview

Game Theory and Linguistics

- Language Evolution
  - Signaling Games
- GT in Lang. Use
- Pragm. Reasoning
  - Signaling Games
  - IBR model
  - SIM

Branches of Game-Theoretic Linguistics
Parikh’s (1991) *Strategic Implicature Model* was the first account to use game theory for matters in pragmatics.

Parikh deals with *particularized implicatures of relevance* he distinguishes between two types of relevance:

- **logical relevance**: a proposition is logical relevant with respect to a set of propositions, if the two together have a 'nontrivial' implication without having it individually (in spirit of Sperber and Wilson, 1986)

- **rational relevance**: a proposition is rational relevant, if it increases the expected utility of a participant’s goal
Example: B’s decision problem

Situation: A and B have to attend a talk at 5 pm and B believes the probability of it’s being time for the talk now is .2 and that there is still time is .8.

The information that it is time for the talk has rational relevance here, since it has a positive value of information: $10 - 6 = 4$. 
A Relevance Implicature

Example

Situation: It is an important talk today (at 5 pm)
A: ’It is 5 pm.’
+> It is time for the talk.

- logical relevance: the proposition (expression) is relevant because it is non-trivial in the given set of propositions (situation, context) by generating the implicature.
- rational relevance: the proposition is rational relevant for the hearer since it increases his expected utility
- note that both types of relevance are essential for Grice’s idea of the cooperation principle!
The SIM Model For The Given Example

- situation s: 'important talk at 5 pm'; and empty context s'
- expression \( \varphi \): 'It is 5 pm.'
- information states t: it is 5 pm and the talk is now; and \( t' \): it is 5 pm
- literal interpretation \( l \) (fits to \( t' \)) and pragmatic interpretation \( p \) (fits to t)
- production and interpretation cost: -1
- incorrect interpretation: -2, pragmatic inference: -1
- information value by rational relevance: +4
The SIM Model’s Local Game

- $B$ could reason that $A$ could have an alternative in situation $s$
- to use a more complex expression $\mu$: ’It is time for the talk now.’
  (extra costs: $-.5$)
- to express meaning $e$: it’s time for the talk
- $A$ could also have an alternative in situation $s'$
- to be silent ($\nu$: empty message)
  that expresses nothing ($e'$: empty meaning)

**Figure 5: The Local Game $LG(\varphi)$**
Solution idea:

- A’s information set is \{s\} and \{s'\}
- B’s information set is \{t, t'\}, \{e\} and \{e'\}
- there are four sender strategies and two receiver strategies
- and therefore 8 strategy profiles
- there is only one unique Nash equilibrium
  \{((s, \varphi), (s', \nu); \{t, t'\}, p)\}
The SIM Model’s Global Game

- the local game results after A has uttered $\varphi$
- if A would utter something else (e.g. $\psi$ or $\mu$: 'the speaker entered the stage', 'the audience is waiting'...), it would emerge a different local game
- each local game has an expected utility value
- A should use the expression that triggers the local game with maximal expected utility

If $LG$ denotes the set of all local games that result from A’s choice, then $\langle GG, LG \rangle$ is called the Strategic Implicature Model.
Conclusion

- the SIM model is (one of) the first model that combines Pragmatics and Game Theory
- the model deals with particularized implicatures (relevance) and therefore takes a given context/situation into account
- the solution concept is a unique *Nash equilibrium* that depicts pragmatic language use
- but: there is criticism for model and solution concept:
  - there are a lot of numbers made up, which differences are critical for the right solution
  - the Nash equilibrium explains why to stay with a strategy, but not how to come to it
  - there is no good solution for the case of multiple Nash equilibria
The Logic of Indirect Speech (Pinker, Nowak & Lee 2007)

Example 1

'Would you like to come up and see my etchings?'

> a sexual come-on

Example 2

'If you could pass the guacamole, that would be awesome.'

> a polite request

Example 3

'Nice store you got here. Would be a real shame if something happened to it.'

> a threat

Example 5

'Gee, officer, is there some way we could take care of the ticket here?'

> a bribe
The Logic of Indirect Speech

- **Direct bribe:** 'If you let me go without a ticket, I’ll pay you 50 dollar.'

- **Indirect bribe:** 'Gee, officer, is there some way we could take care of the ticket here?' (showing a fifty dollar bill)

<table>
<thead>
<tr>
<th></th>
<th>dishonest officer</th>
<th>honest officer</th>
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</thead>
<tbody>
<tr>
<td>Don’t bribe</td>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>Bribe</td>
<td>-50</td>
<td>-500</td>
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<tr>
<td>Indirect bribe</td>
<td>-50</td>
<td>-100</td>
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</table>
The Logic of Indirect Speech

- The *directness* of a proposition is probably continuous:
  - ’If you let me go without a ticket, I’ll pay you 50 dollar.’
  - ’Is there some way we could take care of the ticket here?’
  - ’I’ve learned my lesson; you don’t have to worry about me doing this again.’

- A corrupt cop is more sensitive to recognize bribe than a honest cop.
- There is a threshold for both to recognize the bribe.
People use indirect speech also in nonlegal situations, where there are no financial or legal payoffs and penalties.

Example: bribing a *maître d’* to get seated in a full restaurant directly without reservation.

When relations are ambiguous, a divergent understanding between the parties can lead to the aversive emotion we call awkwardness.

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<tr>
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<th>dishonest maitre d’</th>
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<tbody>
<tr>
<td>Don’t bribe</td>
<td>long wait (D/D)</td>
<td>long wait (D/D)</td>
</tr>
<tr>
<td>Bribe</td>
<td>instant seating (R/R)</td>
<td>awkwardness (R/D)</td>
</tr>
<tr>
<td>Indirect bribe</td>
<td>instant seating (R/R)</td>
<td>long wait (D/D)</td>
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</table>

D: dominance relationship; R: reciprocity relationship.
There are three essential human relationship types:
- boss - employee (dominance)
- good friends (communality)
- business transaction (reciprocity)

Relationship mismatches impose emotional costs (awkwardness)
sometime relationship types are highly unambiguous
indirect speech can help to avoid relationship mismatches by "the benefit of the doubt"
indirect speech merely provides shared individual knowledge, but not *common knowledge*
### Coordination & Signaling

<table>
<thead>
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<th>$R$</th>
<th>$L$</th>
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<tbody>
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<tr>
<td>$L$</td>
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<table>
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<th>$a_S$</th>
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<td>0</td>
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<tr>
<td>$t_S$</td>
<td>0</td>
<td>1</td>
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</table>

**Messages: One or two lanterns?**

- $S_1$: $t_L \rightarrow m_1$
  - $t_S \rightarrow m_2$
- $S_2$: $t_L \times t_L$
  - $m_1 \times m_1$
- $S_3$: $t_L \rightarrow m_1$
  - $t_S \rightarrow m_2$
- $S_4$: $t_L \rightarrow m_1$
  - $t_S \rightarrow m_2$

- $r_1$: $m_1 \rightarrow a_L$
  - $m_2 \rightarrow a_S$
- $r_2$: $m_1 \times a_L$
  - $m_2 \times a_S$
- $r_3$: $m_1 \rightarrow a_L$
  - $m_2 \rightarrow a_S$
- $r_4$: $m_1 \rightarrow a_L$
  - $m_2 \rightarrow a_S$
a signaling game is a tuple $SG = \langle \{S, R\}, T, Pr, M, A, U \rangle$

a Lewis game is defined by:

- $T = \{t_L, t_S\}$
- $M = \{m_1, m_2\}$
- $A = \{a_L, a_S\}$
- $Pr(t_L) = Pr(t_S) = .5$

$U(t_i, a_j) = \begin{cases} 
1 & \text{if } i = j \\
0 & \text{else} 
\end{cases}$

<table>
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<tr>
<th></th>
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<th>$a_S$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
</tr>
<tr>
<td>$t_S$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

- $N$
  - $t_L$.5
  - $t_S$.5
- $S$
  - $m_1$
  - $m_2$
- $R$
  - $a_L$ 1
  - $a_S$ 0
- $R$
  - $a_L$ 1
  - $a_S$ 0
- $R$
  - $a_L$ 0
  - $a_S$ 1
- $R$
  - $a_L$ 0
  - $a_S$ 1

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Branches of Game-Theoretic Linguistics
Pure strategies

Pure strategies are contingency plans, players act according to.

- sender strategy: \( s : T \rightarrow M \)
- receiver strategy: \( r : M \rightarrow A \)

\[
\begin{align*}
S_1: & \quad t_L \rightarrow m_1 & \quad t_S \rightarrow m_2 \\
S_2: & \quad t_L \times m_1 & \quad t_S \times m_2 \\
S_3: & \quad t_L \rightarrow m_1 & \quad t_S \rightarrow m_2 \\
S_4: & \quad t_L \rightarrow m_1 & \quad t_S \rightarrow m_2 \\
R_1: & \quad m_1 \rightarrow a_L & \quad m_2 \rightarrow a_S \\
R_2: & \quad m_1 \times a_L & \quad m_2 \times a_S \\
R_3: & \quad m_1 \rightarrow a_L & \quad m_2 \rightarrow a_S \\
R_4: & \quad m_1 \rightarrow a_L & \quad m_2 \rightarrow a_S
\end{align*}
\]
Signaling Systems

- signaling systems are combinations of pure strategies. The Lewis game has two: $L_1 = \langle s_1, r_1 \rangle$ and $L_2 = \langle s_2, r_2 \rangle$

  $t_L \rightarrow m_1 \rightarrow a_L$

  $t_L \rightarrow m_1 \rightarrow a_L$

  $L_1:$

  $t_s \rightarrow m_2 \rightarrow a_s$

  $t_s \rightarrow m_2 \rightarrow a_s$

  $L_2:$

  $L_1$ and $L_2$ are signaling systems that are strict Nash equilibria of the EU-table:

<table>
<thead>
<tr>
<th></th>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$r_3$</th>
<th>$r_4$</th>
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</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>1</td>
<td>0</td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0</td>
<td>1</td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>$s_3$</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>$s_4$</td>
<td>.5</td>
<td>.5</td>
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</tr>
</tbody>
</table>

- in signaling systems messages associate states and actions uniquely
- signaling systems constitute evolutionary stable states
Behavioral strategies are functions that map choice points to probability distributions over actions available in that choice point.

- behavioral sender strategy
  \[ \sigma : T \to \Delta(M) \]
- behavioral receiver strategy
  \[ \rho : M \to \Delta(A) \]

\[
\sigma = \begin{bmatrix} 
  t_1 & \mapsto & \begin{bmatrix} 
    m_1 & \mapsto & 0.9 \\
    m_2 & \mapsto & 0.1 \\
    m_1 & \mapsto & 0.5 \\
    m_2 & \mapsto & 0.5 
  \end{bmatrix} \\
  t_2 & \mapsto & \begin{bmatrix} 
    m_1 & \mapsto & 0.5 \\
    m_2 & \mapsto & 0.5 
  \end{bmatrix} 
\end{bmatrix}
\]
\[
\rho = \begin{bmatrix} 
  m_1 & \mapsto & \begin{bmatrix} 
    a_1 & \mapsto & 0.33 \\
    a_2 & \mapsto & 0.67 
  \end{bmatrix} \\
  m_2 & \mapsto & \begin{bmatrix} 
    a_1 & \mapsto & 1 \\
    a_2 & \mapsto & 0 
  \end{bmatrix} 
\end{bmatrix}
\]
Learning Dynamics & Signaling Games

Extensions in time:

- agents play the game repeatedly
- agents’ decisions are influenced by previous encounters
- application of learning dynamics like reinforcement learning and belief learning
Playing *Best Response* means to make a choice that maximizes the *Expected Utility*.

\[
EU_S(m|t, \beta) = \sum_{a \in A} \beta(a|m) \times U(t, a)
\]  

(1)

\[
EU_R(a|m, \beta) = \sum_{t \in T} \beta(m|t) \times U(t, a)
\]  

(2)

How does an agent get belief \(\beta\)?
Belief Learning

- The belief is a result of observation
- Example:

\[
SO \quad a_1 \quad a_2 \\
\hline
m_1 \quad 8 \quad 2 \\
m_2 \quad 7 \quad 13 \\
\beta = \begin{bmatrix}
m_1 & \mapsto & \begin{bmatrix} a_1 & \mapsto & .8 \\
a_2 & \mapsto & .2 \end{bmatrix} \\
m_2 & \mapsto & \begin{bmatrix} a_1 & \mapsto & .35 \\
a_2 & \mapsto & .65 \end{bmatrix} \\
\end{bmatrix}
\]

\[
RO \quad t_1 \quad t_2 \\
\hline
m_1 \quad 6 \quad 0 \\
m_2 \quad 4 \quad 4 \\
\beta = \begin{bmatrix}
t_1 & \mapsto & \begin{bmatrix} m_1 & \mapsto & .6 \\
m_2 & \mapsto & .4 \end{bmatrix} \\
t_2 & \mapsto & \begin{bmatrix} m_1 & \mapsto & 0 \\
m_2 & \mapsto & 1 \end{bmatrix} \\
\end{bmatrix}
\]
Best Response as Behavioural Strategy

- **Behavioural sender strategy**
  \[ \sigma : T \rightarrow \Delta(M) \]
  \[ \sigma(m|t) = \begin{cases} 
  \frac{1}{|BR(t)|} & \text{if } m \in BR(t) \\
  0 & \text{else} 
\end{cases} \]

- **Behavioural receiver strategy**
  \[ \rho : M \rightarrow \Delta(A) \]
  \[ \rho(a|m) = \begin{cases} 
  \frac{1}{|BR(m)|} & \text{if } a \in BR(m) \\
  0 & \text{else} 
\end{cases} \]
Reinforcement Learning

- the sender has an urn for each state $t \in T$
- each urn contains balls of each message $m \in M$
- the sender decides by drawing from urn $U_t$
- successful communication $\rightarrow$ urn update
- in general a signaling system emerges over time

- the receiver has an urn for each message $m \in M$
- each urn contains balls of each action $a \in A$
- the receiver decides by drawing from urn $U_t$
Behavourial & Pure Strategies

Pure strategies are a subset of behavioural strategies.

Example:

\[
\begin{align*}
\sigma_2 : & \quad \begin{array}{c}
m_1 \mapsto 0 \\
m_2 \mapsto 1 \\
m_1 \mapsto 1 \\
m_2 \mapsto 0
\end{array} & \quad \begin{array}{c}
t_1 \mapsto m_1 \\
t_2 \mapsto m_2
\end{array}
\end{align*}
\]

\[
\rho_2 : & \quad \begin{array}{c}
a_1 \mapsto 0 \\
a_2 \mapsto 1 \\
a_1 \mapsto 1 \\
a_2 \mapsto 0
\end{array} & \quad \begin{array}{c}
m_1 \mapsto a_1 \\
m_2 \mapsto a_2
\end{array}
\end{align*}
\]

Note: If an agent plays \(\sigma_2\) as sender and \(\rho_2\) as receiver, we say, he has learned the signaling language \(L_2 = \langle \sigma_2, \rho_2 \rangle\).
Conclusion

1. The Strategic Implicature Model (Parikh 1991)
   - is the first model that combines pragmatics and game theory
   - deals (basically) with relevance implicatures
   - uses the Nash equilibrium as solution concept
   - but: there is criticism for model and solution concept:

2. Strategic Language Use
   - is a branch in the tree of research directions that use game theory for issues in linguistics
   - ascribes two essential functions to communication: information transmission and relationship negotiation
   - integrates social costs (awkwardness)

3. EGT and Signaling Games
   - gives a framework to model dynamics of language evolution and change
   - integrates e.g. learning dynamics on repeated plays