

# TRALE feature logic

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# Facts about ALE

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- ▶ ALE stands for Attribute Logic Engine
- ▶ it is the name for both a formalism and its Prolog implementation
- ▶ a logic programming environment in which terms are typed feature structures
- ▶ the feature logic we will have a look at is in essence an attribute-value logic with variables
- ▶ understanding this logic is essential to be able to write grammars in ALE
- ▶ ALE will compile such grammars into parsers

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- ▶ TRALE is an extension of the ALE system that supports some extra functionality
- ▶ on the logical level, the main difference between ALE and TRALE is the view on subtyping
- ▶ since we will mainly be dealing with TRALE, everything presented today will be from the TRALE perspective
- ▶ we will mostly be concerned with TRALE signatures and their meaning

# Outline

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3. Subsumption und unification
4. Enhanced type system, feature appropriateness in signatures
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- ▶ ALE is a language with **strong typing** - every structure it uses comes with a type
- ▶ the user must declare all the of types that will be used
- ▶ types are arranged in an **inheritance hierarchy**
- ▶ many types will typically have **subtypes**
- ▶ constraints on more general types are inherited by their subtypes: **inheritance-based polymorphism**

# Inheritance Hierarchies

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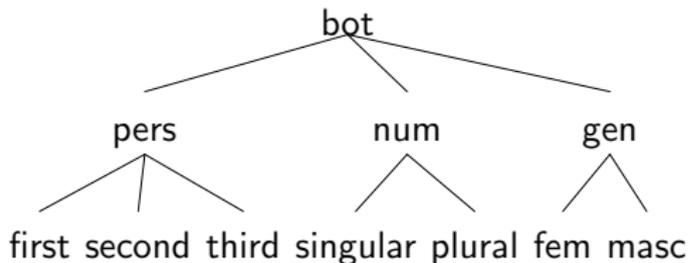
## Subtype covering

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- ▶ subtyping relation only specified by **immediate** subtyping declarations
  - ▶ however, the subtyping itself is **transitive**
  - ▶ it is also **anti-symmetric** ( $a < b \wedge b < a \rightarrow a = b$ )
- subtyping constitutes a **partial order** on the types
- ▶ there are additional restrictions:
    - ▶ there must be a unique most general type named **bot**
    - ▶ type hierarchies must be **bounded complete** (explanation to come)

# A simple example

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# The signature in TRALE format

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```
type_hierarchy
bot
  per
    first
    second
    third
  num
    singular
    plural
  gen
    fem
    masc
```

# Bounded completeness

- ▶ if a poset is **bounded complete**, every subset that has some upper bound must also have a least upper bound
- ▶ for type hierarchies: every collection of types with a common subtype must have a **unique most general common subtype**

Problematic:  
type\_hierarchy  
bot

```

a
  c
  d
b
  c
  d

```

Solution:  
type\_hierarchy  
bot

```

a
  e
    c
    d
b
  e

```

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- ▶ primary representational device in ALE
- ▶ serve as universal data structure
- ▶ a feature structure consists of
  - ▶ a type drawn from the inheritance hierarchy
  - ▶ a finite (possibly empty) set of **feature/value pairs** where each value is in itself a feature structure ( $\rightarrow$  **recursion**)
- ▶ Examples in ALE output notation:

noun	verb
CASE akk	PERS first
NUM pl	NUM pl
GEN f	SUBCAT ne_list
	HD noun
	CASE akk
	TL e_list

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- ▶ a **substructure** is the value of a feature at some level of nesting
- ▶ two different substructures can be **token-identical**  
→ **substructure sharing**
- ▶ a **path** is a sequence of features designating a substructure
- ▶ if two paths are in a structure sharing relation, their values are token-identical
- ▶ structure sharing is indicated by numbered **tags** in the feature structures

# Cyclic structures

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*“This sentence is false.”*

[0] false

ARG1 [0]

- ▶ this is a **legal** feature structure!
- ▶ structure sharing between paths:  $\epsilon$ , ARG1, ARG1 ARG1, ARG1 ARG1 ARG1 ...

*“It is false that this sentence is false.”*

false

ARG1 [0] false

ARG1[0]

- ▶ the two structures are not treated as identical, a cyclic structure is not conflated with its infinite unfolding

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- ▶ feature structures provide partial information (Why?)
- ▶ we can order feature structures based on the amount of information they provide
- ▶ this order is called the **subsumption ordering**, providing a formal notion of **information containment**
- ▶ with subsumption, we can define **information combination (unification)**
- ▶ unification is the standard mechanism to perform computations with feature structures

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- ▶ let  $F, G$  feature structures
- ▶  $F$  subsumes  $G$  iff
  - ▶ the type of  $F$  is more general than the type of  $G$
  - ▶ if a feature  $f$  is defined in  $F$  then  $f$  is also defined in  $G$  such that the value in  $F$  subsumes the value in  $G$
  - ▶ if two parts are shared in  $F$  then they are also shared in  $G$

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$$\begin{array}{l} \text{agr} \\ \text{PERS first} \end{array} < \begin{array}{l} \text{agr} \\ \text{PERS first} \\ \text{NUM plu} \end{array}$$

$$\begin{array}{l} \text{sign} \\ \text{SUBJ agr} \\ \text{PERS first} \\ \text{NUM plu} \\ \text{OBJ agr} \\ \text{PERS first} \\ \text{NUM plu} \end{array} < \begin{array}{l} \text{sign} \\ \text{SUBJ [0] agr} \\ \text{PERS first} \\ \text{NUM plu} \\ \text{OBJ [0]} \end{array}$$

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- ▶ **unification** is an operation defined over pairs of feature structures returning a feature structure combining the information contained in the input structures if they are consistent and fails otherwise
- ▶ the result is the most general feature structure subsumed by both input structures
- ▶ from an operational point of view:
  - ▶ unify the types of the structures according to the hierarchy (unique result required, this is why we need bounded completeness)
  - ▶ recursively unify the values of the features which occur in both structures
  - ▶ if a feature only occurs in one structure, copy it over into the result

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$$\begin{array}{l} \text{agr} \\ \text{PERS first} \end{array} + \begin{array}{l} \text{agr} \\ \text{PERS second} \end{array} = \text{*failure*}$$

$$\begin{array}{l} \text{agr} \\ \text{PERS first} \end{array} + \begin{array}{l} \text{agr} \\ \text{NUM plu} \end{array} = \begin{array}{l} \text{agr} \\ \text{PERS first} \\ \text{NUM plu} \end{array}$$

$$\begin{array}{l} \text{sign} \\ \text{SUBJ agr} \\ \text{PERS 1st} \\ \text{OBJ agr} \\ \text{NUM plu} \end{array} + \begin{array}{l} \text{sign} \\ \text{SUBJ [0] bot} \\ \text{OBJ [0]} \end{array} = \begin{array}{l} \text{sign} \\ \text{SUBJ [0] agr} \\ \text{PERS first} \\ \text{NUM plu} \\ \text{OBJ [0]} \end{array}$$

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$$\begin{array}{c} t \\ F [0] t \\ G [0] \end{array} + \begin{array}{c} t \\ F t \\ F [1] \\ G [1] \end{array} = \begin{array}{c} t \\ F [1] t \\ F [1] \\ G [1] \end{array}$$

$$\text{bot} + t = t$$

$$\text{e\_list} + \begin{array}{c} \text{ne\_list} \\ \text{HD a} \\ \text{TL e\_list} \end{array} = \text{*failure*}$$

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- ▶ so far, our notion of the type system was built almost entirely around the notion of subtyping
- ▶ with the features in place, we can now add the notion of **feature appropriateness**
- ▶ each type must specify which features it can be defined for, and which types of values these features can take
- ▶ these appropriateness specifications are inherited in the type hierarchy

# A TRALE signature with features

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```

type_hierarchy
bot
  list
    e_list
    ne_list hd:bot tl:list
atom
  a
  b
  .

```

# Restrictions on appropriateness conditions

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- ▶ appropriateness conditions must be acyclic
- ▶ every feature must be introduced at a unique most general type
- ▶ each type must specify which features it can be defined for, and which types of values these features can take
- ▶ these appropriateness specifications are inherited in the type hierarchy

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- ▶ every feature structure must respect the appropriateness restrictions
- ▶ this amounts to two conditions on feature structures:
  - ▶ if a feature is defined on a structure, its type must be appropriate for the feature and the value of the feature must have the appropriate type
  - ▶ if a type was declared with a feature, every feature structure of that type must have a value for the feature
- ▶ feature structures fulfilling these conditions are called **totally well-typed**

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- ▶ TRALE assumes that subtypes exhaustively cover their supertypes  
→ every object of a non-maximal type is also of one of the maximal types subsumed by it
- ▶ the following hierarchy fragment illustrates what this implies:

$$t \text{ f:bool } g:\text{bool}$$

$$t_1 \text{ f:+ } g:-$$

$$t_2 \text{ f:- } g:+$$

- ▶ this signature will not allow structures like the following:

$$t$$

$$F +$$

$$G +$$

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- ▶ so far, the feature structures we allowed for were only specified by a signature
- ▶ the constraints we can impose in this way are rather restricted
- ▶ trying to define a language like this inevitably leads to severe overgeneration
- ▶ to write grammars, we need another device that allows us to impose much more complex conditions on valid feature structures
- ▶ to achieve this, we need a syntax to write rules and principles in a way similar to logic programming
- ▶ to generalize over feature structures, we need a description language that allows us to select classes of structures by means of properties that we are interested in
- ▶ for this purpose, we will introduce TRALE descriptions

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- ▶ We know
  - ▶ how signatures are written and what they mean
  - ▶ how feature structures and their unification are defined
  - ▶ a lot of basic facts about TRALE's feature logic
- ▶ In the next session, we will learn about
  - ▶ atomic values and why they are not in the signature
  - ▶ the difference between token identity and structural identity
  - ▶ the syntax of basic descriptions in TRALE, allowing us to start writing grammars

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Thank you.

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- ▶ the set of descriptions used in ALE can be described by the following grammar:

$$\begin{aligned}
 \langle \text{desc} \rangle ::= & \langle \text{type} \rangle \\
 & | \langle \text{variable} \rangle \\
 & | (\langle \text{feature} \rangle : \langle \text{desc} \rangle) \\
 & | (\langle \text{desc} \rangle, \langle \text{desc} \rangle) \\
 & | (\langle \text{desc} \rangle ; \langle \text{desc} \rangle) \\
 & | (= \setminus = \langle \text{desc} \rangle)
 \end{aligned}$$